## Math Objectives

- Students will be able to identify the effect of changing $h$ and $k$ on the graph of the quadratic function in the vertex form
$y=a(x-h)^{2}+k$.
- Students will be able to identify the effect of $a$ and $b$ on the graph of the quadratic function in the standard form $y=a x^{2}+b x+c$.
- Students will make sense of problems and persevere in solving them (CCSS Mathematical Practice).
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).


## Vocabulary

- parabolic
- quadratic
- parameters
- vertex form of a quadratic equation
- standard form of a quadratic equation


## About the Lesson

- This lesson involves manipulating the equation of a quadratic function so that its graph passes through a particular point.
- As a result, students will:
- Use sliders to change the parameters in the quadratic equation and observe the changes to the graph.
- Use sliders to generate new random problems to repeat the exercise and consolidate the deductions.


## TI-Nspire ${ }^{\text {TM }}$ Navigator ${ }^{\text {TM }}$ System

- Use Screen Capture and Live Presenter to examine students' randomly generated graphs. Use Live Presenter to demonstrate the procedure and check on students' progress. Multiple graphs can be manipulated at the same time to allow students to see and discuss the commonalities and differences.
- Use Quick Poll to assess students' understanding.

Use the sliders at the bottom of the screen to change the parameters in the quadratic equation to help the trainer feed the seal. Use the sliders at the top to change the position of the trainer and the seal.

## TI-Nspire ${ }^{\text {TM }}$ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point in a slider
- Click on sliders to obtain a new problem


## Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- You can hide the entry line by pressing ctril $\mathbf{G}$.


## Lesson Materials:

Student Activity
Parabolic_Paths_Student.pdf
Parabolic_Paths_Student.doc
TI-Nspire document
Parabolic_Paths.tns

Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)

## Discussion Points and Possible Answers

Tech Tip: The quadratic equations are set to always pass through the point corresponding to the trainer's position on the vertical axis. Moving the sliders at the bottom of the page will change the graph. Change the values of the parameters until the graph reaches the point corresponding to the seal's position. Use the slider on the left of the page to change the position of the seal and generate a new problem.

## Move to page 1.2.

Use the sliders at the bottom of the screen to change the values of $h$ and $k$ in the quadratic equation $y=a(x-h)^{2}+k$. Help the trainer (on the vertical axis) feed the seals. The seal is pictured along the horizontal axis. The horizontal axis is representing the water; however, the seal can jump up on the rocks (represented
 by the black box). A message will tell you when the seal catches the fish. New problems can be generated by clicking the New slider on the left of the page. Try several problems before answering the questions.

1. What is the effect of changing the value of $h$ ?

Answer: When $h$ is changed, the graph of the quadratic equation changes such that its vertex moves horizontally left and right.
2. What is the effect of changing the value of $k$ ?

Answer: When $k$ is changed, the graph of the quadratic equation changes such that its vertex moves vertically up and down.
3. If the trainer throws the fish over the seal, should you increase or decrease the value of $h$ ? Explain.

Answer: If the graph goes past the point corresponding to the seal, it is necessary to decrease the value of $h$.
4. If the trainer throws the fish just short of the seal, should you increase or decrease the value of $h$ ? Explain.

Answer: If the graph does not reach the point corresponding to the seal, it is necessary to increase the value of $h$.
5. Can there be more than one throw (quadratic equation) which will reach the seal? Explain.

Answer: There are an infinite number of equations that will reach the seal. The graph may reach higher or lower at its vertex point. Two points (the trainer's position and the seal's position) do not define a unique quadratic.

Teacher Tip: You may want to discuss the equation of the parabolic path. Because the trainer's position is fixed and only $h$ and $k$ are being changed using sliders, the value of a will be changing to keep the quadratic going through the trainer's position. This is being done behind the scene. Also make sure students do not make the mistake of thinking $h$ is making the parabolic path wider.

## TI-Nspire Navigator Opportunity: Screen Capture/Live Presenter <br> See Note 1 at the end of this lesson.

## Move to page 2.1.

Use the sliders at the bottom of the screen to change the values of $a$ and $b$ in the quadratic equation $y=a x^{2}+b x+c$. Help the trainer feed the seals. A message will tell you when the seal catches the fish. New problems can be obtained by clicking the NEW slider on the left of the page.Try several problems before answering the
 questions.

Teacher Tip: Depending on students' previous experience, there may need to be some teacher guidance and discussion about the effect of $c$ in the equation.

Teacher Notes Math Nspired
6. What does the value of $c$ in the quadratic equation $y=a x^{2}+b x+c$ represent in this scenario?

Answer: The value of $c$ represents the initial position of the trainer and the start of the throw.
7. What do you notice happens to the graph as you change the value of $a$ ?

Answer: The effect is not as obvious as with the vertex form of the equation. Changing a will stretch or compress the graph.
8. What effects did you notice as you changed the value of $b$ ?

Answer: Changing the value of $b$ will move the graph of the quadratic equation both vertically and horizontally. It is very difficult to come up with a firm conclusion.
9. If the trainer throws the fish over the seal, should you increase or decrease the values of $a$ and $b$ ? Explain.

Answer: If the graph goes past the point corresponding to the seal, it is necessary to increase the values of $a$ and/or decrease the value of $b$.

Teacher Tip: Since the value of $a$ is negative in these examples, decreasing the value of $b$ will change the $x$-coordinate of the vertex $\left(-\frac{b}{2 a}\right)$ and move the graph of the quadratic equation to the left.
10. Can you always reach the seal by changing just one of the parameters?

Answer: There are some scenarios where you must change both the parameters a and $b$ to reach the seal.
11. Comparing the two forms of the quadratic equation, is it easier to find the quadratic equation to feed the seal using vertex form or standard form? Explain.

Answer: It is easier to change the equation using the vertex form as each parameter affects a change in one direction only. The precision is easier to control when changing $h$ and $k$.
12. If a third point was given (for example the vertex of the parabolic path), can there be more than one throw (quadratic equation) which will reach the seal? Explain.

Answer: The third point will anchor the quadratic equation, defining a unique equation.

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The effect on the graph of a quadratic equation in vertex form $\left(y=a(x-h)^{2}+k\right)$ when the parameters $h$ and $k$ are changed.
- The effect on the graph of a quadratic equation in standard form $\left(y=a x^{2}+b x+c\right)$ when the parameters $a$ and $b$ are changed.


## Assessment

You may administer a Quick Poll to assess the students' understanding of the concepts in this activity. Some sample questions are provided below.

## Sample Questions:

1. The trainer throws a fish that falls into the water in front of a seal. What change would you make to the parameters $h$ and $k$ in the equation $y=a(x-h)^{2}+k$, to change the parabolic path to reach the seal?
a) Increase the value of $h$ and/or increase the value of $k$.
b) Increase the value of $h$ and/or decrease the value of $k$.
c) Decrease the value of $h$ and/or increase the value of $k$.
d) Decrease the value of $h$ and/or decrease the value of $k$.
2. The trainer throws a fish that falls into the water behind a seal. What change would you make to the parameters $h$ and $k$ in the equation $y=a(x-h)^{2}+k$, to change the parabolic path to reach the seal?
e) Increase the value of $h$ and/or increase the value of $k$.
f) Increase the value of $h$ and/or decrease the value of $k$.
g) Decrease the value of $h$ and/or increase the value of $k$.
(h) Decrease the value of $h$ and/or decrease the value of $k$.
3. The trainer throws a fish that falls into the water in front of a seal. What change would you make to the parameters $a$ and $b$ in the equation $y=a x^{2}+b x+c$, to change the parabolic path to reach the seal?
i) Increase the value of $a$ and/or increase the value of $b$.
j) Increase the value of $a$ and/or decrease the value of $b$.
k) Decrease the value of $a$ and/or increase the value of $b$.
I) Decrease the value of $a$ and/or decrease the value of $b$.

## TI-Nspire Navigator

## Note 1

Questions 1-5, Screen Capture/Live Presenter: All students will have different randomly generated problems. You may want to use Live Presenter to demonstrate the procedure for using sliders to change the parameters in the equation. Use Screen Capture to monitor the students' progress through the activity. You can show students various examples at the same time and coordinate the changes made to the parameters to see if students can generalize the effects.

