Factors that Count

TEXAS INSTRUMENTS

7 8 9 10 11 12

TI-Nspire CAS · Investigation · Student · 120min

Aim:

The number 12 has six factors: 1, 2, 3, 4, 6 and 12. The number 36 has more factors. Which number would have the greatest number of factors: 36 or 5929? This investigation explores how to determine the quantity of factors for any given number without listing each individual factor.

Equipment:

For this activity you will need:

- Playing cards / Numbered counters
- TI-Nspire CAS
- TI-Nspire CAS file (tns): Factors that Count

Preliminary Investigation:

1. Construct factor trees for each of the following numbers:

a) 36

b) 100

c) 196

d) 441

2. Write down the prime factorisation for each of the numbers 36, 100, 196 and 441.

3. Using a pack of playing cards (or numbered counters), determine all the factors for each of the numbers 36, 100, 196 and 441. Record all arrangements of the cards used to determine the factors for each number.
Remember to include one and the original number in the factor count.

Examples: Factors of 36

2	×	(2	×	3	×	3)		
Factor Pair:	2	×	18					

3	×	(2	×	2	×	3)		
Factor Pair:	3	×	12					

4. What do you notice about the quantity of factors for the numbers 36, 100, 196 and 441?

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Investigation – The factor Sleuth:

5. Use questions 1 to 4 as a guide to investigate the following numbers: 24, 54, 250 and 1029. Determine the prime factorisation and the quantity of factors for each of these numbers.

6. Use questions 1 to 4 as a guide to investigate the following numbers: 80, 162, 405 and 1250. Determine the prime factorisation and the quantity of factors for each of these numbers.

The numbers from Question 1: 36, 100, 196 and 441 all have something in common based on their prime factorisation, let this collection of numbers be identified as 'Group 1'. The numbers from Question 5: 24, 54, 250 and 1029 also have something in common, let this collection of numbers be identified as 'Group 2'. The numbers from Question 6: 80, 162, 405 and 1250 belong to 'Group 3'.

7. Identify which groups each of the following numbers belong to and determine the quantity of factors for each: 88, 104, 875, 484, 1375, 3773 and 3025.

8. Determine three numbers that have exactly nine factors and explain how you found these numbers. *You can not use numbers that have already been included in this activity.*

9. Determine three numbers that have exactly eight factors and explain how you found these numbers. *You can not use numbers that have already been included in this activity.*

10. Explain how the prime factorisation of a number helps identify the quantity of factors.

Investigation – Factors Rule:

It is time to formalise a rule that identifies the quantity of factors for any given number. The rule uses the prime factorisation of the number.

11. Investigate each of the following prime factorisations and determine the quantity of factors for each.

The first one has been done for you.

Number	Prime Factorisation	Factors	Qty of Factors	Indices	Bases
108	$2^2 \times 3^3$	{1,2,3,4,6,9,12,18,27,36,54,108}	12	2, 3	2, 3
675	$5^2 \times 3^3$			2, 3	3, 5
1125	$5^3 \times 3^2$			3, 2	3, 5
392	$7^2 \times 2^3$				2, 7
968	$11^2 \times 2^3$				2, 11
726				1, 1, 2	
132					
198					
396				1, 2, 2	2,3,11
1100					
700					

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12. Study carefully the indices and the number of factors. Write down a 'conjecture' (proposed rule) that relates the indices to the number of factors.
13. Generate your own table of numbers to explore. Explain your selection of numbers and how they were generated.
14. Use your table of numbers to check or validate your rule from question 12.
    Note: If your rule from question 12 does not work, you will need to re-write it.

Technology:

TI-Nspire CAS can be used to explore the prime factorisation of a number:

$$factor(36)$$

From the **Number** (or **Algebra**) menu, select **Factor**.