

Activity 7

On the Rebound

When a ball is dropped on a flat surface, it will rebound to a certain percentage of the original drop height. As the ball continues to bounce up and down, each successive bounce after the first will continue to rebound to approximately this same percentage of the previous bounce's height. You can model this behavior mathematically with an exponential function. Exponential functions are characterized by the property that equal changes in the independent variable produce equal percentage changes in the dependent variable.

Introduction


In this activity, you will collect motion data for a bouncing ball using TI InterActive!™ and a CBL™ or CBL 2™ unit. You will then analyze this data and attempt to find the exponential relationship between the bounce number and the maximum height that the bounce reaches.

Equipment Required

- ♦ Computer
- ♦ TI InterActive! software
- ♦ CBL/CBL 2
- ♦ Motion detector
- ♦ TI-GRAPH LINK™ cable
- ♦ Ball

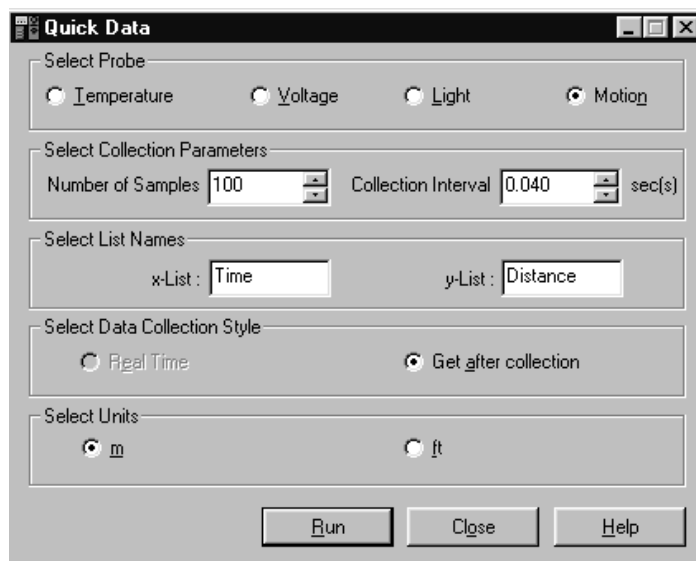
Instructions

1. Plug the TI-GRAPH LINK cable into your computer.
2. Plug the other end of the TI-GRAPH LINK cable into the CBL/CBL 2 unit.
3. If you are using a CBL, plug the motion detector into the **Sonic** port. If you are using a CBL 2, plug the motion detector in to the **DIG/SONIC** port.
4. Start TI InterActive! The software opens to a new, blank document.
5. Title your document *Rebound* and add your name and the date. Click the

Save button  to save and name your document.

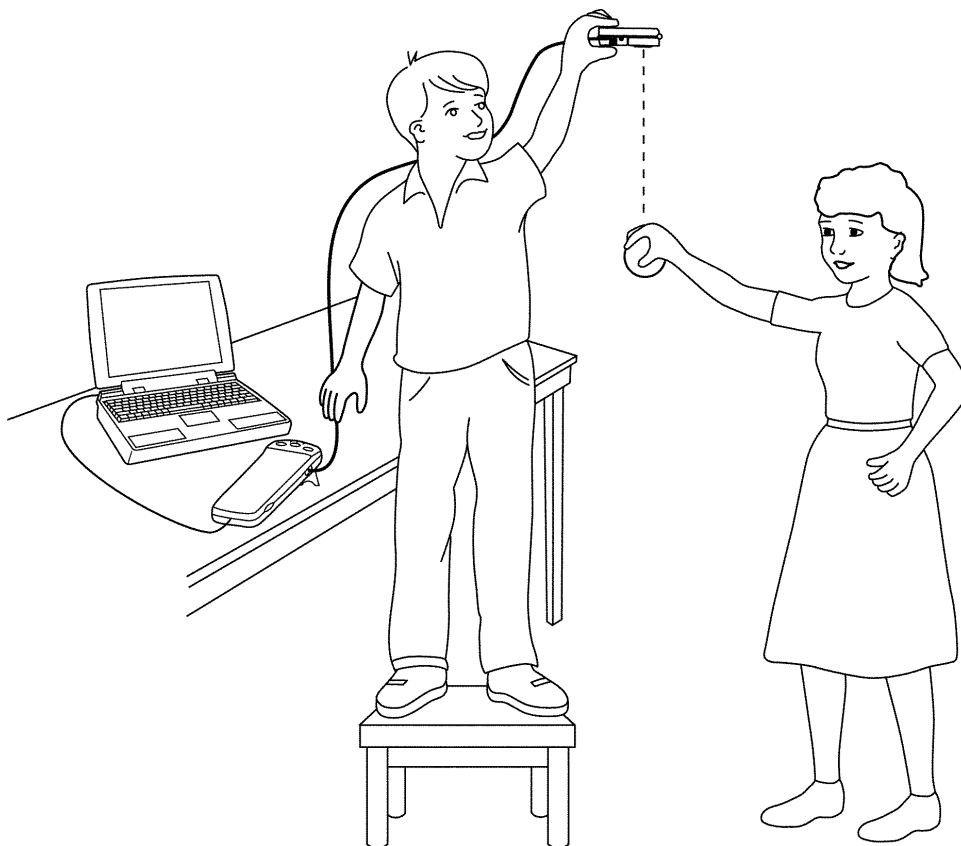
6. Click the List button  to open the Data Editor.


- From the **Data** menu, choose **Quick Data**.
- Adjust the Quick Data dialog box so the settings match the ones shown below.

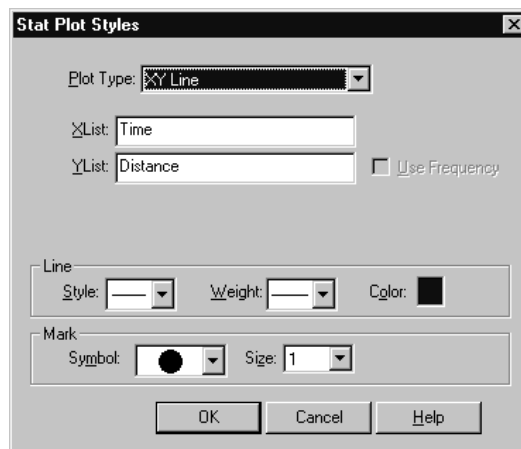



Collecting the Data

- Have a student hold the motion detector approximately 1.5 to 2 meters above the floor and parallel to it, as shown in the illustration below.





- A second student should hold the ball approximately .5 meters below the detector and prepare to release it when the unit begins to click. Be careful that the hands are immediately moved out from under the detector as soon as the ball is released.
- If you are using a CBL™, turn it on by pressing the red **ON/HALT** button.
- Click on the **Run** box on the Quick Data screen to begin data collection.
- When data collection is done, click the Zoom Statistics button  in the Graph window. The viewing boundaries adjust automatically to show all the plotted data.
- If the scatter plot looks confusing, click the Stat Plots tab of the Functions dialog box, then click the box to the right of the check box. This will bring up the Stat Plot Styles dialog box shown below. Change the Plot Type to **XY Line** and click **OK**.







- Your plot should appear to be a number of parabolas opening upward and rising from left to right. If you are satisfied with your data, click on Save to Document  to save your graph in your TI InterActive!™ document. If you are not satisfied, close the Graph window and return to step 2 to collect a new data set.

Recording the Data

- The plot you recorded is a distance versus time graph. Convert the recorded distances to heights by subtracting each value from the distance to the floor. The distance to the floor will be the maximum distance point that you collected.
- Double-click on listname **L1**, enter **time** in the **Formula** box, and click **OK**. Double-click on listname **L2**, enter **max(distance)-distance** in the **Formula** box, and click **OK**. This will convert your distance readings to heights.
- Click the Graph button  to open the Graph window and Functions dialog box. Click the Stat Plots tab to display **L1** and **L2** and click Enter to turn on the plot.
- Click the Zoom Statistics button . The viewing boundaries adjust automatically to show the plot of height versus time.
- Press **Trace** and move along the data plot. Record your initial drop height as bounce zero. Continue to record the maximum heights of each successive bounce in the table on the next page.




Bounce #	Height (m)
0	
1	
2	
3	
4	
5	
6	

6. Close the Trace Value dialog box and click the Save to Document button  to return to the Data Editor.
7. Click the empty cell at the top of list **L3**. Type the bounce number and then press the down-arrow key on the keyboard or press Enter to move to the next cell. Continue until you have entered all of the bounce numbers from the table above into **L3**.
8. Click the empty cell at the top of list **L4**. Type the corresponding height for the bounce in list **L3** and then press the down-arrow key or press Enter to move to the next cell. Continue until you have entered all of the height readings from the table above into **L4**.
9. Click the Graph button , and then click the Stat Plots tab. In the uppermost text box, type **L3** to specify it as the list containing the x -coordinates. Press the Tab key and move to the second text box. Type **L4** to specify the list containing the y -coordinates.
10. Press Enter, and then click the Zoom Statistics button . The viewing boundaries are adjusted automatically to show your plot of bounce number versus height.
11. Your bounce number versus height plot should appear to be a graph of an exponential decay. Click on Save to Document  to copy this plot to your TI InterActive! document.

Analysis and Questions

1. The theoretical model for the bouncing ball data recorded on the axes above is exponential. You will attempt to fit this data set with a curve of the form: $y = AB^x$ where A is the initial height of the ball and B is the ratio of the successive maximum heights of each bounce. Use the information you recorded in the table above to find the initial height:

$A =$

2. Click Save to Document  to return to the Data Editor. Highlight the values you entered in L3 and L4, click Copy , and then click on the spreadsheet tab at the bottom of the Data Editor. The cursor will be in cell A1. Click Paste . This pastes the contents of lists 3 and 4 into columns A and B.

The variable B in the equation $y = AB^x$ represents the ratio of the maximum height of each bounce in relation to the preceding bounce. To find the ratio of the first bounce to the original drop height, move to cell C1, type $=B2/B1$, then press Enter. To find the remaining ratios, click on cell C1. Move the cursor to the lower right hand corner of the cell, then click and drag until all cells adjacent to the filled cells in column B except for the last are selected. Release the click and all the ratios will be calculated. Record these ratios in the table below.

3. Move the cursor to an empty cell and find the average of these ratios. Type $=\text{average}(c1,cx)$ where x is the number of the last cell containing a ratio, and press Enter. Record the average in the space below.

$$B =$$

4. Use the values you found for A and B to write an equation modeling bounce height as a function of bounce number in the form $y = AB^x$.

$$y =$$

5. Close the Data Editor and double-click on the graph of bounce number versus height in the TI InterActive! document to refresh the Graph window. Click the $f(x)$ tab in the Functions dialog box and enter the formula that you found in step 5 above. Press Enter to turn on the function. How well does your formula fit the data?

6. Use your model to predict the height of the ball on its 8th bounce.
7. According to your model, how many bounces will take place before the ball no longer reaches a maximum height of 5 centimeters?
8. Save and print your TI InterActive!™ document.

Extensions

- ◆ Find the total distance the ball would travel over the course of 8 bounces.
- ◆ Have TI InterActive! find a model for this activity by performing an exponential regression on the data. How does this model compare to the one you found in this activity?
- ◆ Repeat this activity using different balls, drop heights, and surfaces, and observe how each affects the values of A and B in your model.
- ◆ Connect the TI-GRAPH LINK™ cable to your calculator and use TI InterActive! to send the data you collected from the computer to the calculator. Analyze the data on your calculator using the same methods applied in this activity.

Teacher Notes

Activity 7: On the Rebound



Math Concepts

- ◆ CBL™/CBL 2™
- ◆ Spreadsheets
- ◆ List Manipulation
- ◆ Ratios
- ◆ Exponential Function

Activity Notes

- ◆ If there is not enough room near the computer to collect this data, it can be collected using the calculator with a CBL or CBL 2. The data can then be transferred to a TI InterActive!™ document.
- ◆ Caution students to be sure that nothing blocks the path between the motion detector and the ball. Hands and cords are common obstructions.

Sample Data

Note: Due to the large number of data points collected, the sample data contains only the values of the maximum heights of the bounces recorded in the activity.

Bounce #	Height (m)
0	.840
1	.683
2	.535
3	.418
4	.340
5	.266
6	.209

Analysis and Questions - Key

1. The initial height of the ball was .84 meters.

2.

Cell C
0.813
0.783
0.781
0.813
0.782
0.785

3. The average of the ratios in the table is .799.
4. The equation is $y = .84(.799)^x$.

5. The model is an almost perfect fit.
6. The predicted height on the 8th bounce is .14 meters.
7. On the 13th bounce, the ball will fail to reach .05 meters.