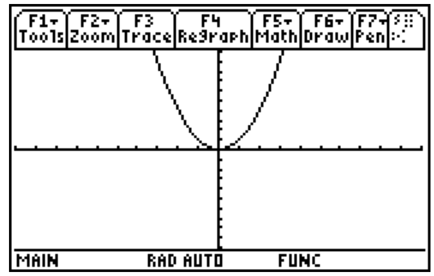
Name ______________________

Class ______________________

Problem 1 – Explore and discover



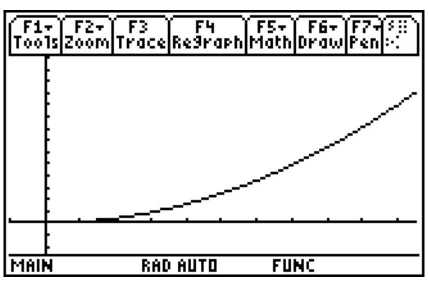
The graph at the right is the curve $y = x^2$.

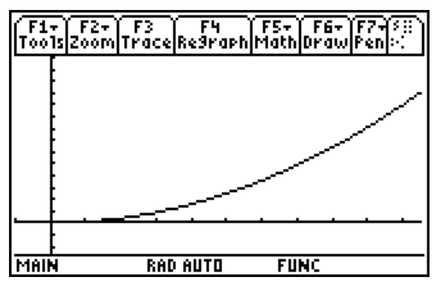
Your challenge is to think of *at least* two ways to estimate the area bounded by the curve $y = x^2$ and the x-axis on the interval [0, 1] using rectangles. Use the following guidelines:

- all rectangles must have the same width
- you must build all your rectangles using the same methods
- the base of each rectangle must lie on the x-axis

Graph $y = x^2$ and set your window to [–0.1, 1] for x and [–0.2, 1.3] for y. Draw your first and second method on the graphs below. For each method calculate the following:

- Number of rectangles
- Height and width of each one
- Area of each
- Sum of the area





- Which method did a better job?

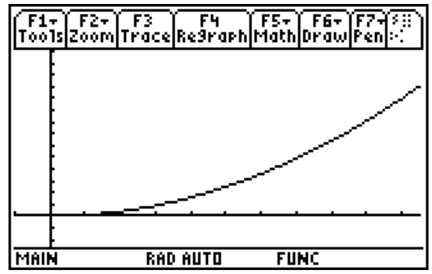
- How could you improve on it?

In the following problem, you will examine three common techniques that use rectangles to find the approximate area under a curve. Perhaps you discovered some of these techniques during your exploration in the above problem. The first problem uses rectangles whose right-endpoints lie on the curve $y = x^2$.

Problem 2 – Using five right-endpoint rectangles

Divide the interval [0, 1] into five equal pieces. Enter the information for each interval or rectangle in the table below. Remember that the right endpoint is the *x*-value and the height is the *y*-value of the right endpoint on the curve.

Interval	Right Endpoint	Height	Area



The formula that can be used to express the total area is:

$R_5 = 0.2 \cdot f1(0.2) + 0.2 \cdot f1(0.4) + 0.2 \cdot f1(0.6) + 0.2 \cdot f1(0.8) + 0.2 \cdot f1(1.0)$

or

$R_5 = 0.2[f1(0.2) + f1(0.4) + f1(0.6) + f1(0.8) + f1(1.0)]$

- Calculate this sum.

- Now add up the numbers in the Area column.

- Are these two numbers the same or different?

Another way to find the area of the rectangles is using sigma notation.

- Write the notation in the $\sum_{x=1}^{5} x^2$ form. Adjust what is being summed.

To sum it on the calculator, use **Home > F3:Calc > 4:Σ(sum** for the command with the format:

$\sum(\text{expression}, \text{variable}, \text{ lower limit}, \text{ upper limit})$

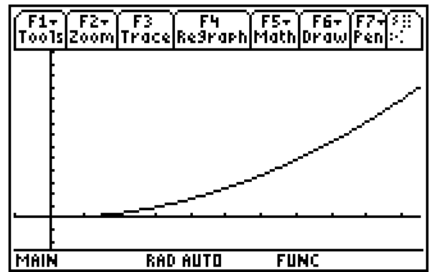
- Does this agree with the answer for the area you found previously?

Problem 3 – Using five left-endpoint rectangles

This problem uses rectangles whose left-endpoints lie on the curve $y = x^2$.

Divide the interval [0, 1] into five equal pieces. Enter the information for each interval in the table below. Remember that the left endpoint is the x-value and the height is the y-value of the left endpoint on the curve.

Interval	Left Endpoint	Height	Area



The formula that can be used to express this area is:

$L_5 = 0.2 \cdot f1(0) + 0.2 \cdot f1(0.2) + 0.2 \cdot f1(0.4) + 0.2 \cdot f1(0.6) + 0.2 \cdot f1(0.8)$ or

$L_5 = 0.2[f1(0) + f1(0.2) + f1(0.4) + f1(0.6) + f1(0.8)]$

- Calculate this sum.

- Now add up the numbers in the Area column.

- Are these two numbers the same or different?

- What is the sigma notation for the area of the rectangles?

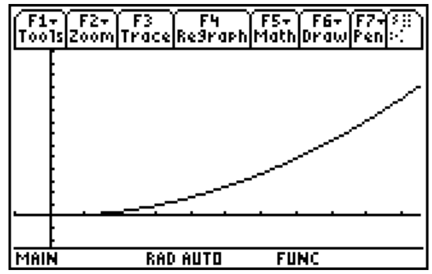
- Use the calculator to find the sum. Does this result agree with the answer above?

Problem 4 – Using five midpoint rectangles

We will now investigate a *midpoint* approximation. How would you draw five rectangles, with equal width, such that their midpoints lie on the curve $y = x^2$?

Divide the interval [0, 1] into five equal pieces. Enter the information for each interval or rectangle in the table below. Remember that the midpoint is the x-value and the height is the y-value of the midpoint on the curve.

Interval	Midpoint	Height	Area



The formula that can be used to express this area is:

$M_5 = 0.2 \cdot f1(0.1) + 0.2 \cdot f1(0.3) + 0.2 \cdot f1(0.5) + 0.2 \cdot f1(0.7) + 0.2 \cdot f1(0.9)$ or

$M_5 = 0.2[f1(0.1) + f1(0.3) + f1(0.5) + f1(0.7) + f1(0.9)]$

- Calculate this sum.

- Now add up the numbers in the Area column.

- Are these two numbers the same or different?

- What is the sigma notation for the area of the rectangles?

- Use the calculator to find the sum. Does this result agree with the answer above?

Problem 5- Summarize your findings

In this activity, you explored three different methods for approximating the area under a curve. The exact area under the curve $y = x^2$ on the interval [0, 1] is $\frac{1}{3}$ or 0.333.

- Which approximation produced the best estimate for the actual area under the curve?

- Describe which factors contribute to left, right, and midpoint rectangles giving *overestimates* versus *underestimates*.

- What can you do to ensure that all three of these techniques produce an answer that is very close to $\frac{1}{3}$? Test your conjecture by using evaluating a sum that produces a much more accurate answer.