

# EXPLORATIONS

## Chapter 5

### Features Used

`[ ]`, `abs()`, `real()`, `imag()`,  
`conj()`, `[f]`, `[a]`, `NewProb`, `[Z]`,  
 Program Editor, `[Y=]`,  
`exp▶list()`

### Setup

`[ ]` 1, `NewFold power`,  
`setMode("Angle", "Degree")`  
`setMode("Complex Format",  
 "Polar")`

**Power Engineering** This chapter shows how to handle phasor algebra, complex power, power factor corrections, and unbalanced three-phase calculations using the TI-89.

### Topic 19: Phasor Algebra

The sinusoidal, single-frequency voltages and currents of power systems are usually written as phasors — complex numbers in magnitude and phase form. This form is particularly useful in three-phase calculations when phasors must be added or subtracted. For Y-configurations, the line-to-neutral voltages,  $v_{an}$ ,  $v_{bn}$ , and  $v_{cn}$ , (often called the phase voltages) are combined to give the line-to-line voltages,  $v_{ac}$ ,  $v_{ba}$ , and  $v_{cb}$  as shown in Figure 1.

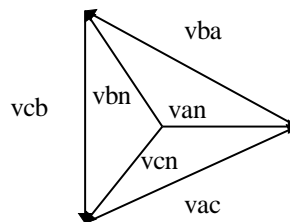


Figure 1. Three-phase line and phase voltages

For a positive phase sequence, the phasor forms of the line to neutral voltages are given as

$$v_{an} = 110$$

$$v_{bn} = v_{an} \angle 120^\circ = 110 \angle 120^\circ$$

$$v_{cn} = v_{an} \angle 240^\circ = 110 \angle 240^\circ = 110 \angle -120^\circ$$

A negative phase sequence gives the opposite signs for the phase angles.

1. Clear the TI-89 by pressing  $\text{[2nd]} \text{[F6]} \text{2:NewProb} \text{[ENTER]}$ .
2. The function **vphase()** uses **van** and the sign of the phase sequence to calculate the phase voltages (see screen 1). Press  $\text{[APPS]} \text{7:Program Editor} \text{3:New}$ , select **2:Function** for **Type**, and enter **vphase** for **Variable**. Then type the function lines as shown in screen 1.

See Tips and Generalizations for shortcuts on entering functions.

3. Calculate the phase voltages for **van=110** with a positive phase sequence. Return to the Home screen, and use the function **vphase()** with arguments of 110 and 1 as shown in screen 2.

Two entries are displayed in screen 2. The first entry shows the returned answer; the second shows the answer scrolled to display the right side.

4. For a negative phase sequence, use arguments of 110 and -1 (screen 3).

If different results are displayed, press  $\text{[MODE]}$  and set the modes as shown in the Setup section.

5. Each line-to-line voltage is expressed as the phasor difference of the two adjacent phase voltages, for example, **vac=van-vcn**. The function **phas2lin()** returns the line-to-line voltages in a list **{vac, vba, vcb}** for **van** and the sign of the phase sequence.

Enter the function as shown in screen 4. The function **vphase()** is used to calculate the phase voltages which then are used to calculate and return the list of line voltages.

6. To calculate the line-to-line voltages for a phase voltage **van=110** with a positive phase sequence, use **phas2lin()** with arguments of 110 and 1 (screen 5). Two entries are displayed again to show the complete answer.

```

F1- F2- F3- F4- F5- F6-
Tools Control I/O Var Find... Mode
vphase(van, sqnc)
Func
(van, van*(1 ∠ sqnc*120), v
an*(1 ∠ sqnc*240))
EndFunc
    
```

(1) POWER DEGAUTO FUNC

**Note:** To enter  $\angle$ , press  $\text{[2nd]} \text{[∠]}$ .

```

F1- F2- F3- F4- F5- F6-
Tools n13eBrq Calc Other Pr3mID Clean Up
NewProb Done
vphase(110, 1)
(110 (110 ∠ 120) (110 ∠
vphase(110, 1)
(110 ∠ 120) (110 ∠ -120))
vphase(110, 1)
    
```

(2) POWER DEGAUTO FUNC 3/30

```

F1- F2- F3- F4- F5- F6-
Tools n13eBrq Calc Other Pr3mID Clean Up
vphase(110, 1)
(110 ∠ 120) (110 ∠ -120))
vphase(110, -1)
(110 (110 ∠ -120) (110
vphase(110, -1)
(110 ∠ -120) (110 ∠ 120))
vphase(110, -1)
    
```

(3) POWER DEGAUTO FUNC 5/30

```

F1- F2- F3- F4- F5- F6-
Tools Control I/O Var Find... Mode
phas2lin(van, sqnc)
Func
Local phase
vphase(van, sqnc)→phase
(phase[1]-phase[3], phase[
2]-phase[1], phase[3]-phas
e[2])
EndFunc
    
```

(4) POWER DEGAUTO FUNC

```

F1- F2- F3- F4- F5- F6-
Tools n13eBrq Calc Other Pr3mID Clean Up
vphase(110, -1)
(110 ∠ -120) (110 ∠ 120))
phas2lin(110, 1)
(110·√3 ∠ 30) (110·√3 ∠
phas2lin(110, 1)
(√3 ∠ 150) (110·√3 ∠ -90))
phas2lin(110, 1)
    
```

(5) POWER DEGAUTO FUNC 7/30

- Phase voltages are calculated from line-to-line voltages as the difference of the adjacent line voltages, for example,  $v_{an}=(v_{ac}-v_{ba})/3$ . The function **lin2phas()** returns the phase voltages given the line-to-line voltages and the sign of the phase sequence.

Enter the function shown in screen 6. The **vac** line to line voltage and phase sequence are used to calculate **van** which is used to calculate the other two phase voltages.

```

F1- F2- F3- F4- F5- F6-
Tools Control I/O Warn Find... Mode
: lin2phas(vac, snc)
: Func
: Local van
: (vac-vac*(1 < snc*120))/
: 3->van
: (van, van*(1 < snc*120), v
: an*(1 < snc*240))
: EndFunc
    
```

(6) POWER DEGAUTO FUNC

- Screen 7 shows the calculation of the phase voltages for  $v_{ac}=110\sqrt{3}\angle 30^\circ$  and a positive phase sequence.

```

F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up
: PHAS2LIN(110, 1)
: <-.√3 < 150) ( 110·√3 < -90))
: lin2phas(( 110·√3 < 30), 1)
: (110 (110 < 120) (110 <
: lin2phas(( 110·√3 < 30), 1)
: <( 110 < 120) (110 < -120))
: lin2phas((110*√(3)<30), 1)
    
```

(7) POWER DEGAUTO FUNC 9/30

### Topic 20: Average Power

Instantaneous power is defined as  $p(t)=i(t)v(t)$  for real signals where  $i(t)$  is the current which flows in the direction of the voltage drop across the element,  $v(t)$ .  $p(t)$  varies with time as the signal varies. On the other hand, the average power of many signals is constant and often a more useful parameter. It is defined as

$$P_{AVG} = \frac{1}{T} \int_{t=t_0}^{t_0+T} v(t)i(t)dt$$

where T is the period of the signal.

- Set the **Angle** mode to **Radian**.
- To calculate the average power dissipated in a 1 kΩ resistor with a voltage of  $v(t)=10\sin(2\pi 60t)$  V across it, first calculate the current as shown in screen 8.

Since  $i=v/r$ , the current is  $i(t)=10\sin(2\pi 60t)$  (bottom of screen 8). The period of this signal is calculated from the relationship  $T=1/f=1/60$  s.

```

F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up
: setMode("angle", "radian")
: "DEGREE"
: 10·sin(2·π·60·t) → v(t) Done
: v(t)
: 1000 → i(t) Done
: v(t)/1000 → i(t)
    
```

(8) POWER RAD AUTO FUNC 12/30

- Enter the average power,  $P_{AVG}$ , as shown in screen 9.

```

2nd [f] v ( t ) i ( t ) , t , 0 , 1 ÷ 60 ) ÷ ( 1 ÷
60 )
    
```

The average power is calculated as  $1/20=50$  mw.

```

F1- F2- F3- F4- F5- F6-
Tools Algebra Calc Other Pr3mID Clean Up Done
: v(t)
: 1000 → i(t) Done
: ∫ 1/60 (v(t)·i(t))dt
: 1/60 1/20
: v(t)i(t), t, 0, 1/60) / (1/60)
    
```

(9) POWER RAD AUTO FUNC 13/30

4. The average power dissipated in a 1  $\mu\text{F}$  capacitor with a voltage of  $v(t)=10\sin(2\pi 60t)$  V across it is calculated in a similar way. However, for a capacitor  $i(t)=Cdv/dt$ . The calculator can do the work of calculating the derivative as well as the power. To do this, enter the three expressions as shown in screen 10.

1 [EE] [(-)] 6 [STO] c  
 c [2nd] [d] v [ ( ] t [ ) ] , t [ ) ] [STO] i [ ( ] t [ ) ]  
 and  
 [2nd] [∫] v [ ( ] t [ ) ] i [ ( ] t [ ) ] , t [ , ] 0 [ , ] 1 [ ÷ ] 60 [ ) ] ÷ [ ( ] 1 [ ÷ ] 60 [ ) ]

The average power dissipated in a capacitor is always zero!

5. The integral calculation of power is valid when the voltage and current are not so nicely related. For example, consider an unusual device that has a periodic pulse train voltage across it with one cycle defined with the “when” function as shown on the top of screen 11.

[CATALOG] when( t [2nd] [ < ] 0.01 [ , ] 10 [ , ] 2 [ ) ] [STO] v [ ( ] t [ ) ]

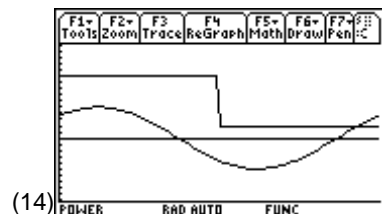
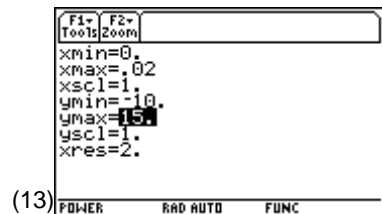
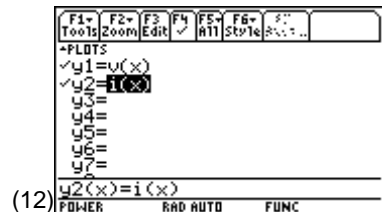
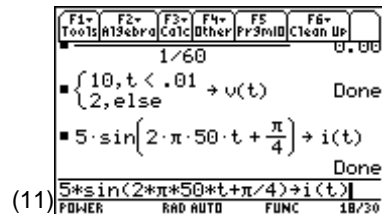
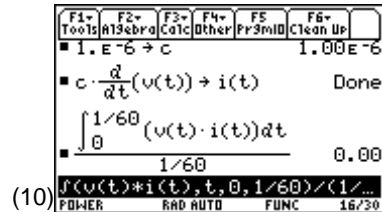
This device also has a sinusoidal current given by  $i(t)=5\sin(2\pi 50t+\pi/4)$ , as shown on the bottom of screen 11. These expressions are entered as voltage and current.

6. Note that the two signals must be periodic with the same period  $T=0.02$  seconds. To graph the two signals over one period, enter  $v(x)$  and  $i(x)$  in the Y= Editor (screen 12).

7. Enter the graphing parameters in the Window Editor screen as shown in screen 13.

8. Press [GRAPH] to display the results in the graph in screen 14.

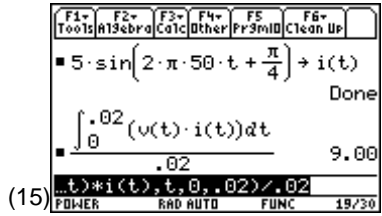
**Note:** The integral entered before can be copied to the entry line by moving the cursor into the history section with  $\ominus$ , highlighting the integral, and pressing [ENTER].



9. The average power is found by using the same integral over the period of 0.02 seconds. Return to the Home screen, and enter this as shown in screen 15.

$\boxed{2\text{nd}} \boxed{[f]} \boxed{v} \boxed{[ ]} \boxed{t} \boxed{[ ]} \boxed{\times} \boxed{i} \boxed{[ ]} \boxed{t} \boxed{[ ]} \boxed{,} \boxed{t} \boxed{[ ]} \boxed{0} \boxed{,} \boxed{0.02} \boxed{[ ]} \boxed{\div} \boxed{0.02}$

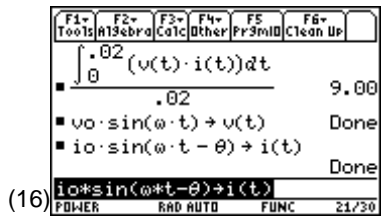
Therefore, the average power dissipated in a device with the square wave voltage and sinusoidal current is 9.00 W.



### Topic 21: Complex Power

For steady-state, sinusoidal signals with  $v(t)=v_o \sin\omega t$  and  $i(t)=i_o \sin(\omega t-\theta)$ , the average power is calculated with the integral as before where  $T=2\pi/\omega$ .

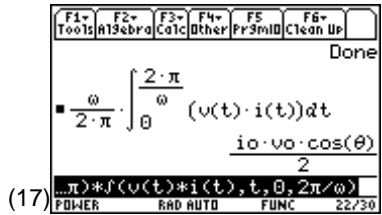
1. Enter  $v(t)$  and  $i(t)$  as shown in screen 16.



**Note:** To enter  $\omega$ , press  $\boxed{\alpha} \boxed{w}$ .

2. Find the average power as shown in screen 17.

$\boxed{\alpha} \boxed{[ ]} \boxed{\alpha} \boxed{w} \boxed{\div} \boxed{[ ]} \boxed{2} \boxed{[ ]} \boxed{2\text{nd}} \boxed{[ \pi ]} \boxed{[ ]} \boxed{\times} \boxed{[ ]} \boxed{2\text{nd}} \boxed{[f]} \boxed{v} \boxed{[ ]} \boxed{t} \boxed{[ ]} \boxed{\times} \boxed{i} \boxed{[ ]} \boxed{t} \boxed{[ ]} \boxed{,} \boxed{t} \boxed{[ ]} \boxed{0} \boxed{,} \boxed{2} \boxed{[ ]} \boxed{2\text{nd}} \boxed{[ \pi ]} \boxed{\div} \boxed{\alpha} \boxed{[ ]} \boxed{\alpha} \boxed{w} \boxed{[ ]}$



This form provides the basis for the common expression for average power

$$P_{AVG} = \frac{v_o}{\sqrt{2}} \frac{i_o}{\sqrt{2}} \cos\theta$$

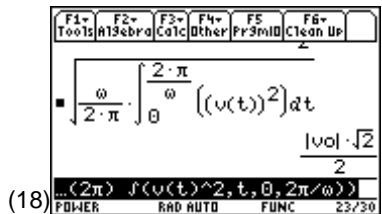
where  $\theta$  is the angle by which the current lags the voltage. The terms  $(v_o/\sqrt{2})$  and  $(i_o/\sqrt{2})$  are known as the root-mean-square (abbreviated as RMS) voltage and current, respectively. RMS quantities are defined as the square root of the mean (or average) of the quantity squared.

$$x_{rms} = \sqrt{\frac{\int_{t=0}^T x^2(t) dt}{T}}$$

3. Calculate the RMS voltage of  $v(t)=v_o \sin(\omega t)$  as shown in screen 18.

$\boxed{2\text{nd}} \boxed{[ \sqrt{ } ]} \boxed{\alpha} \boxed{[ ]} \boxed{\alpha} \boxed{w} \boxed{\div} \boxed{[ ]} \boxed{2} \boxed{[ ]} \boxed{2\text{nd}} \boxed{[ \pi ]} \boxed{[ ]} \boxed{\times} \boxed{[ ]} \boxed{2\text{nd}} \boxed{[f]} \boxed{v} \boxed{[ ]} \boxed{t} \boxed{[ ]} \boxed{\wedge} \boxed{2} \boxed{,} \boxed{t} \boxed{[ ]} \boxed{0} \boxed{,} \boxed{2} \boxed{[ ]} \boxed{2\text{nd}} \boxed{[ \pi ]} \boxed{\div} \boxed{\alpha} \boxed{[ ]} \boxed{\alpha} \boxed{w} \boxed{[ ]}$

The result shows that the RMS value of any sinusoidal signal is  $\frac{\sqrt{2}}{2}$  times its peak magnitude.



When sinusoidal voltage and current are written in phasor form,  $v_o \angle 0$  and  $i_o \angle -\theta$ , the average power is given as  $P_{AVG} = \text{Real}(v_o \cdot \text{conj}(i_o \angle -\theta)) / 2$ .

- Calculate the average power for  $v(t) = v_o \sin(\omega t)$  and  $i(t) = i_o \sin(\omega t - \theta)$  using the phasor method.

First enter the voltage phasor as shown in screen 19.

`[ ] [vo] [÷] [2nd] [√] [2] [ ] [2nd] [∠] [0] [ ] [STO] vphasor`

- Enter the current phasor (screen 20).

`[ ] [io] [÷] [2nd] [√] [2] [ ] [2nd] [∠] [(-)] [ ] [θ] [ ] [STO] iphasor`

- Calculate the average power (screen 21).

`[CATALOG] real( vphasor [x] [CATALOG] conj( iphasor [ ] [ ]`

The results are identical with the time-domain averaging.

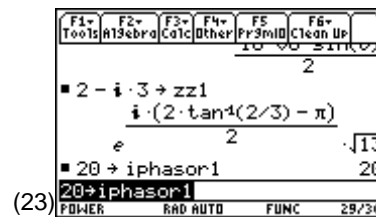
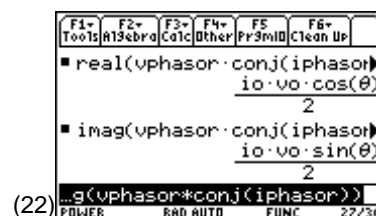
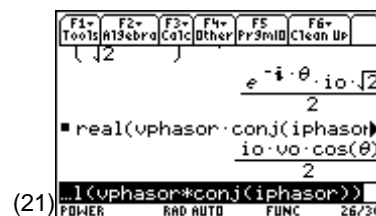
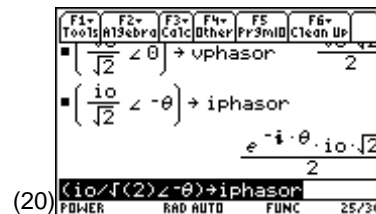
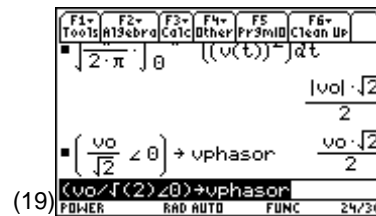
This leads to the generalized concept of complex power. The complex power,  $S$ , is defined as  $S = P + jQ$  where the real part  $P$  is identical to average power and is expressed in watts, the imaginary part  $Q$  is known as reactive power expressed in VARs (volt-amperes reactive), and the complex power  $S$  has units of VA (volt-amperes).  $S$  is calculated by  $S = v_o \angle 0 \cdot \text{conj}(i_o \angle -\theta)$ . The average power is  $P = \text{real}(S)$ ; the reactive power is  $Q = \text{imag}(S)$ .

- Find the reactive power as shown in screen 22.

`[CATALOG] imag( vphasor [x] [CATALOG] conj( iphasor [ ] [ ]`

- Calculation of the complex power for a load of  $z_1 = 2 - j3 \Omega$  with a current  $i_{phasor1} = 20$  A rms proceeds as follows.

- First, enter the values of  $z_1$  and  $i_{phasor1}$  (screen 23).



- b. The voltage of the load is  $v_{\text{phasor1}} = i_{\text{phasor1}} \cdot z_{z1}$  and the complex power is  $S1 = v_{\text{phasor1}} \cdot \text{conj}(i_{\text{phasor1}})$  (screen 24).

```
[ ] vphasor1 [alpha] [x] [CATALOG] conj( iphasor1 [ ] [ ]
[STO] s1 [2nd] [:] s1 [CATALOG] ►Rect
```

The calculation gives  $s1 = 800 - j1200$  VA.



**Note:** To enter  $j = \sqrt{-1}$ , press [2nd] [i].

**Note:** Two or more commands can be entered on one line using the colon ([2nd] [:]) to separate them.

9. Calculation of the complex power associated with a load given by  $z_{z2} = 3 + j4 \Omega$  with a voltage of  $v_{\text{phasor2}} = 110$  Vrms, applied across it proceeds as follows.

- a. Press [MODE] and change the **Complex Format** mode to **RECTANGULAR**.

- b. First enter **zz2** and **vphasor2** (screen 25).

- c. The rms load current is  $v_{\text{phasor2}} / z_{z2} = i_{\text{phasor2}}$ ; the complex power is  $S2 = v_{\text{phasor2}} \cdot \text{conj}(i_{\text{phasor2}})$ .

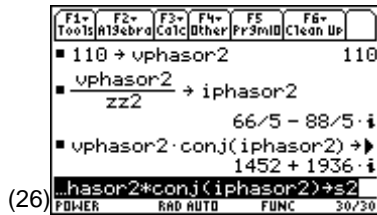
Enter the complex power as shown in screen 26.

```
vphasor2 [x] [CATALOG] conj( iphasor2 [ ] [STO] s2
```

The complex power is calculated as  $s2 = 1452 + j1936$  VA. The average power which does work or produces heat in the  $3 \Omega$  resistive part of the load is 1452 W. The reactive power which represents the rate of change of stored energy in the  $j4 \Omega$  reactive part of the load is 1936 VARs.



**Note:** setMode("Complex Format", "Rectangular") was used since the rest of the examples are best displayed in rectangular mode. This can be set with [MODE] to minimize the typing.



### Topic 22: Power Factor

Complex power  $S$  is supplied to a load. The real portion of the power,  $P$ , is available to do work or produce heat; the imaginary portion of the power,  $Q$ , is unusable. The fraction of complex power which is available to do work is given by the power factor

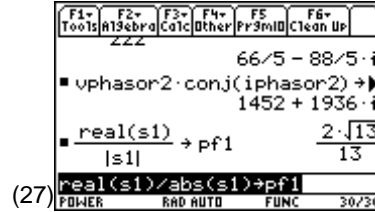
$$pf = \frac{\text{Real Power}}{\text{Complex Power}} = \frac{P}{S} = \cos \theta$$

For positive phase angles  $\theta$ , the power factor is called leading; for negative phase angles, lagging. Resistive loads have a unity power factor, that is,  $pf = 1$ ; reactive loads have a zero power factor. The angle  $\theta$  is the same angle as the impedance phase angle. Since  $Y = 1/Z$ , the admittance phase angle is the negative of the impedance phase angle and can be used to calculate the power factor as well.

The complex power for the load  $z_{z1} = 2 - j3 \Omega$  in Topic 21 is  $s1 = 800 - j1200$  VARs.

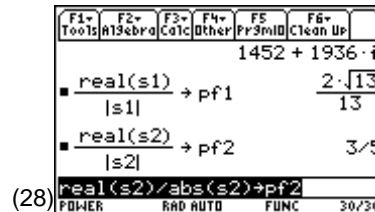
1. Find the power factor, pf1 (screen 27).

The power factor of  $pf1 = \frac{2 \cdot \sqrt{13}}{13}$  is lagging since it is a capacitive load, that is, the reactive component is negative.



2. The complex power for the load  $zz2=3+j4 \Omega$  in Topic 21 is  $s2=1452+j1936$  VARs.

Screen 28 shows the power factor of  $pf2 = 3/5$  is leading for the inductive load.



### Topic 23: Power Factor Correction Using Impedances

Reduction of the reactive power improves the power factor which means that more of the power generated by the electric utility can be sold. Therefore, rate incentives are offered for industrial users to improve or “correct” their power factor. Most large users have a leading power due to the inductive nature of motors. A leading power factor can be “corrected,” that is, brought closer to unity, by adding capacitors in parallel. The capacitive susceptance,  $j2\pi fC$ , cancels part of the inductive susceptance of the load,  $-jB_L$ , and makes the angle  $\theta$  smaller and the  $pf = \cos\theta$  closer to unity. See Figure 2.

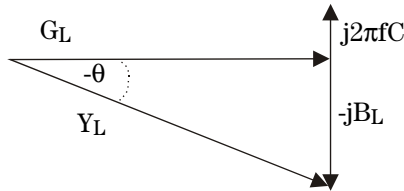
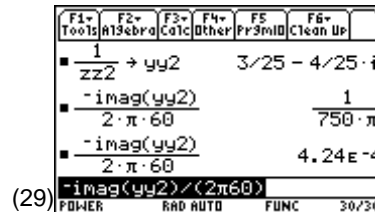


Figure 2. Admittance triangle

The procedure for calculating the amount of parallel capacitance needed to correct the power factor to unity for the inductive load  $zz2=3+j4 \Omega = 5\angle 53.1^\circ \Omega$  (Topic 21) follows. This load has  $pf2 = \cos(53.1^\circ) = 3/5 = 0.6$  (Topic 22).

1. Convert the impedance to admittance,  $yy2 = 1/zz2 = 3/25 - j4/25 = 0.12 - j0.16$  S (top of screen 29).
2. The added parallel capacitive susceptance must cancel the inductive susceptance so that  $j(0.16) = j2\pi(60)C$  which gives  $C = 424 \mu F$  (bottom of screen 29).



**Note:** Calculate the floating-point result by pressing  $\square [=]$ .

Unity power factor is often prohibitively expensive to achieve so the rates are set to offer incentive for partial correction.

Repeat the last example for the load  $zz2=3+j4 \Omega$  but with a final power factor of  $pf=0.9$  leading. The load conductance does not change with the addition of the parallel capacitor. To obtain the specified power factor, the added capacitive susceptance must result in a leading  $pf=0.9 = \cos\theta$  so that  $\tan\theta = \tan(\cos^{-1}(0.9)) = -0.484$ .



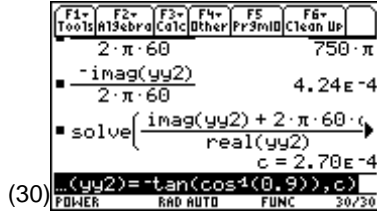
With the addition of the parallel capacitor, the total susceptance of the load is given as  $b_{tot} = \text{imag}(yy2) + 2\pi 60c$ .

The ratio of  $\text{imag}(s)/\text{real}(s) = (\text{imag}(yy2) + 2\pi 60c) / \text{real}(yy2) = \tan\theta = \tan(\cos^{-1}(0.9))$ .

Find the desired value of capacitance as shown in screen 30.

`CATALOG solve( ( CATALOG imag( yy2 ) + 2 [2nd] [π] 60c ) ÷`  
`CATALOG real( yy2 ) = ( - ) [2nd] [TAN] * [COS-1] 0.9 ) , c )`

The result is  $C = 270 \mu\text{F}$ , somewhat less than that needed for unity power factor and therefore less expensive.



### Topic 24: Power Factor Correction Using Power Triangle

Alternatively, power factor calculations can be made in terms of complex power as well, but these calculations require the voltage or current.

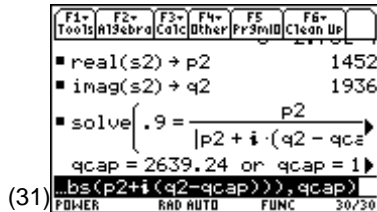
To recalculate the parallel capacitance needed to bring the power factor to 0.9 leading when the complex power is  $S = 1452 + j1936 \text{ VA}$ , recall that  $P = \text{real}(S)$  and  $Q = \text{imag}(S)$  (top of screen 31).

Some negative reactive power is introduced by the added parallel capacitance  $\text{pf} = 0.9 = P/S = P / (P + j(Q - Q_{\text{CAP}}))$ .

1. Enter this as shown in screen 31.

`CATALOG solve( 0.9 = p2 ÷ ( CATALOG abs( p2 + [2nd] [i]`  
`( q2 - qcap ) , qcap )`

The complete answer is  $q_{\text{cap}} = 2639.24$  or  $q_{\text{cap}} = 1232.76$



2. Enter the second solution to obtain

$$Q_{\text{CAP}} = 1232.8 = 2\pi 60 C |v|^2 = 2\pi 60 C (|v_{\text{phasor2}}|^2)$$

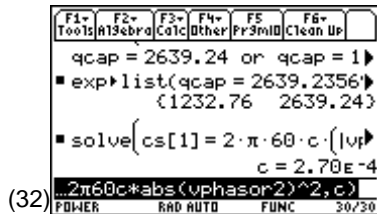
`CATALOG exp>list( [2nd] [ANS] , qcap ) [STO] cs`

and

`CATALOG solve( cs [2nd] [t] 1 [2nd] [j] = 2 [2nd] [π] 60c x`  
`CATALOG abs( vphasor2 ) ^ 2 , c )`

Screen 32 shows this leads to  $c = 270 \mu\text{F}$ , as before.

Note that the second solution  $\text{cs}[2] = 2639.2$  gives a power factor of 0.9, but lagging. Since this requires a capacitor twice as large, it is not an economical solution.



**Note:** `exp>list(ans(1),qcap) [STO] cs` is used to convert the results of the first solve into a list which is stored in **cs**. **cs[1]** uses the first solution to solve for **c**.

### Topic 25: Y-Δ and Δ-Y Transformations

Many circuits are simplified by transforming a circuit from a Y configuration to a Δ or a Δ configuration to a Y as shown in Figure 3. The transformation is particularly useful in simplifying circuits for later series-parallel combinations.

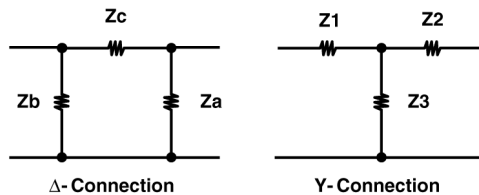


Figure 3. Δ and Y Configurations

The transformations are expressed as

$$Z_1 = \frac{Z_b Z_c}{Z_a + Z_b + Z_c} \quad Z_a = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_1}$$

$$Z_2 = \frac{Z_c Z_a}{Z_a + Z_b + Z_c} \quad Z_b = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_2}$$

$$Z_3 = \frac{Z_a Z_b}{Z_a + Z_b + Z_c} \quad Z_c = \frac{Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1}{Z_3}$$

Since there are multiple inputs and multiple results, functions work well to make these transformations.

1. Press **[APPS]** **7:Program Editor 3:New** and define the two functions, **delta2y()** and **y2delta()**, to implement these equations (screens 33 and 34).

```

F1- F2- F3- F4- F5- F6-
Tools Control I/O Var Find... Mode
:delta2y(z)
:Func
:Local den,za,zb,zzc
:z[1]+za:z[2]+zb:z[3]+zcc
:za+zb+zcc+den
:(zb*zcc,zc*za,za*zb)/den
:EndFunc
POWER RAD AUTO FUNC
    
```

(33)

2. Return to the Home screen, and use the **delta2y()** function to calculate the Y-configuration elements for a delta-configured circuit with  $z_a\Omega = 3+j$ ,  $z_b=5+j5\Omega$ , and  $z_c=2-j4\Omega$  as shown in the middle of screen 35.

```

delta2y ([ 2nd [ ( ] 3. + [ 2nd [ i ] , 5 + [ 2nd [ i ] 5 ] 2 - [ 2nd
[ i ] 4 [ 2nd [ ) ] ]
    
```

The **delta2y()** function gives the results  $z_1=2.69-j1.54\Omega$ ,  $z_2=0.77-j1.15\Omega$ , and  $z_3=1.35+j1.73\Omega$  (middle of screen 35).

3. The inverse transformation, **y2delta()**, returns the original impedance values (bottom of screen 35).

```

F1- F2- F3- F4- F5- F6-
Tools Control I/O Var Find... Mode
:y2delta(z)
:Func
:Local num
:z[1]*z[2]+z[2]*z[3]+z[3]*
z[1]+num
:(num/z[1],num/z[2],nu
m/z[3])
:EndFunc
POWER RAD AUTO FUNC
    
```

(34)

```

F1- F2- F3- F4- F5- F6-
Tools A13eBrd Calc Other Pr3rd Clcn Up
solve(cs[1]=2*pi*60*c:|v|
c=2.70E-4
delta2y((3.+i 5+i.5)
(2.69-1.54.i .77-1.15)
y2delta((2.6923076923077)
(3.00+1.00.i 5.00+5.0)
y2delta(ans(1))
POWER RAD AUTO FUNC 30/30
    
```

(35)

### Topic 26: Unbalanced Three-Phase Systems

Solving balanced three-phase Y-systems is quite simple. Since all phases have the same sources and loads, their voltages and currents differ from each other only by 120°. The solution for a single phase is just shifted by ±120° to get the solutions for the other phases. Although the solutions are more complicated for unbalanced three-phase Y-systems, they are easy with the TI-89's matrix operations.

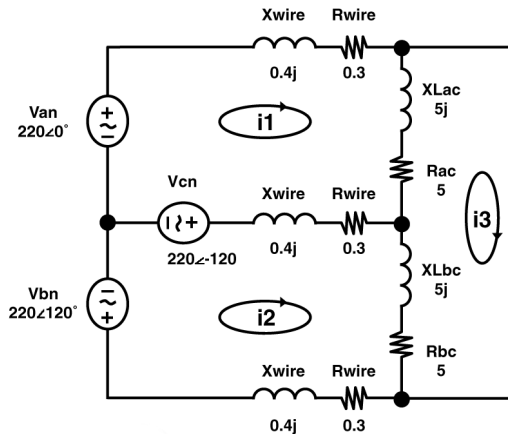


Figure 4. Unbalanced, three-phase circuit

The three-phase system of Figure 4 has a short-circuit across one phase. The currents can be calculated by using Kirchhoff's voltage law to write three mesh equations for the circuit.

$$\begin{aligned}
 i1(5.6 + j5.8) - i2(0.3 + j0.4) - i3(5.0 + j5.0) &= 220 - (220\angle -120) \\
 -i1(0.3 + j0.4) + i2(5.6 + j5.8) - i3(5.0 + j5.0) &= (220\angle 120) + (220\angle -120) \\
 -i1(5 + j5) - i2(5 + j5) + i3(10 + j10) &= 0
 \end{aligned}$$

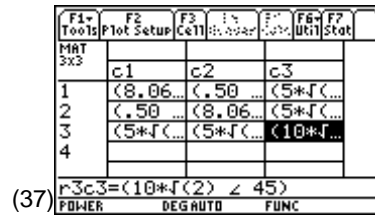
In matrix form, the equations are

$$\begin{bmatrix}
 5.6 + j5.8 & -.3 - j.4 & -5 - j5 \\
 -.3 - j.4 & 5.6 + j5.8 & -5 - j5 \\
 -5 - j5 & -5 - j5 & 10 + j10
 \end{bmatrix}
 \begin{bmatrix}
 i1 \\
 i2 \\
 i3
 \end{bmatrix}
 =
 \begin{bmatrix}
 220 - (220\angle -120) \\
 (220\angle 120) + (220\angle -120) \\
 0
 \end{bmatrix}$$

1. Press **MODE**. Then set **Angle** mode to **DEGREE** and **Complex Format** mode to **POLAR**.
2. Create a new matrix variable. Press **APPS** **6:Data/Matrix Editor** **3:New**. Select **2:Matrix** for **Type**, name the variable **unsymnet**, and set the row and column dimensions to 3 (screen 36). Press **ENTER** to display the matrix.



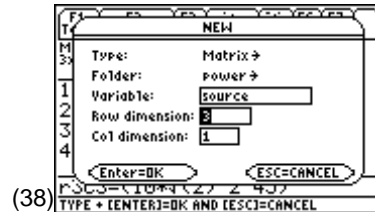
- Enter the elements from the matrix equation above. Note that you can enter the values in rectangular form. They will automatically be displayed in polar form (screen 37). For  $j$ , press  $\boxed{2nd} \boxed{[i]}$ .



(37)

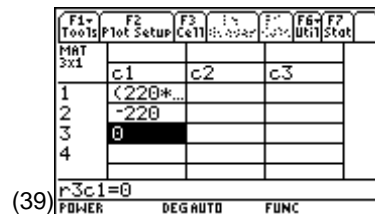
**Note:** Press  $\boxed{\blacklozenge} \boxed{[1]}$  and set the column width to 6 to get the display above.

- To enter the 3x1 source matrix on the right hand side of the equation, create a new matrix by pressing  $\boxed{F1} \boxed{3} \text{New}$  (screen 38). Select **2:Matrix** as the type, name the variable **source**, and set the row dimension to 3 and column dimension to 1. Press  $\boxed{ENTER}$ .



(38)

- Enter the values from the right hand side of the matrix equation in the same manner (screen 39).

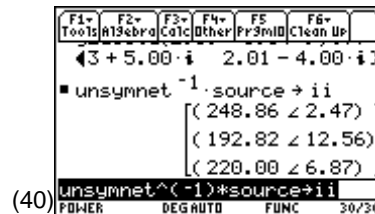


(39)

**Note:** When the first element of a complex number is preceded by a minus sign it must be entered with  $\boxed{(-)}$ . A minus sign on a following element is entered as  $\boxed{-}$ .

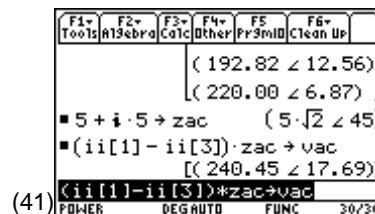
- Once the matrices are entered, matrix math is all that's needed to find the solution for the current matrix,  $ii$ . Return to the Home screen, and enter this as shown in screen 40.

$$\text{unsymnet} \boxed{\wedge} \boxed{[1]} \boxed{(-)} \boxed{1} \boxed{[1]} \boxed{\times} \text{source} \boxed{STO} \boxed{\triangleright} ii$$



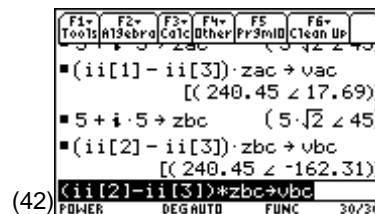
(40)

- The load voltage across the short circuit is zero. The load voltage across the  $z_{ac}$  is  $v_{ac} = (i_1 - i_3)(5 + j5)$  (screen 41).



(41)

- The load voltage across  $z_{bc}$  is  $v_{bc} = (i_2 - i_3)(5 + j5)$ . Enter the expressions as shown in screen 42. As expected, the two non-zero voltages are the same magnitude, but 180° out of phase.



(42)

## Tips and Generalizations

There are many shortcuts for entering functions. Check under the **F2**, **F3**, and **F4** menus while in the Program Editor. For example, **Local** can be entered by pressing **F4 3:Local**.

The string **phase** is entered often in the examples in this chapter. To save typing, enter it once and just after pressing **e**, press and hold **↑** while pressing **↓** five times (once for each letter in **phase**).

Finally, release the **↑** key and press **♦ [COPY]**. This will save a copy of the highlighted letters so that they can be quickly entered by pressing **♦ [PASTE]**.

The TI-89 can easily handle Greek symbols such as  $\alpha$  and  $\omega$ . However, they take four keystrokes each to enter (**♦ [alpha] a** and **♦ [alpha] w**). If convenience is important, simply use the English equivalents (**a, w**) which take only two keystrokes (**[alpha] a** or **[alpha] w**) or fewer if [a-lock] is already set.

The concept of building a simple function to do repetitive tasks was introduced. Don't underestimate the power of this method. For example, the whole power factor correction example could be written as a function that takes various circuit parameters as an input and returns the capacitor value needed to correct the power factor. Think of the time savings!

Another analysis technique involves the Laplace transform and the s-domain. Chapter 6 shows how the symbolic capabilities of the TI-89 make it the ideal tool for dealing with all of those s's.

