EXPLORATIONS

Chapter 11

Confidence Intervals

Topic 22 covers confidence intervals for a proportion and includes a simulation. Topic 23 addresses confidence intervals for a mean. Differences between two proportions are covered in Topic 24. The confidence interval for the difference between two means is discussed in Topic 25.

It is important that you understand the following two examples using the standard normal probability distribution (the z distribution).

Example 1: Find the z value that separates the top 0.025 of the area under the distribution from the bottom 0.975. This is called the *critical value* and designated as $z_{.025}$ or $z_{a/2}$. (In this case $a = 0.05$.)

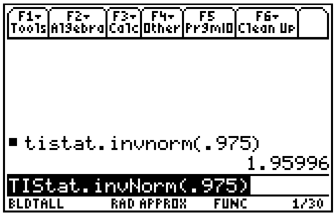
From the Stats/List Editor:

1. Press [F5] **Distr**, **2:Inverse**, and then choose **1:Inverse Normal** with Area: **.975**, $\mu = 0$ and $\sigma = 1$. Observe the result of **1.95996**.

From the Home screen:

2. Press [CATALOG], [F3] **Flash Apps**. Press **I** and use ⊝ to highlight **invNorm**.
3. Press [ENTER] to complete the input.
4. Type **.975)** and press [ENTER] for **tistat.invnorm(.975) = 1.95996** (screen 1).

(1)



Example 2: Find the area in the tail of the distribution using **NormalCdf**, both from the Stats/List Editor and the Home screen (Topic 18).

1. From the Stats/List Editor, press [F5] **Distr, 4:NormalCdf** with Lower Value: **1.95996**, Upper Value: ∞, μ = **0**, and σ = **1**.
2. Press [ENTER] for the result of .025.

From the Home screen:

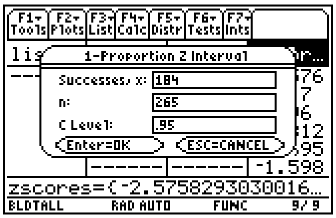
1. Press [CATALOG], [F3] **Flash Apps** and **normcdf(**.
2. Type **1.95996, ∞, 0, 1)** to complete the entry.
3. Press [ENTER] to observe the result of .025000162887.

Topic 22—Large Sample Confidence Interval for a Proportion and Simulation to Clarify the Meaning of a Confidence Interval

Example: In a simple random sample of 265 people drawn from a population of interest, 69.4% of them agreed with a particular public policy question. What is the 95% confidence interval for the proportion in the entire population who would agree?

Note: 265 ∗ .694 = 184 ≥ 10 *and* 265 ∗ (1 - .694) = 265 - 184 = 81 ≥ 10, *so a normal distribution can be used to approximate a binomial, as discussed in Topic 19.*

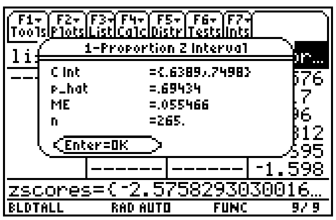
1. In the Stats/List Editor, press [2nd] [F7] **Ints**, **5:1-PropZInt**, with Successes, x: **184**, n: **265**, and C Level: **.95** (screen 2).



(2)

Note: *If for successes* 265 ∗ .694 *were used, a Domain error would result since* 265 ∗ .694 = 183.91, *which is not an integer. Entries must be rounded to the nearest integer.*

2. Press [ENTER] to view screen 3 with the 95% C Int: **{.6389, .7498}** or 63.9% ≤ p ≤ 75.0%, or 0.694 ± .0555, with p-hat: **0.69434**, and the margin of error ME: **0.055466**. You are 95% confident that between 64% and 75% of the population agree with the public policy question.



(3)

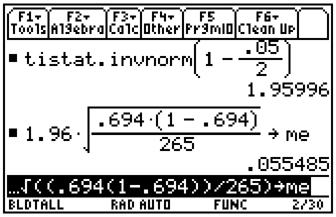
As you complete this topic and subsequent ones, there will be various Home screen calculations at appropriate points. Enter these calculations from the Home screen to verify the results.

Note: 184 ÷ 265 = .69434 = p-hat ≈ 69.4% *in the original problem.*

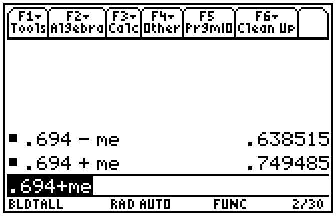
Home screen calculation:

$$\text{ME} = z_{a/2} * \sqrt{\frac{\hat{p}*(1-\hat{p})}{n}} = 1.96\sqrt{\frac{.694(1-.694)}{265}} = 0.055485$$

as in screen 4, with the lower and upper values calculated as in screen 5 (.694 ± ME).



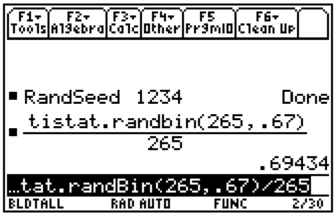
(4)



(5)

Simulation to Clarify the Meaning of a Confidence Interval

What does it mean to be 80% confident that an interval contains the real population value? The $\hat{p}$ in the previous example was randomly generated from a binomial distribution with n = **265** and p = **.67**, (screen 6), so the confidence interval of 0.6389 to 0.7498 did, in fact, contain the population proportion of 0.67. However, you should take a sample to estimate the population proportion because you do not know it. How confident can you be of your answer?



(6)

From the Home screen:

1. Set **RandSeed 23** (screen 7).
2. Enter the following functions, being sure to place the colons (:) between the functions:

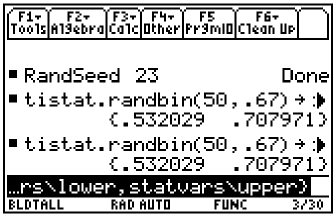
   tistat.randbin(50,.67)→x:tistat.zInt_1p(x,50,.80): {statvar\lower, statvars\upper}

   This is all on one line, only part of which is displayed in screen 7.

   a. **tistat.randbin(50,.67)→x** generates the number of Successes, x: for a sample of **50** people.

   b. **tistat.zInt_1p(x,50,.80)** calculates the 80% confidence interval.

   c. **{statvars\lower, statvars\upper}** displays the results.
3. Press ENTER to display the results **{.532029, .707971}** in the middle of screen 7.

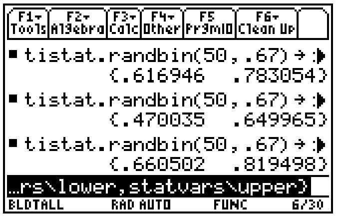


(7)

4. Press [ENTER] four more times to view screens 7 and 8.

   Notice that four of the five intervals (80%) contain the population value 0.67. Only the second interval on screen 8 (.470 to 0.650) does not. You should not expect to get exactly four out of each five intervals for every five intervals generated, but you could be confident that, on average, about 80 out of 100 would. From Topic 19 you know the distribution of proportions, $\hat{p}$ has mean = p and standard deviation $\sqrt{\dfrac{p(1-p)}{n}}$, so the ME = $z_{a/2}\sqrt{\dfrac{p(1-p)}{n}}$ is long enough to reach the mean p if $\hat{p}$ is not in the tails of the distribution. In your example this occurs 80% of the time. Investigate further by trying different confidence levels, sample sizes, and population p's.

(8)



Topic 23—Large Sample Confidence Interval for a Mean

Example: A random sample size of 30 people in the population has heights given below. Enter the following data from the Home screen.

{63.0, 63.6, 66.3, 67.9, 69.3, 66.0, 68.7, 64.2, 66.9, 66.7, 65.3, 62.5, 67.7, 63.9, 65.6, 62.0, 61.0, 65.5, 65.8, 66.9, 61.8, 62.2, 62.6, 66.0, 63.6, 65.5, 64.9, 63.7, 69.4, 64.8} → list1.

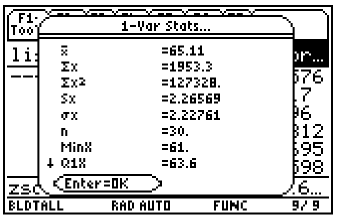
What is the 90% confidence interval for the population mean?

From the Central Limit Theorem, a random sample of size ≥ 30 is considered sufficiently large for estimating means (not proportions) and assures that the distribution of sample means is approximately normally distributed. A sample size of 30 is also large enough to be able to replace σ with s_x.

1. From the Stats/List Editor, press [F4] **Calc**, **1:1-Var Stats**. Use List: **list1**, Freq: **1**, and clear the Category List: and Include Categories: fields.

2. Press [ENTER] to display screen 1 with $\bar{x}$ = **65.11**, s_x = **2.26569**, and n = **30**.

(9)



In the steps that follow, you do two techniques side by side. The first technique is used if σ is known. You will assume it is known, that σ = **2.27**, and calculate a z Interval. The second technique assumes you do not know σ, s_x = **2.26567** ≈ **2.27** is used, and calculate a t Interval.

Stats Input Method

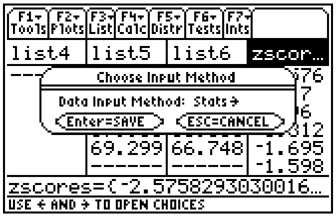
Using the **Stats** variable in the Data Input Method field:

1. Calculate a z Interval:

   a. Assume σ = **2.27** and press [2nd] [F7] **Ints**, **1:ZInterval**, and choose Data Input Method: **Stats** (screen 10).

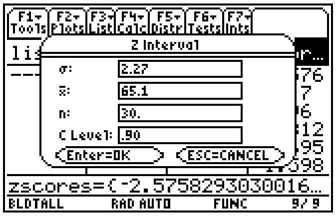
   Or calculate a t Interval:

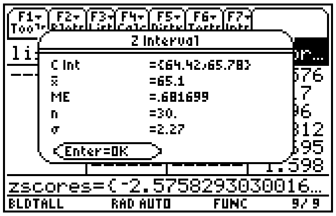
   b. If σ is unknown, press [2nd] [F7] **Ints**, **2:TInterval**, and choose Data Input Method: **Stats** for a screen similar to screen 10.

(10)


2. Calculate a z Interval:

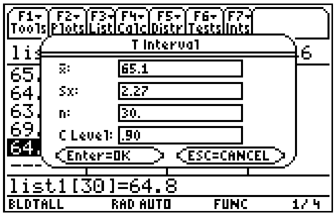
   a. Assume σ = **2.27** and press [ENTER], with inputs σ = **2.27**, $\bar{x}$ = **65.1**, n: **30**, and C Level: **0.90** (screens 11 and 12).

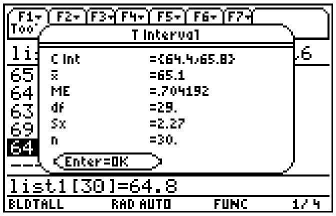
(11)


(12)


   Or calculate a t Interval:

   b. If σ is unknown, press [ENTER], with inputs $\bar{x}$: **65.1**, s_x: **2.27**, n: **30**, and C Level: **0.90** (screens 13 and 14).

(13)


(14)


Both techniques give basically the same results if s_x replaces σ ($s_x = \sigma$) and $n \geq$ **30** as you see in the Home screen calculations in the *Data Input Method* section.

You are 90% confident that the mean population height is between 64.4 and 65.8 inches, or $64.4 \le \mu \le 65.8$, or 65.1 ± 0.7 inches, with $\bar{x}$ = **65.1** and the margin of error ME ≈ **0.7**.

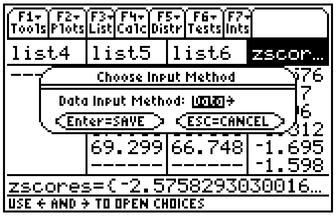
Data Input Method

Using the **Data** variable in the Data Input Method field:

1. Calculate a z Interval:

   a. Assume σ is unknown and s_x = **2.26567** ≈ **2.27**.

   Press [2nd] [F7] **Ints**, **1:ZInterval**, and choose Data Input Method: **Data** (screen 15).

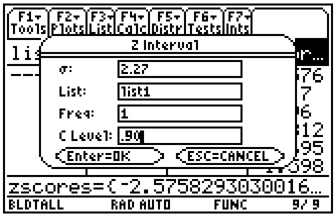
   

   (15)

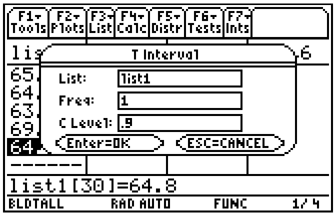
   Or calculate a t interval:

   b. If σ is not known, press [2nd] [F7] **Ints**, **2:Tinterval** and choose Data Input Method: **Data**.

   c. Choose List: **list1**, Freq: **1**, and C Level: **0.90** (screen 16). Screen 16 needs σ, but screen 17 does *not* need s_x.

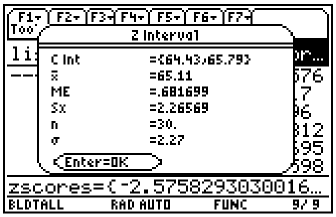
   

   (16)

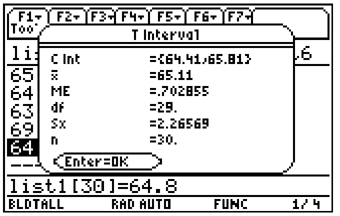
   (17)

2. Press [ENTER] to display screens 18 or 19, which differs from screens 12 and 14 respectively only in that s_x was calculated from the data.



(18)

The *TInterval technique* is easier to use in the case where σ is unknown, even for large samples, because you do not need an input for σ. The *ZInterval technique* for large samples with unknown σ requires that you first calculate s_x and input s_x for σ.

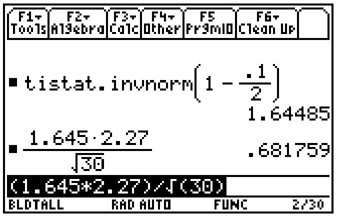


(19)

Home screen calculations:

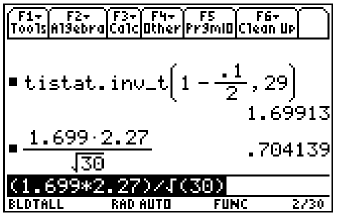
a. $\text{ME} = z_{a/2} * \dfrac{s}{\sqrt{n}} = \dfrac{1.645 * 2.27}{\sqrt{30}}$ =0.68 ≈ 0.7 as in screen 18 and in screen 20.

b. $\text{ME} = t_{a/2} * \dfrac{s_x}{\sqrt{n}} = \dfrac{1.699 * 2.27}{\sqrt{30}}$ = 0.70 as in screen 19 and in screen 21.



(20)

As the sample size gets larger, $z_{a/2}$ and $t_{a/2}$ become closer (see Topic 31). Therefore, when you replace σ with s_x, these techniques are basically the same.



(21)

Small Sample Confidence Intervals for the Mean

Small sample confidence intervals for the mean are calculated the same as in previous examples if the samples are from populations that are normally distributed. Use the *ZInterval technique* if σ is known and the *TInterval technique* if σ is unknown, but s_x can be calculated. Do not use a ZInterval by approximating σ with s_x for small samples. (See Topic 31.)

Topic 24—Large Sample Confidence Interval for the Difference Between Two Proportions

Example: A polling organization found that in a simple random sample of 936 women, 694 (694/936 = 74.1%) agreed with a particular public policy question. In a simple random sample of 941 men, 645 (68.5%) agreed with the same public policy question. Find the 95% confidence interval for the difference in proportions between the two populations sampled.

Note: *n_1 = 936 = 694 + 242 (both values > 5), n_2 = 941 = 645 + 296 (both values > 5), therefore a normal distribution can be used to approximate the binomials.*

In this example, the values in x1 and n1 represent women, and x2 and n2 represent men.

1. In the Stats/List Editor, press [2nd] [F7] **Ints**, **6:2-PropZInt**.
2. Enter these values and press [ENTER]:

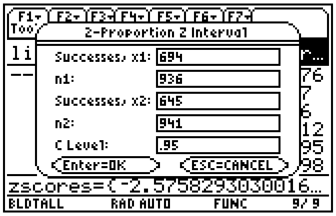
   Successes, x1: **694**

   n1: **936**

   Successes, x2: **645**

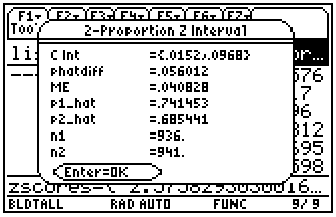
   n2: **941**

   C Level: **0.95** (screen 22).



(22)

3. Press [ENTER] to display screen 23 with the 95% confidence interval of (.0152 to 0.0968), or $1.5\% \le p_1 - p_2 \le 9.7\%$, or $(\hat{p}_1 - \hat{p}_2) \pm \text{ME} = 5.6\% \pm 4.1\%$.



(23)

You are 95% confident that the difference in population proportions lies between 1.5 and 9.7 percent. Because zero is not in the interval, this indicates that a significantly larger proportion of women are in agreement with the policy question than are men.

Home screen calculation:

$z_{\alpha/2}$ = tistat.invnorm(1-.05/2) = 1.95996 is used for the margin of error.

$$ME = 1.96\sqrt{\frac{.7415*(1-.7415)}{936} + \frac{.6854*(1-.6854)}{941}} = .0408$$

Topic 25—Large Sample Confidence Interval for the Difference Between Two Means (Unpaired and Paired)

Unpaired or Independent Samples

Example: A study designed to estimate the difference in the mean test scores that result from using two different teaching methods for a block of material obtained the following data from two random samples of students. Find the 90% confidence interval for the differences of the mean test scores for the two teaching methods.

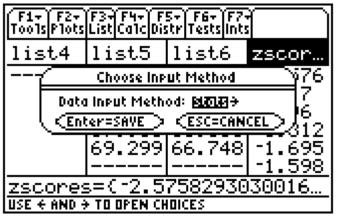
	Mean	stdDev	*n*
Method A	75.2	8.42	32
Method B	78.4	8.13	30

Since n_1 and n_2 are both ≥ 30, substitute stdDevA and stdDevB for σ_1 and σ_2, and the Central Limit Theorem assures you that the distribution of differences of sample means is normally distributed.

Note: *Modified boxplots of the original data showed no outliers.*

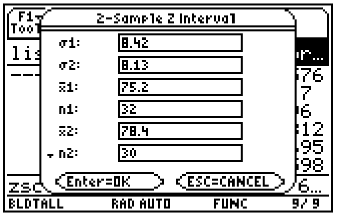
For these large samples if stdDevA = S_A *instead of* σ_A *and* stdDevB = S_B *instead of* σ_B*, then the 2-SampZInt procedure that follows gives approximately the same answer as the 2-SampTInt procedure of Topic 32 (for the reason explained in Topic 23).*

1. From the Stats/List Editor, press [2nd] [F7] **Ints**, **3:2-SampZInt**, with Data Input Method: **Stats** (screen 24).



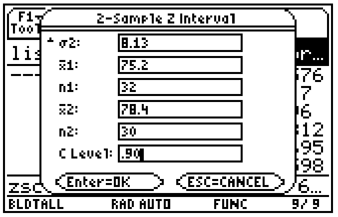
(24)

2. Press [ENTER] to display screen 25, which gives the input with **8.42** substituted for σ_1 and **8.13** substituted for σ_2. Enter **75.2** for $\bar{x}_1$, **32** for n_1, **78.4** for $\bar{x}_2$ and **30** for n_2.



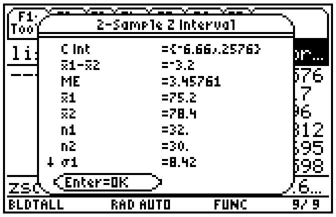
(25)

3. Press [↓] to get to the last line shown in screen 26.
4. Enter **0.90** in the C Level: field for a 90% confidence interval.



(26)

5. Press [ENTER] to display screen 27 with the 90% confidence interval from -6.66 to 0.25763, or $-6.66 < \mu_1 - \mu_2 < 0.26$, or -3.2 ± 3.46.



(27)

Home screen calculation:

There is a difference of 3.2 points in the mean scores of the two methods (or 75.2 – 78.4 = -3.2) with a margin of error

$$1.645 * \sqrt{\frac{8.42^2}{32} + \frac{8.13^2}{30}} = 3.46 \text{ when invNorm}(1 - 0.10/2) = 1.645.$$

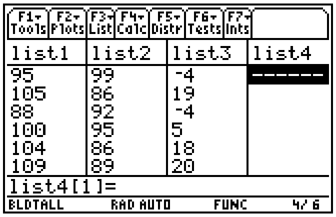
Conclusion

Because zero is in the interval, the observed difference between the sample means is not statistically significant. Method A could have a higher population mean than Method B (a positive difference), or Method B could have a higher population mean than Method A (a negative difference.)

Matched Paired Samples

Example: To test blood pressure medication, the diastolic blood pressure readings for a random sample of 30 people with high blood pressure were recorded. After a few weeks on the medication, their pressures were recorded again. The data is recorded in the table and stored in **list1** and **list2** and the differences are stored in **list 3** (screen 28).

Calculate the 90% confidence interval for the difference in blood pressure after taking the medication.



(28)

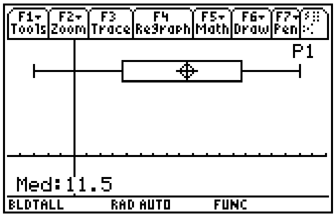
Note: *To continue, you can save time by just entering the differences in* **list3**.

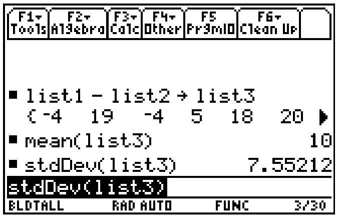
Subject	Before (list1)	After (list2)	(list1-list2)→list3
1	95	99	-4
2	105	86	19
3	88	92	-4
4	100	95	5
5	104	86	18
6	109	89	20
7	89	87	2
8	93	85	8
9	100	87	13
10	97	86	11
11	87	88	-1
12	104	90	14
13	106	89	17
14	103	91	12
15	105	87	18
16	101	93	8
17	108	95	13
18	107	90	17
19	105	90	15
20	92	95	-3
21	106	97	9
22	103	90	13
23	97	97	0
24	103	95	8
25	92	80	12
26	108	85	23
27	102	98	4
28	100	91	9
29	106	88	18
30	96	90	6
$\bar{x}$	100.4	90.4	10
s_x	6.35	4.51	7.55

1. Set up and define **Plot 1** as a modified boxplot with Mark: **Square**, x: **list3**, and then press [F5] **ZoomData**.

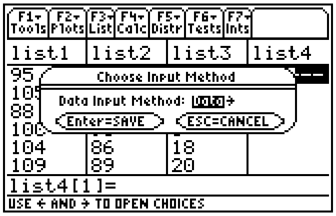
2. Press [F3] **Trace** (screen 29). The boxplot shows there are no obvious outliers, so with sample size $n = 30 \geq 30$ you can assume that the distribution of the mean of the differences $\bar{d}$ ($\bar{d} = 10$ for this sample) are normally distributed). The sample size is large enough that you can replace σ_d with $s_d = 7.55$, as justified in Topic 23. (See Topic 32 for the t Interval procedure.)

   From the Home screen, enter **mean(list3)** and **stdDev(list3)** to observe the values in screen 30. Notice that the mean blood pressure before is greater than the mean pressure after the medication is taken.

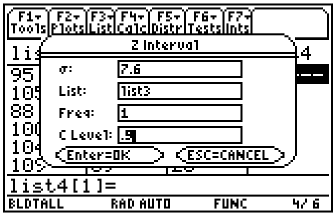
(29)


(30)


3. In the Stats/List Editor, press [2nd] [F7] **Ints**, **1:ZInterval**, and change to Data Input Method: **Data** (screen 31).

(31)


4. Press [ENTER] to display screen 32.

(32)


5. Enter σ: **7.6** (from **stdDev(list3)**), List: **list3**, Freq: **1**, C Level: **.90**, and then press [ENTER] (screen 33).

You are 90% confident of a 7.7 to 12.3 point drop in the blood pressure, or $7.7 < \mu_d < 12.3$, or 10 ± 2.28, or

$$10 \pm 1.645 \frac{\sigma_d}{\sqrt{n}} = 10 \pm 1.645 * \frac{7.6}{\sqrt{30}} = 10 \pm 2.3$$

(33)