One Sided Limits

ID: 10994

Time required
15 minutes

Activity Overview

*Students will graph piecewise functions and evaluate both the left hand limit and the right hand limit of the function as x approaches a given number, c. The **Trace** feature will be used to graphically estimate the one sided limit. Students will also use a table of values of each function to numerically verify that the values of the function to left and right of c are approaching the same number.*

Topic: Limits

- *One Sided Limits*

Teacher Preparation and Notes

- *Students should already have been introduced to one-sided limits.*
- *Students should know that a limit exists if and only if the left hand limit and the right hand limit are equal.*
- *Upgrade the TI-89 Titanium to OS Version 3.10 so that "Discontinuity Detection" can be utilized. On a graph, press F1 > Format to turn Discontinuity Detection ON.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "10994" in the keyword search box.***

Associated Materials

- *OneSidedLimits_Student.doc*

Suggested Related Activities

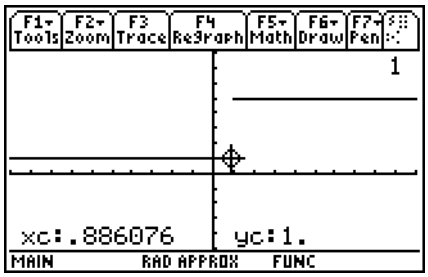
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Limits (TI-Nspire CAS technology) — 8997*
- *Continuity and Differentiability of Functions (TI-Nspire technology) — 8498*

The student worksheet gives key press instructions to set up the window so that their graphs look like the following.

Problem 1

Before changing the value of ***a***, students will graphically estimate the limit of *y*1(*x*) as *x* approaches 1 from the left and the right. Students will also use the table to numerically estimate the value of ***a*** that will ensure that the limit of *y*1(*x*) as *x* approaches one exists.

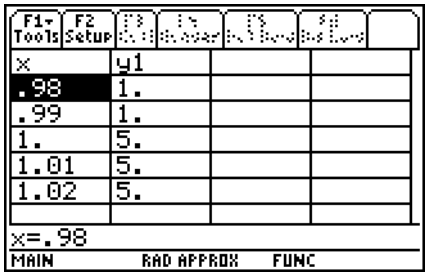


Student Worksheet solutions

$\lim_{x\to 1^-} y1(x) \approx 1;$

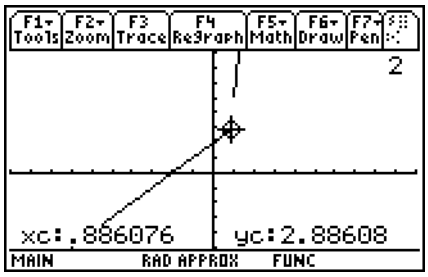
$\lim_{x\to 1^+} y1(x) \approx 5;$

$a \approx 1$



Problem 2

Problem 1 is repeated for a different function. Before changing the value of ***a***, students will graphically estimate the limit of *y*2(*x*) as *x* approaches 1 from the left and the right. Students will also use the table to numerically estimate the value of ***a*** that will ensure that the limit of *y*2(*x*) as *x* approaches one exists. Here the algebraic calculations for the left and right hand limits are to be shown.



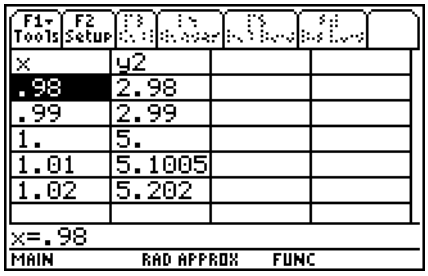
Student Worksheet solutions

$\lim_{x\to 1^-} y2(x) \approx 3;\ \lim_{x\to 1^+} y2(x) \approx 5;\ a \approx 3$

$\lim_{x\to 1^-} y2(x) = 1 + 2 = 3$

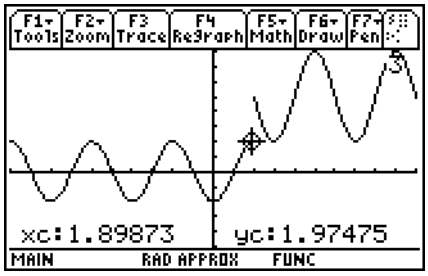
$\lim_{x\to 1^+} y2(x) = a \cdot (1^2) = a$

So $1 + 2 = a \cdot 1^2;\quad a = 3$

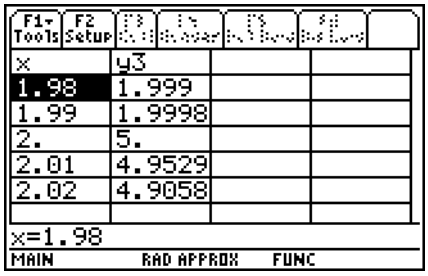


Problem 3

Problems 1and 2 are repeated for a different function. Before changing the value of ***a***, students will graphically estimate the limit of *y3*(*x*) as *x* approaches 2 from the left and the right. Students will also use the table to numerically estimate the value of ***a*** that will ensure that the limit of *y3*(*x*) as *x* approaches two exists. Here the algebraic calculations for the left and right hand limits are to be shown.



Students should view the table near $x = 2$ instead of 1.



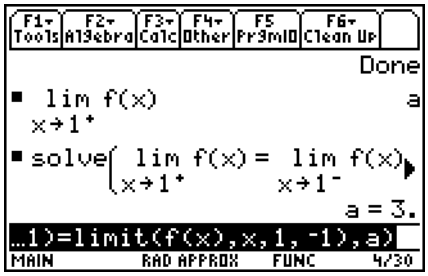
Student Worksheet Solutions

$\lim_{x\to 2^-} f1(x) \approx 2$; $\lim_{x\to 2^+} f1(x) \approx 5$; $a \approx 2$;

$$2\sin\left(\frac{\pi}{2}(2-1)\right) = 3\sin\left(\frac{\pi}{2}(2-4)\right) + a; a = 2$$

Extension – Continuity

Students are introduced to the concept of continuity and are asked to prove the functions in Problems 2 and 3 are continuous. They are also instructed how to use CAS to algebraically solve for ***a*** that makes the limit exist.



Student Worksheet Solutions

Students must show that all three conditions are met in order to satisfy the criteria for continuity.