

ACTIVITY 2

Flipping a Penny

Two functions are *inverses* if the inputs and outputs of one function are *reversed* for the second function. As an example, suppose 2 is added to 3 to obtain 5 ($2 + 3 = 5$). To “reverse” this answer and obtain the original value of 3, 2 is subtracted from 5 ($5 - 2 = 3$). Thus, adding 2 and subtracting 2 are inverses of each other. This means that an inverse will “reverse” an operation and the original number will be obtained.

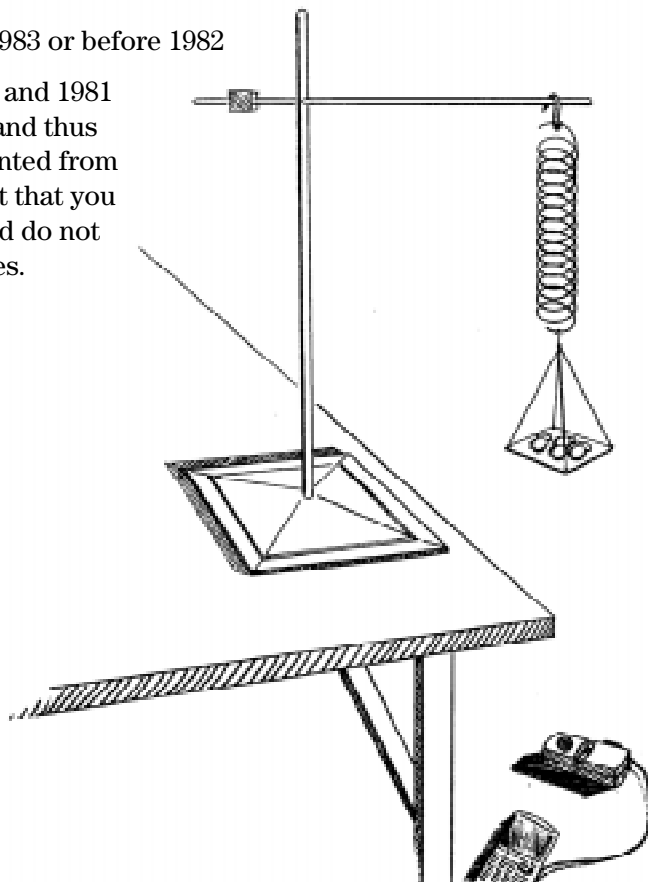
In this activity, you will explore two functions which are inverses of each other. You will explore their characteristics and learn how they “reverse” each other’s operation.

You’ll Need

- ◆ 1 CBR unit
- ◆ 1 TI-83 or TI-82 Graphing Calculator
- ◆ *BIG* handful of pennies dated after 1983 or before 1982

Note: Pennies minted between 1959 and 1981 have a higher percentage of copper and thus have a greater mass than pennies minted from 1983 to the present, so it is important that you sort the pennies before you begin and do not mix the two different types of pennies.

- ◆ Spring or slinky
- ◆ Paper bowl or plate
- ◆ Ring stand or hook



Instructions

1. Attach the paper bowl or plate to the spring. Hang the spring from the ceiling or a ring stand.
2. Position the CBR face up under the plate .
3. Run the **RANGER** program on your calculator.
4. Enter the setup instructions.
 - a. From the **MAIN MENU** select **1:SETUP/SAMPLE** to access the setup menu.
 - b. Press **[ENTER]** until the **REALTIME** option reads **no**.
 - c. Press **[↓]** (the down arrow) to select the next line **TIME (S)** and press **[ENTER]** **4** **[ENTER]** to change the time to 4 seconds.
 - d. Press **[↓]** to select the next line. Correct or verify the settings and press **[ENTER]**. Repeat until the options for each line read as shown at right.
 - e. Press **[↓]** to move the cursor to the **START NOW** command. Press **[ENTER]** and follow the directions on the calculator screen.

MAIN MENU	▶START NOW
REALTIME:	NO
TIME(S):	4
DISPLAY:	DIST
BEGIN ON:	[ENTER]
SMOOTHING:	LIGHT
UNITS:	METERS

5. The graph should be a horizontal line. If you are not satisfied with your results, press **[ENTER]** and select **5:REPEAT SAMPLE**. Trace along the graph to approximate the distance between the plate and the CBR. Record this distance in the table below.
6. Add 5 pennies to the plate. Press **[ENTER]** and select **5:REPEAT SAMPLE**.
7. Repeat step 6 until a total of 20 pennies have been added to the plate.
8. Press **[ENTER]** and select **7:QUIT** to exit the **RANGER** program.

Data Collection

1. Convert the distance to the plate from meters to centimeters and record both in the table.

Number of Pennies	Distance to the Plate (meters)	Distance to the Plate (centimeters)
0		
5		
10		
15		
20		

2. Enter the collected data into lists 1 and 2 by first clearing the lists. To do this, press **[STAT]** then select **4:ClrList** and type **[2nd] [L1] [,] [2nd] [L2] [ENTER]**. Press **[STAT]** and select **1:Edit** and enter the number of pennies collected in each trial in **L1**. Then enter the distance to the plate in centimeters in **L2**.

Questions

1. To set up a scatter plot, press $\boxed{2\text{nd}}$ **[STAT PLOT]** and select **1:Plot1**. Highlight **On** and press $\boxed{\text{ENTER}}$. Select $\boxed{\wedge}$ for the **Type** of plot, **L1** for the **Xlist**, **L2** for the **Ylist**, and the **square** for the **Mark**. Press $\boxed{\text{ZOOM}}$ and select **9:ZoomStat**. Sketch the scatter plot in the space provided.



2. Find the linear regression of the line.

For the TI-83: Press $\boxed{\text{STAT}}$ \blacktriangleright and select **4:LinReg (ax+b)**. Press $\boxed{2\text{nd}}$ $\boxed{[\text{L1}]}$ $\boxed{,}$ $\boxed{2\text{nd}}$ $\boxed{[\text{L2}]}$ $\boxed{\text{ENTER}}$.

For the TI-82: Press $\boxed{\text{STAT}}$ \blacktriangleright and select **5:LinReg (ax+b)**. Press $\boxed{2\text{nd}}$ $\boxed{[\text{L1}]}$ $\boxed{,}$ $\boxed{2\text{nd}}$ $\boxed{[\text{L2}]}$ $\boxed{\text{ENTER}}$.

Record the equation.

$y =$ _____

3. Identify the slope of the line. Put into words the meaning of the slope of this line.

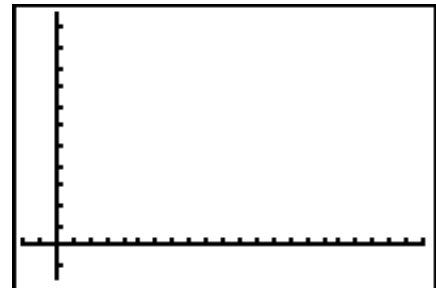
slope = _____

4. Identify the y -intercept of the line and explain its meaning in words.

y -intercept = _____

5. Press $\boxed{\text{Y=}}$ and enter the equation in **Y1**. Press $\boxed{\text{GRAPH}}$. How well does the graph of the line model the data?

6. To plot the number of pennies versus the distance to the plate from the original collected data, press $\boxed{2\text{nd}}$ **[STAT PLOT]** and select **2:Plot2**. Highlight **On** and press $\boxed{\text{ENTER}}$. Select $\boxed{\wedge}$ for the **Type** of plot, **L2** for the **Xlist**, **L1** for the **Ylist**, and the **square** for the **Mark**. Press $\boxed{\text{ZOOM}}$ and select **9:ZoomStat**. Sketch the scatter plot in the space provided.



7. Find the linear regression of the line.

For the TI-83: Press **[STAT]** **[▶]** and select **4:LinReg (ax+b)**. Press **[2nd]** **[L2]** **[,]** **[2nd]** **[L1]** **[ENTER]**.

For the TI-82: Press **[STAT]** **[▶]** and select **5:LinReg (ax+b)**. Press **[2nd]** **[L2]** **[,]** **[2nd]** **[L1]** **[ENTER]**.

Record the equation:

$$y = \underline{\hspace{4cm}}$$

This is the inverse of the equation found in question 3 since the values of the independent and dependent variables have been switched.

8. Identify the slope of the line. Put into words the meaning of the slope of this line.

$$\text{slope} = \underline{\hspace{4cm}}$$

9. Identify the y -intercept of the line and explain its meaning in words.

$$y\text{-intercept} = \underline{\hspace{4cm}}$$

10. Press **[Y=]** and enter the equation in **Y2**. Press **[GRAPH]** and compare the two graphs.

11. In the table below, find the value of **Y1** for the indicated number of pennies which would be the distance between the CBR and the plate for that number of pennies. Then evaluate **Y2** at those distances.

P Number of Pennies	Y1 (P)	Y2 (Ans)
3		
9		
18		
75		

12. Describe the pattern that you find when you examine the table from question 11.

13. This pattern is true for any function and its inverse. To verify this relationship for all values of x for this function and its inverse, press $\boxed{Y=}$ and enter $Y1(Y2)$ in $Y3$.

For the TI-83: Press $\boxed{\text{VAR}} \blacktriangleright$ 1:FUNCTION 1:Y1 $\boxed{}$ $\boxed{\text{VAR}} \blacktriangleright$ 1:FUNCTION 2:Y2 $\boxed{}$.
Press $\boxed{\text{GRAPH}}$.

For the TI-82: Press $\boxed{2\text{nd}}$ $\boxed{\text{[Y-VARS]}}$ 1:FUNCTION 1:Y1 $\boxed{}$ $\boxed{2\text{nd}}$ $\boxed{\text{[Y-VARS]}}$ 1:FUNCTION
2:Y2 $\boxed{}$. Press $\boxed{\text{GRAPH}}$.

What is the function that models the graph in $Y3$?

14. Press $\boxed{Y=}$ and enter $Y2(Y1)$ in $Y4$.

For the TI-83: Press $\boxed{\text{VAR}} \blacktriangleright$ 1:FUNCTION 2:Y2 $\boxed{}$ $\boxed{\text{VAR}} \blacktriangleright$ 1:FUNCTION 1:Y1 $\boxed{}$.
Press $\boxed{\text{GRAPH}}$.

For the TI-82: Press $\boxed{2\text{nd}}$ $\boxed{\text{[Y-VARS]}}$ 1:FUNCTION 2:Y2 $\boxed{}$ $\boxed{2\text{nd}}$ $\boxed{\text{[Y-VARS]}}$ 1:FUNCTION
1:Y1 $\boxed{}$. Press $\boxed{\text{GRAPH}}$.

What is the function that models the graph in $Y4$?

15. How do the graphs in $Y3$ and $Y4$ compare?

16. How does the comparison found in question 15 verify the pattern found in question 12?
