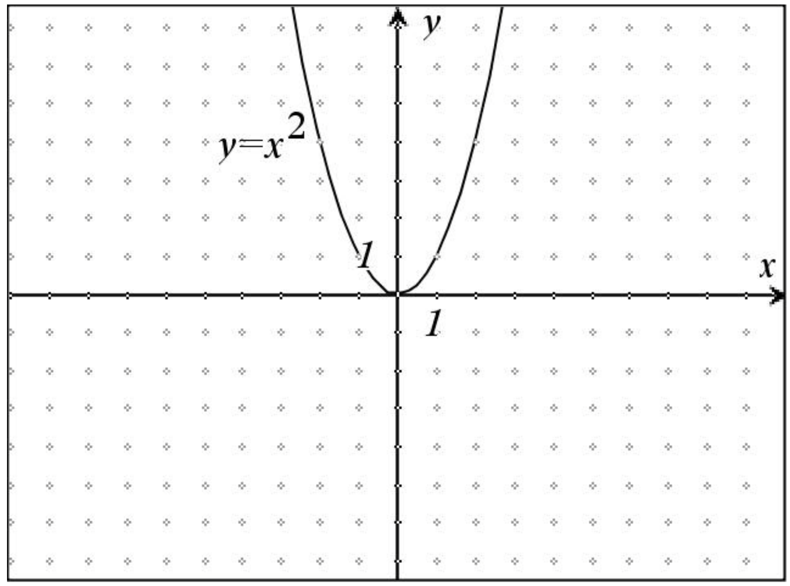
Name ______________________

Class ______________________

Problem 1 – Draw A Tangent Line by Hand

On the graph to the right, draw a line tangent to $y = x^2$ in the first quadrant.



- Approximate the slope of the line. Show your work.

- Write the equation of your line.

Problem 2 – Draw and Explore Tangent Line with Technology

Press [ON]. Go to [Y=] by pressing [◆] [F1]. Turn off all function by either using [F4] to uncheck the functions or select **F1:Tools > 8:Clear Functions**. After **y1=** press [X] [^] [2].

Use the standard window by selecting **F2:Zoom > 6:ZoomStd**.

To draw a tangent line on the graph in the first quadrant use **F5:Math > A:Tangent**. Type in an *x*-value between 0 and 3.1. The tangent line and its equation now appear. Press [◆] [F1] and enter this function in **y2=**.

Let's zoom in and observe the behavior. Press **F2:Zoom > 2:ZoomIn**. Move the cursor near the point of tangency and press [ENTER]. Zoom in again by pressing **F2:Zoom > 2:ZoomIn** and [ENTER]. Repeat the process of zooming in on the point of tangency a few more times to observe what happens.

- Write your observation of how your tangent line and the graph **y1(*x*)=*x*²** compare when examined close up?

- **Conjecture** – Will this type of behavior occur for all other functions? Explain your reasoning.

(You may want to try it for another function. You can choose your own or try $y = \sin(x)$.)

Problem 3 – Graph a piecewise function to explore local linearity

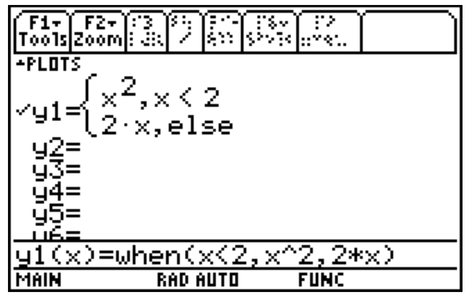
A function is said to be linear over an interval (i.e. locally linear over a small interval) if the slope is constant. Let's discover if all functions have a constant slope when they are examined in a small enough interval.

On your calculator, graph $y = \begin{cases} x^2, x \le 2 \\ 2x, x > 2 \end{cases}$ in **y1** using the command **when(**.

Press [CATALOG], find **when(** quickly by pressing [alpha] [.]. Note the formatting hints provided at the bottom of the catalog screen. Press [ENTER].

This 2-piece piecewise function is entered by pressing *x* [2nd] [<] [=] 2 [,] *x* [^] 2 [,] 2 *x* [)] [ENTER].

First observe the graph with the standard window. Then press **F2:Zoom > 6:ZoomStd**.



Discover if all functions have the property of local linearity by zooming in on the point (2, 4). This point is called a cusp. (Recall the **F2:Zoom > 2:ZoomIn** process.)

- Explain your observations. Use words like "slope" and "local linearity" to explain if, in the neighborhood of (2, 4), the function becomes one straight line.

Problem 4 – Graph another piecewise function

To explore if all piecewise functions lack the property of local linearity, beginning with **Zoom Standard**, zoom in on (2, 4) of the function $f(x) = \begin{cases} x^2 & , x < 2 \\ 4x - 4, & x \ge 2 \end{cases}$

- Does this function appear to be locally linear in the neighborhood of (2, 4)? Compare and contrast this function to the one graphed and explored in Problem 3.

Problem 5 – Conclusion

You know the slope is $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$. For the function $f(x)$, this can be written as $\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$. If you were finding the slope of function in the interval of a repeatedly zoomed in graph, describe what happens to $\Delta x = x_2 - x_1$.

