

Solutions to Differential Equations

ID: 8998

Name _____

Class _____

In this activity, you will explore:

- differential equations

Open the file *CalcAct15_DifferentialEqns_EN.tns* on your handheld. Use this document as a guide to the activity and a place to record your answers.

Introduction

Chances are, you have already had some experience with differential equations.

Consider the equation $\frac{dy}{dx} = 2x$. What does this equation mean?

Can you find a function y whose derivative with respect to x is equal to $2x$?


Can you find a different function with the same derivative?

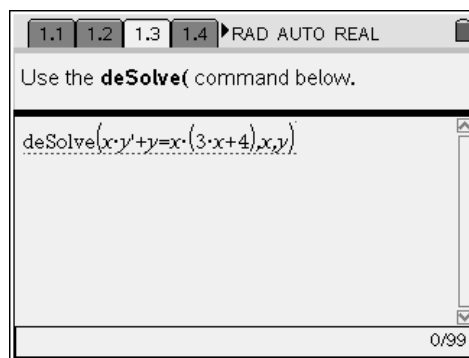
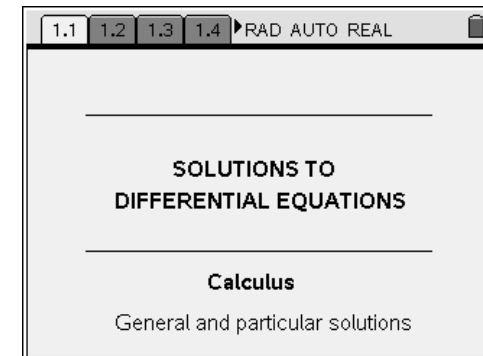
How might you write a **general solution** to this differential equation?

Now consider the equation shown on page 1.2: $xy' + y = x(3x + 4)$. This is also an example of a differential equation. Can you see how this equation might be more difficult to solve than the one in the previous paragraph?

Nonetheless, this differential equation does have a solution and the solution is of the form $y = f(x)$. In this activity, you will investigate the **general** and **particular** solutions to this differential equation.

Problem 1 – Finding and verifying a general solution to $xy' + y = x(3x + 4)$

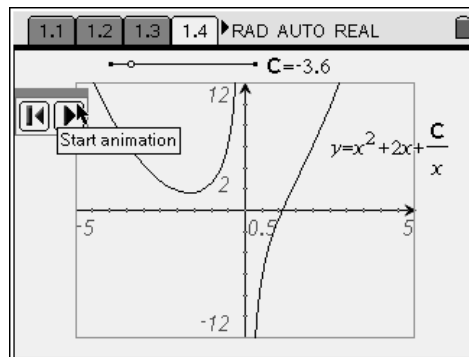
Advance to the *Calculator* application on page 1.3. Here, you will use the **deSolve(** command to solve the differential equation $xy' + y = x(3x + 4)$. Select **MENU > Calculus > Differential Equation Solver** and enter the arguments as shown to the right. The first argument is the differential equation, the second is the independent variable, and the third is the dependent variable. Press  to execute the command, and inspect the result.



Note: Be sure to include the multiplication signs as shown above. You may also wish to use the **expand(** command (**MENU > Algebra > Expand**) to express the result as three separate terms.

Notice that the solution contains a constant. Since this constant can take on any value, we call this result the **general solution** to our differential equation.

To view this general solution graphically, advance to page 1.4. The slider at the top of the screen has been configured so that **C** will take on values from -5 to 5 . Click play on the animation tool to start the animation, as shown in the screen to the right. After pressing play, you can pause the animation at any time by pressing the pause button that appears.



When finding solutions to algebraic equations, recall that you can check your solutions by substituting them into the original equation. If the resulting statement is true, then you know that you have found a correct solution. You will now use a similar process to verify that the correct solution to the differential equation

$$xy' + y = x(3x + 4) \quad (1)$$

is given by the equation below.

$$y = x^2 + 2x + \frac{C}{x} \quad (2)$$

- Begin by finding the derivative of equation (2), our proposed solution to the differential equation (1), with respect to x .

$$y' = \underline{\hspace{2cm}} \quad (3)$$

- Substitute y and y' from equations (2) and (3) in to equation (1) to transform the differential equation so it is written strictly in terms of x . Then simplify both sides of the equation.

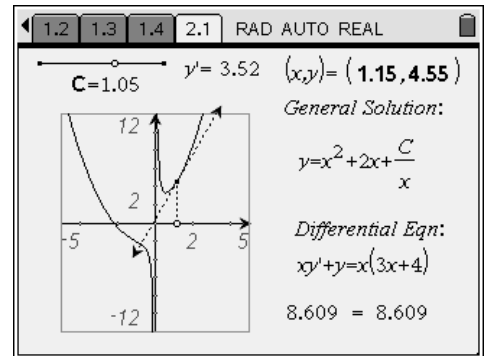
By showing that both sides of the simplified equation are equal, you will have verified that $y = x^2 + 2x + \frac{C}{x}$ is the **general solution** to the differential equation $xy' + y = x(3x + 4)$.

Problem 2 – Investigating particular solutions to $xy' + y = x(3x + 4)$

In the general solution to a differential equation, assigning a specific value to C results in a **particular solution** to the differential equation.

Advance to page 2.1 (also shown to the right). Here, you find C equal to 1.05, and therefore this particular solution is given by the equation $y = x^2 + 2x + \frac{1.05}{x}$.

Also notice that a tangent line is drawn to the curve and that the slope of this line and the coordinates of the point of tangency are given at the top of the screen.



- Explain how these specific values of C , y' , x , and y may be used to verify this particular solution by obtaining the true statement, $8.609 = 8.609$, at the bottom of the screen.

Move the tangent line along the curve by dragging the open point located on the x -axis.

- How does this action help verify the particular solution $y = x^2 + 2x + \frac{1.05}{x}$?

Now adjust the value of C by dragging the open point on the slider. You may also change the position of the tangent line as before.

- What changes in the graph as you adjust the value of C ?
What *does not* change?
- What does the result shown at the bottom right corner of the screen tell you about this new value of C ?
- Suppose you know that a particular solution passes through the point $(2, 10)$. Use this information to determine the value of C in the general solution $y = x^2 + 2x + \frac{C}{x}$.
- In Problem 1, you found derivatives and used substitution to verify the general solution. Explain why this means that the particular solution therefore does not need to be verified.

