CHAPTER 2

Advanced Graphing and the GRAPH MATH Menu

In this chapter, we will graph a variety of different functions by building upon the graphing capabilities developed in Chapter 1. We will develop capabilities for analyzing a graph using several of the items contained in the GRAPH MATH menu. We assume the reader has mastered the material covered in Chapter 1.

Preliminaries

1. For the first portion of this chapter the mode settings should be as in (2.0.1). *Recall that we press* [2nd] [MODE] *to access the mode settings.*

(2.0.1)

2. The graph format settings should be as in (2.0.2). *We press* [GRAPH] [MORE] ⟨**FORMAT**⟩ *to access the format settings.*

(2.0.2)

3. Clear all functions in the ⟨**y(x)=**⟩ graph editor by using the ⟨**DELf**⟩ selection. Next, enter

   $y1 = x^4 - 5x^2 + x + 3$ as shown in (2.0.3).

(2.0.3)

4. Change the viewing window to $[-5, 5, 1] \times [-7, 7, 1]$ as shown in (2.0.4).

(2.0.4)

5. Then select ⟨**GRAPH**⟩ to obtain (2.0.5).
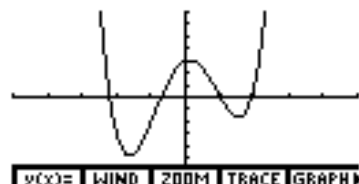
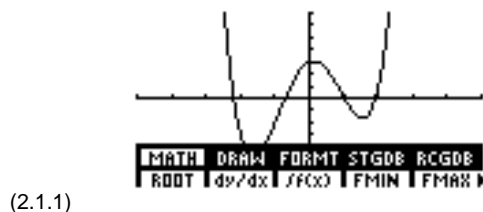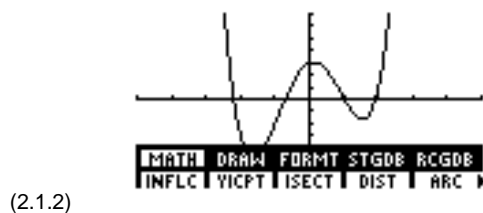(2.0.5)

Advanced Graphing and the GRAPH MATH Menu (Continued)
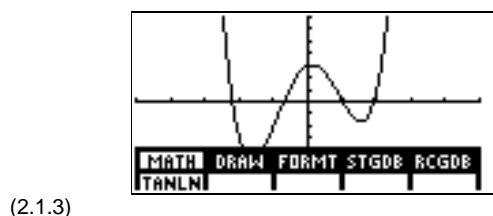
§1 – Using the GRAPH MATH Menu

1. With the menu line as in (2.0.5), press MORE
    ⟨**MATH**⟩ to obtain the portion of the GRAPH MATH
   menu shown in (2.1.1).

(2.1.1)

2. Press MORE to obtain (2.1.2).

(2.1.2)

3. Press MORE again to obtain (2.1.3). Press MORE again
    to return to (2.1.1). Press CLEAR to remove the two
   menu lines. Press EXIT to recover the two menu
    lines.

(2.1.3)

Next, we will investigate the ⟨**ROOT**⟩, ⟨**FMIN**⟩, ⟨**FMAX**⟩, and ⟨**ISECT**⟩ options. These options are used to analyze a graph for x-values between the settings given by the user in response to prompts for a **Left Bound** and a **Right Bound**. As will be seen, there are several instances in which the TI-86 will also prompt the user for an optional **Initial Guess** value.

§2 – Using ⟨ROOT⟩ to Find Zeros of a Function

1. We will use the ⟨**ROOT**⟩ option to find the solutions
    (roots) of $x^4 - 5x^2 + x + 3 = 0$, i.e., the points x
    where the graph of $y1$ intersects the horizontal axis.

    With the display as in (2.1.1), select ⟨**ROOT**⟩. You
    will obtain (2.2.1).

(2.2.1)
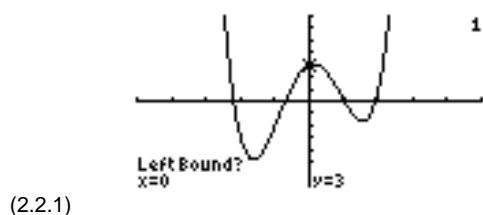
2. Note that the calculator prompts us for a
    Left Bound value. Type in –1.5 and press ENTER.
    Figure (2.2.2) shows the resulting screen image.

(2.2.2)
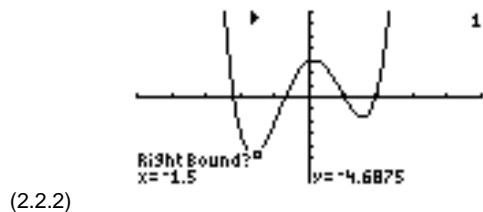
3. Enter 0.2 in response to the **Right Bound?** prompt,
    and press ENTER to obtain (2.2.3).

(2.2.3)
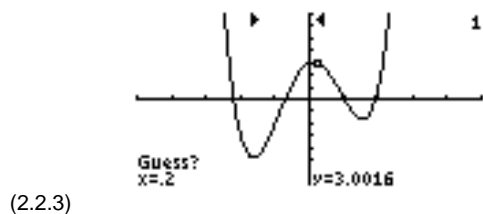
Advanced Graphing and the GRAPH MATH Menu (Continued)

4. We should note that selecting ⟨**ROOT**⟩ placed the TI-86 in the trace mode. An alternative to typing in values for **Left Bound** and **Right Bound** is to just trace to an appropriate point on the curve and press ENTER. In either case, providing values for **Left Bound** and **Right Bound** gives a display similar to (2.2.3).

5. Giving a response to **Guess?** is optional here; but if a value is given, then it must be in the interval [**Left Bound**, **Right Bound**]. To see that the value for **Guess** is optional, from (2.2.3) just press ENTER. After a while, we obtain (2.2.4), which indicates that $x = -0.713537935$ is a root of $y1$.

6. Press EXIT to return to the GRAPH menu. Select ⟨**MATH**⟩ and then ⟨**ROOT**⟩ again. Figure (2.2.5) is the result of using a **Left Bound** of 0.5 and a **Right Bound** value of 1.7 with a **Guess** value of 0.8. Thus, we see that $x = 1$ is also a root of $y1$. The process could be repeated twice more to find the remaining two roots of $y1$, namely $x = 1.9122291785$ and $x = -2.198691244$.

*Note: The values for **Left Bound** and **Right Bound** are indicated on the graph by two wedge symbols. These symbols disappear automatically when we complete this ⟨**ROOT**⟩ process.*

(2.2.4)

(2.2.5)
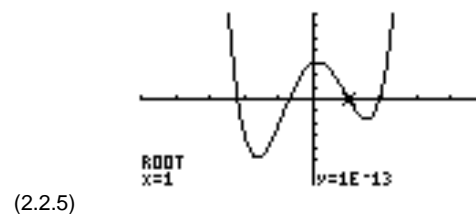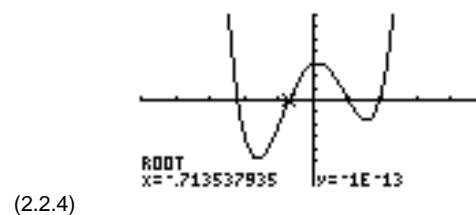
§3 – Using the Absolute Value Function in Graphing

1. Next we enter $y2 = |x - 1|$ in the ⟨**y(x)=**⟩ graph editor. On the TI-86, the absolute value function is given by *abs*. Thus, we want to enter $y2 = abs(x - 1)$. We accomplish this as follows: With the display as in (2.2.5), press GRAPH ⟨**y(x)=**⟩ and the down arrow key to obtain (2.3.1).

(2.3.1)

2. Then, with the blinking cursor in the position shown in (2.3.1), type *abs* by pressing 2nd [MATH] ⟨**NUM**⟩ ⟨**abs**⟩ to obtain (2.3.2).

(2.3.2)

3. Then press (x-VAR − 1) ENTER to obtain (2.3.3).

(2.3.3)

Advanced Graphing and the GRAPH MATH Menu (Continued)

4. Proceed in a similar fashion to enter the function
$y3 = -\textbf{abs}\,x$ as shown in (2.3.4). Remember to use
the $\boxed{(-)}$ key rather than the $\boxed{-}$ key when you enter $y3$.

```
Plot1 Plot2 Plot3
\y1■x^4-5x²+x+3
\y2■abs (x-1)
\y3■ -abs x
```

x	y	INSf	DELf	SELCT
round	iPart	fPart	int	abs ▶

(2.3.4)

5. Press $\boxed{\text{GRAPH}}$ ⟨**GRAPH**⟩ to obtain the graphs of $y1$,
$y2$, and $y3$ shown in (2.3.5).

y(x)=	WIND	ZOOM	TRACE	GRAPH ▶
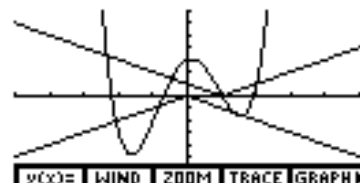
(2.3.5)

§4 – Using ⟨ISECT⟩ to Find an Intersection Point of Two Curves

The ⟨**ISECT**⟩ option is used to find the points of intersection of two graphs between the vertical lines
$x =$ **Left Bound** and $x =$ **Right Bound**, where the values for these two quantities are provided by the
user in response to prompts from the calculator.

1. As a first application of ⟨**ISECT**⟩, we obtain the
point near $(x, y) = (-2, 3)$, where $y1$ and $y2$
intersect. With the display as in (2.3.5), press $\boxed{\text{MORE}}$
⟨**MATH**⟩ $\boxed{\text{MORE}}$ ⟨**ISECT**⟩ to obtain (2.4.1).

```
First curve?
X=0            y=3
```
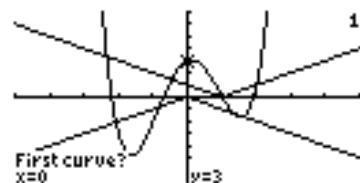
(2.4.1)

2. Note the 1 in the upper right-hand corner of the
display and the **First curve?** prompt in the lower
left portion of the screen. Press $\boxed{\text{ENTER}}$ to select $y1$
as one of the two graphs. You obtain (2.4.2).

```
Second curve?
X=0            y=1
```
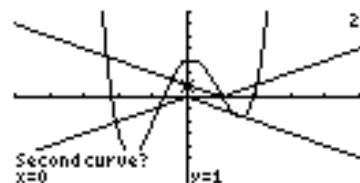
(2.4.2)

3. Note the 2 in the upper right-hand corner and the
Second curve? prompt. Press $\boxed{\text{ENTER}}$ to select $y2$ as
the second curve. Since the calculator is now in the
trace mode, use the left arrow key to move the
cursor along the curve until it is near the desired
intersection point, as shown in (2.4.3).

```
Guess?
X=-1.904761905   y=2.904761904B
```
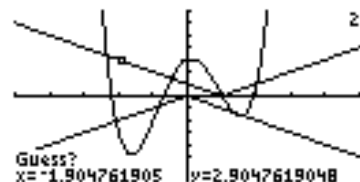
(2.4.3)

4. Then press $\boxed{\text{ENTER}}$ to find the point of intersection.
See (2.4.4). We conclude that $y1$ and $y2$ intersect at
the point $(x, y) = (-2.342923083, 3.3429230828)$.

```
Intersection
X=-2.3429230B3   y=3.342923082B
```
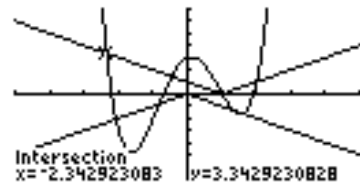
(2.4.4)

Advanced Graphing and the GRAPH MATH Menu (Continued)

§5 – Another Example Using ⟨ISECT⟩

1. As a second application of ⟨**ISECT**⟩, we find the intersection of $y1$ and $y3$ near $(x, y) = (-2, -2)$. With the display as in (2.4.4), press EXIT to obtain (2.5.1).
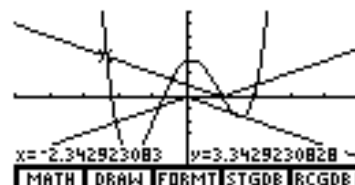
(2.5.1)

2. Select ⟨**MATH**⟩, then press MORE and select ⟨**ISECT**⟩ to obtain (2.5.2). As in §4, press ENTER to choose $y1$ as one of the two functions.

(2.5.2)
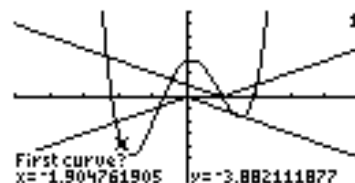
3. Press the down arrow key once to obtain (2.5.3). Press ENTER to select $y3$ as the second curve.

(2.5.3)
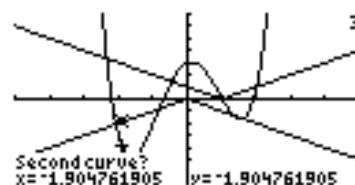
4. Then use the left arrow key to move the cursor near the desired point of intersections as in (2.5.4).

(2.5.4)
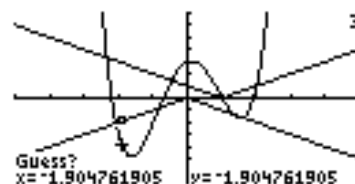
5. Finally, press ENTER to find the point of intersection. From (2.5.5), we conclude that $y1$ and $y3$ intersect at the point with coordinates

$$(x, y) = (-2.074313293, -2.074313293).$$

(2.5.5)

Advanced Graphing and the GRAPH MATH Menu (Continued)

§6 – Introduction of the ⟨FMIN⟩ and ⟨FMAX⟩ Options

The ⟨**FMAX**⟩ option is used to find a local maximum (interior or endpoint) of a function in the closed x-interval [**Left Bound**, **Right Bound**]. If there is more than one local maximum within the interval, then the one found by ⟨**FMAX**⟩ is dependent upon the value given for **Guess**.
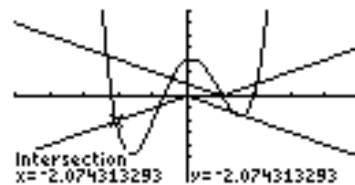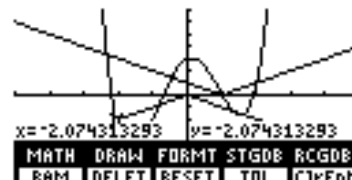
Thus, it is generally advantageous to set **Left Bound** and **Right Bound** so that the interval [**Left Bound**, **Right Bound**] will contain only one local maximum. Similar comments hold for the ⟨**FMIN**⟩ option.

1. The accuracy of the ⟨**FMAX**⟩ and ⟨**FMIN**⟩ options is controlled by the variable **tol**, which is a menu item in [2nd] [MEM]. From (2.5.5), press [2nd] [MEM] to obtain (2.6.1).

(2.6.1)

2. Unless you have previously changed **tol** or **δ**, pressing [F3] should display the current value of **tol** as in (2.6.2). *The value* **tol** = 1×10^{-5} = 0.00001 *is the default value.* The **tol** value must be a positive number, and it is changed the same way the viewing window variables **xMin**, **xMax**, etc. are changed.


(2.6.2)

As a general rule, the computed x-value of a local extremum given by ⟨**FMAX**⟩ or ⟨**FMIN**⟩ differs from the actual x-value by, at most, the value of **tol**. For example, if **tol** = 1×10^{-3} = 0.001, then the computed x-value differs from the actual x-value by, at most, 0.001. This general rule holds true when **tol** is at least as large as 1×10^{-5}. For **tol** smaller than 1×10^{-5}, the general rule no longer holds true. Indeed, in most cases, the TI-86 can find the x-value of a local extremum correct to only within 1×10^{-7}. Thus, a **tol** setting smaller than 1×10^{-7} will not produce greater accuracy. We suggest you stay with the default **tol** = 1×10^{-5}. The default **tol** value is used for the remainder of this chapter.

§7 – Using ⟨FMIN⟩ and ⟨FMAX⟩

1. Press [GRAPH] to return to the basic GRAPH menu. Select ⟨**y(x)=**⟩, and then deselect $y2$ and $y3$, leaving only $y1$ selected for graphing. Then select ⟨**GRAPH**⟩ to obtain (2.0.5) again, the graph of $y1 = x^4 - 5x^2 + x + 3$ in the viewing window [−5, 5, 1] × [−7, 7, 1]. Then press [MORE] ⟨**MATH**⟩ ⟨**FMIN**⟩ to get (2.7.1).
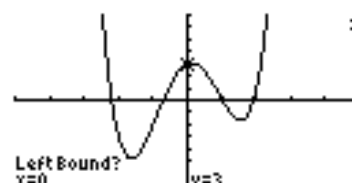
(2.7.1)

2. Enter −4 for **Left Bound** and 4 for **Right Bound**. Omit the optional value for **Guess**, and press [ENTER]. As indicated in (2.7.2), we conclude that $y1$ has a local minimum value of

$$y = -4.855394472 \text{ at } x = -1.628948507.$$
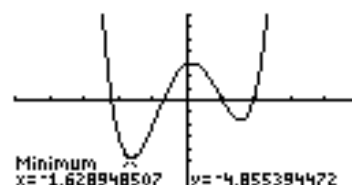
(2.7.2)

Advanced Graphing and the GRAPH MATH Menu (Continued)

3. To see that the value of the initial guess is significant, we suggest that the reader repeat the above process but with **Guess** taken to be 2. Figure (2.7.3) is the result.

(2.7.3)

4. Now press $\boxed{\text{EXIT}}$ ⟨**MATH**⟩ ⟨**FMAX**⟩ to obtain (2.7.4).

(2.7.4)
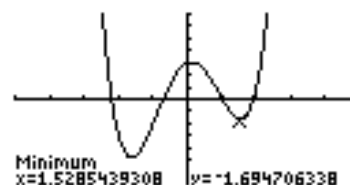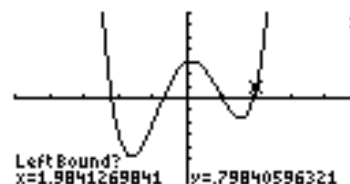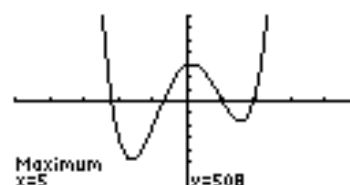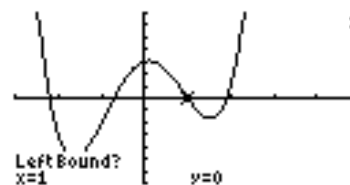
5. Enter −5 for **Left Bound** and 5 for **Right Bound** and again omit the optional **Guess** value. Since **tol** = 1×10^{-5}, we are confident that in the interval [−5, 5], $y1$ has an endpoint maximum value of 508 at $x = 5$. See (2.7.5)

(2.7.5)

§8 – Further Use of ⟨FMIN⟩ and ⟨FMAX⟩

1. Press $\boxed{\text{GRAPH}}$ to return to the basic GRAPH menu. Then select ⟨**WIND**⟩ and change the viewing window to [−3, 5, 1] × [−7, 7, 1]. Then press $\boxed{\text{MORE}}$ ⟨**MATH**⟩ ⟨**FMIN**⟩. The graph is drawn, and we obtain (2.8.1).

(2.8.1)

2. Enter −3 for **Left Bound** and 5 for **Right Bound**. and enter 1 for **Guess**. We can conclude that $y1$ has a local minimum value of −1.694706338 at $x = 1.5285440418$. See (2.8.2).

(2.8.2)
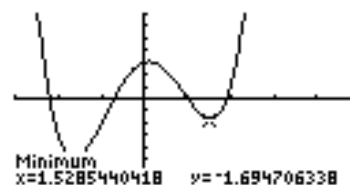
3. Press $\boxed{\text{EXIT}}$ ⟨**MATH**⟩ ⟨**FMAX**⟩, enter −1 and 0.7 for **Left Bound** and **Right Bound**, respectively, and omit a value for **Guess**. The result will be (2.8.3). We deduce that $y1$ has a local maximum value of 3.0501008097 at $x = 0.10040487237$.

(2.8.3)
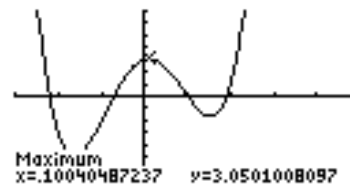
4. In §6 we remarked that the ⟨**FMIN**⟩ and ⟨**FMAX**⟩ options are sensitive to the value of **Guess** as to which local minimum or local maximum they locate. For example, (2.8.4) is the result of using FMIN with **Left Bound** = −3 and **Right Bound** = 5, with **Guess** = −2.
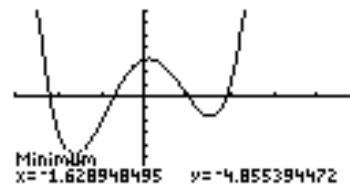
(2.8.4)

Advanced Graphing and the GRAPH MATH Menu (Continued)

§9 – A Side Note About CATLG-VARS

1. With the display as in (2.8.4), press 2nd [QUIT] to return to the home screen. If needed, press CLEAR to clear the home screen. Access the CATALOG/VARIABLES menu by pressing 2nd [CATLG-VARS]. See (2.9.1).

    ```
    CATALOG/VARIABLES




    CATLG  ALL   REAL   CPLX   LIST ▶
    ```
    (2.9.1)

As discussed in §7 of Chapter 1, the CATALOG/VARIABLES menu can be used to type names of instructions, functions, or variables on the home screen. For example, if you have forgotten that *abs* can be found by pressing 2nd [MATH] ⟨**NUM**⟩, one alternative is to find it in the CATALOG/VARIABLES menu. As a follow up to §8, we will use the CATALOG/VARIABLES menu to give the TI-86 an **fMax** command on the home screen to again locate the local maximum that the function $y1$ has near $x = 0.1$, $y = 3.05$. See (2.8.3).

2. From (2.9.1), press ⟨**CATLG**⟩. Typing ALPHA **F** will move the cursor to the first catalog item beginning with the letter F. Paging down once from there should locate the **fMax** command, as indicated in (2.9.2).

    ```
    CATALOG
    ▶fMax(
     fMin(
     fnInt(
     FnOff
     FnOn
     For(
    PAGE↓ PAGE↑ CUSTM BLANK
    ```
    (2.9.2)

3. Press ENTER to paste this command on the home screen. Then complete the command instructions as shown in (2.9.3), where we need to specify the function we want to deal with, the variable for that function, and the end points of the interval over which we want to maximize the function.

    ```
    fMax(y1,x,-1,.7)
    ```
    (2.9.3)

4. Pressing ENTER will give (2.9.4). Notice that the value reported is the x-coordinate of the local maximum.

    ```
    fMax(y1,x,-1,.7)
               .100404877251
    ```
    (2.9.4)

5. To compute the y-coordinate, we make use of the functional notation feature of the TI-86 and press 2nd ALPHA **y** 1 (2nd [ANS]) ENTER. The resulting information displayed in (2.9.5) is consistent with the results in (2.8.3).

    ```
    fMax(y1,x,-1,.7)
               .100404877251
    y1(Ans)
               3.05010080974
    ```
    (2.9.5)

Note: The reader is also encouraged to consult Appendix C to learn how to make a customized menu to be accessed by using the CUSTOM key.

Advanced Graphing and the GRAPH MATH Menu (Continued)

§10 – Using Tables to Compare x^3 and 3^x

1. Access the ⟨**y(x)=**⟩ editor, enter

    $$y4 = x^3, \quad y5 = 3^x, \quad y6 = y4 - y5,$$

    and select only $y4$ and $y5$ to be graphed. See
    (2.10.1).

    Plot1 Plot2 Plot3
    \y2=abs (x-1)
    \y3=⁻abs x
    \y4▣x^3
    \y5▣3^x
    \y6=y4-y5

    | y(x)= | WIND | ZOOM | TRACE | GRAPH |
    | x | y | INSf | DELf | SELCT▶ |

    (2.10.1)

2. Change the viewing window to $[0, 5, 1] \times [0, 50, 10]$
    as shown in (2.10.2).

    WINDOW
    xMin=0
    xMax=5
    xScl=1
    yMin=0
    yMax=50
    ↓yScl=10

    | y(x)= | WIND | ZOOM | TRACE | GRAPH▶ |

    (2.10.2)

3. Then graph $y4$ and $y5$ in this viewing window. Press
    CUSTOM to remove the menu line. We obtain (2.10.3).
    From this graph, we conjecture (but are certainly
    not sure) that $y4$ and $y5$ intersect one or more times
    between $x = 2$ and $x = 3.5$ in the viewing window.

    (2.10.3)

4. It is also worthwhile to consider this intersection
    problem using the TABLE feature of the TI-86. With
    the display as in (2.10.3), press GRAPH ⟨**TBLST**⟩ and
    enter 2.3 for **TblStart** and 0.05 for ∆**Tbl**. See
    (2.10.4).

    TABLE SETUP
    TblStart=2.3
    ∆Tbl=.05
    Indent: **Auto** Ask

    | TABLE | | | | |

    (2.10.4)

5. From (2.10.4), select ⟨**TABLE**⟩ to obtain (2.10.5) and
    to see numerical evidence that the graphs of these
    two functions cross at least once between the
    x-values of 2.3 and 2.55.

x	y4	y5
2.3	12.167	12.5135
2.35	12.97788	13.22011
2.4	13.824	13.96661
2.45	14.70613	14.75527
2.5	15.625	15.58846
2.55	16.58138	16.4687

    x=2.3
    | TBLST | SELCT | x | y | |

    (2.10.5)

§11 – Algebraic Considerations in Solving $3^x = x^3$ Graphically

Since $y4$ and $y5$ intersect at a point having x-value in the interval $[0, 5]$ if and only if $y6 = y4 - y5$ has
this x-value as a root, then perhaps the points of intersection of $y4$ and $y5$ are best determined by
analyzing the graph of $y6$.

1. Press ⟨**y(x)=**⟩ to return to the ⟨**y(x)=**⟩ editor as in
    (2.11.1).

    Plot1 Plot2 Plot3
    \y2=abs (x-1)
    \y3=⁻abs x
    \y4=x^3
    \y5=3^x
    \y6▣y4-y5

 | y(x)= | WIND | ZOOM | TRACE | GRAPH |
    | x | y | INSf | DELf | SELCT▶ |

    (2.11.1)

Advanced Graphing and the GRAPH MATH Menu (Continued)

2.   Select $y6$ as the only function to be graphed, and change the viewing window to $[0, 5, 1] \times [-3, 3, 1]$.

```
WINDOW
 xMin=0
 xMax=5
 xScl=1
 yMin=-3
 yMax=3
↓yScl=1
```
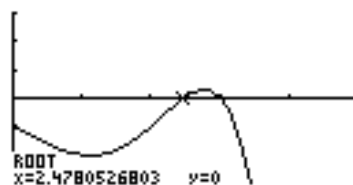 y(x)= | WIND | ZOOM | TRACE | GRAPH ▶

(2.11.2)

3.   Graph $y6$ to obtain (2.11.3). From the graph it is clear that $y6$ has two roots in the x-interval $[0, 5]$.

 y(x)= | WIND | ZOOM | TRACE | GRAPH ▶
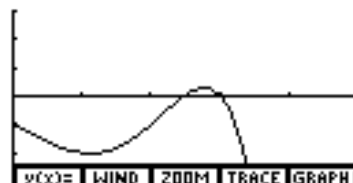
(2.11.3)

§12 – Solving $3^x = x^3$

1.   Use ⟨**ROOT**⟩ to find the first of the two roots as shown in (2.12.1).

```
ROOT
x=2.4780526803    y=0
```

(2.12.1)

2.   With the display as in (2.12.1), press   to return to the home screen. Press CUSTOM to clear the home screen. Then, as shown in (2.12.2), display the current x-value (the root just found in §12.1) and the current values of $y4$ and $y5$ for this x-value. We deduce that $y4$ and $y5$ intersect at the point

(2.47805268029, 15.21708982).

```
×              2.47805268029
y4               15.21708982
y5               15.21708982
```

(2.12.2)

3.   Note that in (2.12.3) the standard functional notation can also be used to evaluate $y4$ and $y5$ at the current value of x. ***Note: The option of using functional notation is a new feature of the TI-86, which is not available on the TI-85.***

```
×              2.47805268029
y4(x)            15.21708982
y5(x)            15.21708982
```

(2.12.3)

§13 – Using ⟨ROOT⟩ to Find a Multiple Root

The TI-86 algorithms for ⟨**ROOT**⟩, ⟨**ISECT**⟩, ⟨**FMAX**⟩, and ⟨**FMIN**⟩ are designed to work under a wide variety of circumstances. However, like most numerical algorithms, each has its limitations and sometimes does not locate the desired quantity to the level of certainty required by the algorithm. Even in these cases, however, the algorithm is usually headed in the right direction; and if the operator uses good judgment in interpreting the results, then a correct answer can be deduced. We will now consider such an example.

Advanced Graphing and the GRAPH MATH Menu (Continued)

1. Graph $y7 = 5x^2(x - 2)$ in the viewing window [−1, 3, 1] × [−7, 4, 1] as shown in (2.13.1).

    (2.13.1)

    ERROR 28 ITERATIONS
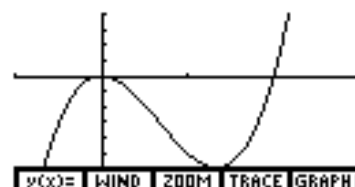
2. Then use ⟨**ROOT**⟩ to try and get the TI-86 to find the root at $x = 0$. No matter how tightly we try to bracket the origin with the [**Left Bound, Right Bound**] interval before executing the ROOT algorithm, if we don't provide 0 as the value of **Guess**, the TI-86 fails to find the root at $x = 0$. See (2.13.2).

    (2.13.2)

As one attempt to have the machine find the root at $x = 0$, use ⟨**ROOT**⟩ with

> **Left Bound** = −1, **Right Bound** = 1.1, and **Guess** = −0.5.

After a moment, you will again obtain (2.13.2) for a display. Thus, the TI-86 could not find to its satisfaction a root between **Left Bound** = −1 and **Right Bound** = 1.1. However, as we shall now see, the algorithm was on the right track.

3. Select ⟨**QUIT**⟩ to return to the home screen, and press CLEAR to clear the display. Then press x-VAR ENTER to see what the TI-86 thought a root in the interval [−1, 1.1] might have been. See (2.13.3). This x-value is so close to $x = 0$ that (even if we did not know 0 was a root) our conjecture would certainly be that 0 was a root.

    $$\times \quad 1.49224497968\text{E}^{-107}$$

    (2.13.3)

§14 – Graphing Piecewise Functions

Piecewise functions are important functions in a number of settings in precalculus and calculus, and we can make use of the logic features found in the 2nd[TEST] menu of the TI-86 to graph this type of function. For example, consider the following function:

$$f(x) = \begin{cases} 0, & \text{if} & x < 0.5 \\ 2, & \text{if} & 0.5 \le x \le 0.75 \\ 0, & \text{if} & x > 0.75 \end{cases}$$

There are many ways to graph this piecewise function using the TI-86. We will take the approach that this is a single function and, hence, should be defined and graphed as a single function. The TI-86 evaluates $(x < a)$ to be 1 when x is less than a, and to be 0 when x is greater than or equal to a. Similarly, on the TI-86, the expression $(x \ge a)$ evaluates to 1 when x is greater than or equal to a, and is 0 when x is less than a. Thus, it is reasonable to set

$$y8 = 2(x \ge 0.5)(x \le 0.75).$$

Advanced Graphing and the GRAPH MATH Menu (Continued)

1. Enter the above piecewise function as $y8$, as shown in (2.14.1), where the symbols $\leq$ **and** $\geq$ are found in the [2nd][TEST] menu. Also note that we have selected the Thick style for $y8$.
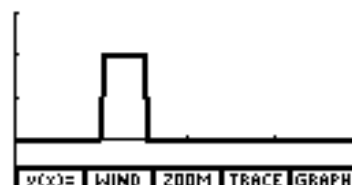
```
Plot1 Plot2 Plot3
\y4=x^3
\y5=3^x
\y6=y4-y5
\y7=5x²(x-2)
\y8◼2(x≥0.5)(x≤0.75)
y(x)=  WIND  ZOOM  TRACE  GRAPH
ALL+  ALL-  STYLE
```

(2.14.1)

2. After setting the viewing window as in (2.14.2), graph to obtain the graph shown in (2.14.3).

```
WINDOW
 xMin=0
 xMax=2
 xScl=.5
 yMin=-1
 yMax=3
↓yScl=1
y(x)=  WIND  ZOOM  TRACE  GRAPH ▶
```
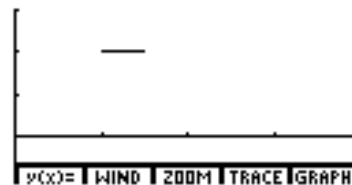
(2.14.2)

(2.14.3)

3. Changing the graph style of $y8$ to Dot style, as shown in (2.14.4), will give the more accurate version of the graph of $y8$ seen in (2.14.5).

```
Plot1 Plot2 Plot3
\y4=x^3
\y5=3^x
\y6=y4-y5
\y7=5x²(x-2)
∙y8◼2(x≥0.5)(x≤0.75)
y(x)=  WIND  ZOOM  TRACE  GRAPH
ALL+  ALL-  STYLE
```

(2.14.4)

(2.14.5)

Experimenting with TRACE verifies the accuracy of the graph of the original piecewise function. In Chapter 6 this particular function will be revisited in a study of the built-in integration algorithm of the TI-86.

§15 – Another Piecewise Function

A somewhat more common piecewise function is given by

$$g(x) = \begin{cases} 5-x, & \text{if } x \leq 2 \\ -x^2+6x-5, & \text{if } x > 2 \end{cases}$$

1. Again making careful use of the logic features of the TI-86, we enter this function in the $\langle \mathbf{y(x)=} \rangle$ editor as $y9 = (5-x)(x \leq 2) + (-x^2+6x-5)(x > 2)$, as is indicated in (2.15.1).

```
Plot1 Plot2 Plot3
\y5=3^x
\y6=y4-y5
\y7=5x²(x-2)
∙y8=2(x≥0.5)(x≤0.75)
\y9◼...-x²+6 x-5)(x>2)
  x   y   INSf  DELf  SELCT
 ==     <     >     ≤     ≥   ▶
```

(2.15.1)

## Advanced Graphing and the GRAPH MATH Menu (Continued)

2. We then set the viewing window to be [–3, 7, 1] × [–5, 8, 1], as in (2.15.2).

```
WINDOW
 xMin=-3
 xMax=7
 xScl=1
 yMin=-5
 yMax=8
↓yScl=1
│y(x)=│WIND│ZOOM│TRACE│GRAPH▶
```

(2.15.2)

3. The resulting graph is shown in (2.15.3). A careful study of the behavior of this piecewise function at x = 2 can be undertaken at this stage from a precalculus graphical point of view and, at a later date, from a calculus point of view.
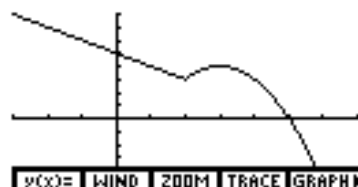
(2.15.3)

§16 – Piecewise Functions and Complex Arithmetic

The ability of the TI-86 to do complex arithmetic can cause unusual things to happen with certain types of piecewise functions. For example, consider the function given by

$$f(x) = \begin{cases} 3-x, & \text{if} \quad x < 2 \\ \sqrt{x-1}, & \text{if} \quad x \ge 2 \end{cases}$$

From a mathematical point of view this is a real valued function defined for all real numbers. *However, whenever the TI-86 reports the value of a function in complex (ordered pair) form, even if the imaginary part is zero, the TI-86 will not plot the point$(x, f(x))$ when graphing the function.*

1. It would be reasonable to enter the above function in the ⟨**y(x)=**⟩ editor as

$$y10 = (3-x)(x<2) + \left(\sqrt{(x-1)}\right)(x \ge 2).$$

But if we do so and try to graph this function on the viewing window [–5, 5, 1] × [–7, 7, 1], then the result will be as in (2.16.1).
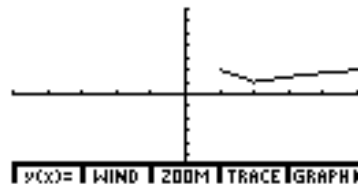
(2.16.1)

2. Part of the graph is missing since for x less than 1 the TI-86 computes the value of $\left(\sqrt{x-1}\right)(x \ge 2)$ to be the complex number (0,0); i.e., on the TI-86 a complex number times the real number zero produces a complex number result. In this particular case w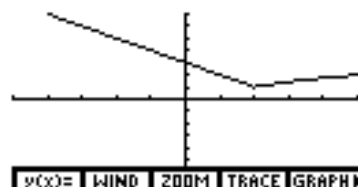e can obtain the expected graph of $f(x)$ by replacing the $\sqrt{x-1}$ part of $y10$ by $\sqrt{abs(x-1)}$. See (2.16.2).

```
Plot1  Plot2  Plot3
\y6=y4-y5
\y7=5x²(x-2)
\·y8=2(x≥0.5)(x≤0.75)
\y9=(5-x)(x≤2)+(-x²+…
\y10B…)(x<2)+(√abs  (…
│y(x)=│WIND│ZOOM│TRACE│GRAPH│
│  x  │  y  │INSf│DELf│SELCT▶
```

(2.16.2)

3. Selecting ⟨**GRAPH**⟩ produces the graph in (2.16.3).

(2.16.3)

Advanced Graphing and the GRAPH MATH Menu (Continued)

§17 – Using Zoom Box

In this section we will consider an application of one of the ⟨**ZOOM**⟩ features which was not considered in Chapter 1, namely the ⟨**BOX**⟩ submenu item of the ⟨**ZOOM**⟩ menu. In particular, we will consider the question graphically as to whether the graphs of $f(x) = 2x^2$ and $g(x) = \sin(5x^2 - 2) + 0.3$ have a first-quadrant point of intersection.

1. Press GRAPH F1 to access the ⟨**y(x)=**⟩ editor, and use ⟨**DELf**⟩ to delete all the functions in the editor. Then enter $y1 = 2x^2$ and $y2 = \sin(5x^2 - 2) + 0.3$. See (2.17.1).

(2.17.1)

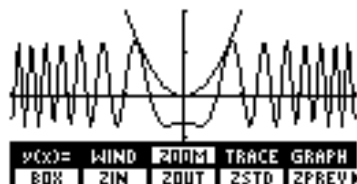2. Next set the viewing window as indicated in (2.17.2).

(2.17.2)

3. Selecting ⟨**GRAPH**⟩ produces the graph shown in (2.17.3). Notice that it is not clear if these graphs intersect in the first quadrant.
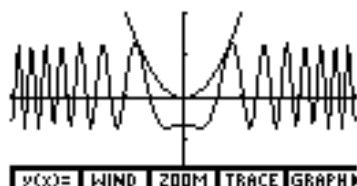

(2.17.3)

4. A BOX ZOOM may help decide the issue. So with the display as in (2.17.3), select ⟨**ZOOM**⟩ to obtain (2.17.4).


(2.17.4)

5. Selecting ⟨**BOX**⟩ will result in a screen similar to (2.17.5) in which the cursor is in the free moving mode. The TI-86 will need to be given a pair of diagonally opposite corners of a viewing box to zoom in on for this graph. Use the cursor arrows to move the free moving cursor to a point similar to the location of the cursor in (2.17.5), and press ENTER.
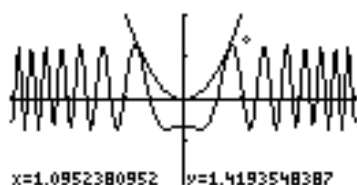
(2.17.5)

6. Now use the ◄ and ▼ and keys to position the other corner of the box as in (2.17.6).
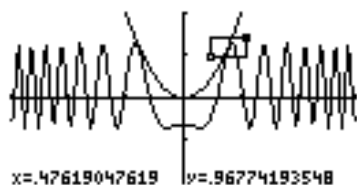
(2.17.6)

Advanced Graphing and the GRAPH MATH Menu (Continued)

7. Finally, press ENTER to obtain a graph similar to the graph in (2.17.7). It is now clear that the graphs of these two functions do not cross in the first quadrant.
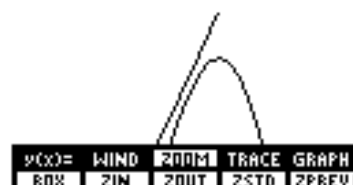
(2.17.7)

§18 – Graphing Rational Functions

We close this chapter with an example of a rational function whose graph has two vertical asymptotes, namely

$$f(x) = \frac{x^3 - 5x + 1}{(x-1)(x+2)}$$

1. From (2.17.7), select ⟨**y(x)=**⟩, and delete the $y1$ and $y2$ functions used in §17.

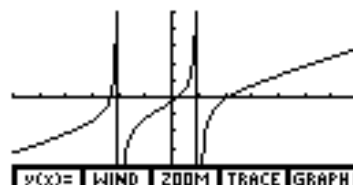   Enter $y1$ as $y1 = (x^{\wedge}3 - 5x + 1) / ((x-1)(x+2))$.

    See (2.18.1).

(2.18.1)

2. Figure (2.18.2) gives the graph of $y1$ in the viewing window $[-6, 7, 1] \times [-10, 10, 2]$. It appears that the TI-86 was smart enough to sketch in the two vertical asymptotes, but this is not the case.

(2.18.2)

3. In fact, near each asymptote, the calculator plotted a point for an x-value to the left of the asymptote, plotted another point for an x-value to the right of the asymptote, and then connected the two points with a line segment which appears nearly vertical. To verify this phenomenon, we change the graph style of $y1$ to the Dot style as indicated in (2.18.3).

(2.18.3)

4. Regraphing gives the graph shown in (2.18.4).
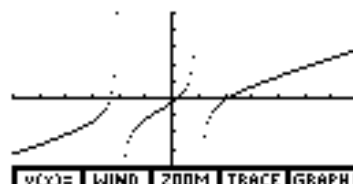
(2.18.4)

5. In the Connected style, the only way to avoid the drawing of these false asymptotes is to force the x-value where the vertical asymptote occurs to be one of the values the calculator uses in plotting points for the graph. For example, since $x = 1$ is midway between –4 and 6, graphing $y1$ using the Connected style in the window $[-4, 6, 1] \times [-10, 10, 2]$ will result in the graph shown in (2.18.5).
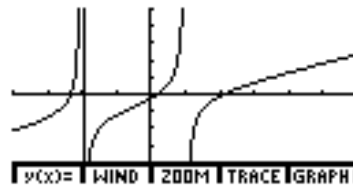
(2.18.5)

Advanced Graphing and the GRAPH MATH Menu (Continued)

Exercises

1. As a follow-up to §2, use ⟨**ROOT**⟩ and verify that the remaining two roots of $y = x^4 - 5x^2 + x + 3$ are approximately $x = 1.9122291785$ and $x = -2.198691244$.

2. Use a BOX ZOOM from figure (2.4.3) to graphically show that $y = -|x|$ and $y = x^4 - 5x^2 + x + 3$ have two fourth-quadrant points of intersection. Then use ⟨**ISECT**⟩ to find them.

3. Find the fourth quadrant local minimum value of $y = x^4 - 5x^2 + x + 3$.

4. Consider $f(x) = \dfrac{x^3 - 5x + 1}{(x-1)(x+2)}$.

    Find a viewing window which includes the two vertical asymptotes but which has the property that, when $f(x)$ is graphed in the Connected style, neither of the vertical asymptotes is falsely drawn.

5. (a) Construct a piecewise function $y1$ which has the following graph.
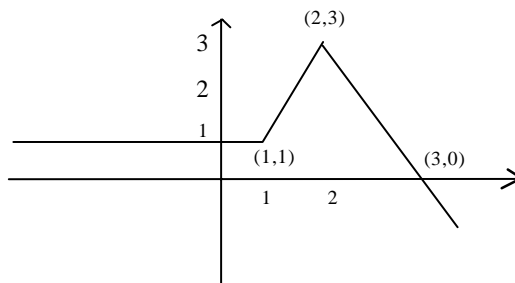
    (b) Use ⟨**FMAX**⟩ to see if the TI-86 can locate the local maximum at the point (2,3).

    (c) Use ⟨**ROOT**⟩ see if the TI-86 can find the root at (3,0).

6. As a follow-up to Exercise 5, let $y2 = y1(x - 1)$ and graph $y2$ on the viewing window $[-1, 5, 1] \times [-3, 5, 1]$. Explain the nature of the resulting shift of the graph of $y1$. Do the same with $y3 = y1(x + 1) + 2$. Finally, use ⟨**ISECT**⟩ to find the point of intersection of $y2$ and $y3$.