The Exponential Derivative

Name ______________________

Class ______________________

We want to find the derivative of the function $f(x) = e^x$. We want to look at a constant (positive) base and variable exponent. The easiest function is the function above where e is the number we found before. What is the definition of e? Does this definition help us with the derivative?

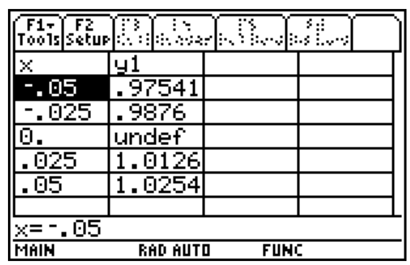
Problem 1 – The Derivative of $y = e^x$

So, we start with the definition of a derivative $f'(x) = \lim\limits_{x \to \infty} \dfrac{f(x+h) - f(x)}{h}$ and we use $f(x) = e$ in that definition: $f'(x) = \lim\limits_{h \to 0} \dfrac{e^{x+h} - e^x}{h} = \lim\limits_{h \to 0} \dfrac{e^x e^h - e^x}{h} = \lim\limits_{h \to 0} \dfrac{e^x(e^h - 1)}{h}$.

To get the answer we want, we need to evaluate $\lim\limits_{h \to 0} \dfrac{e^h - 1}{h}$. Do you know what that limit is?

We will use two methods to evaluate it.

When we try to evaluate this limit and replace h with zero we get the indeterminate form 0/0. To use L'Hôpital's rule, we would have to know the derivative of our exponential function and we do not know that yet.



Set up a table to see the possibilities.

Use the table set function for the function

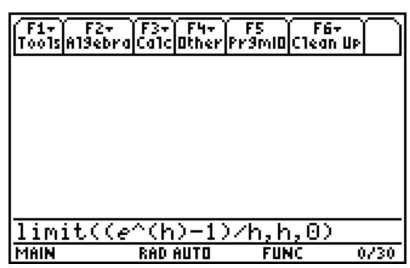
$y1 = \dfrac{e^x - 1}{x}$

With x starting at –0.05 and $\Delta x = 0.025$.

Your table should look like the screen to the right.

Notice that the calculator does not compute the value at 0.

What does the value of $y1$ seem to approach at 0?



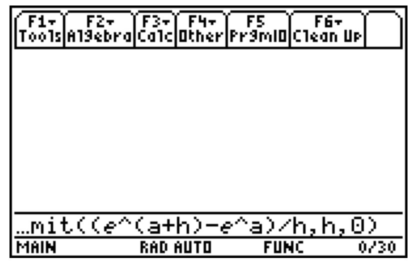
So let's use the **limit** command for this expression and see the result. What is your answer?

$\lim\limits_{h \to 0} \dfrac{(e^h - 1)}{h} =$

Now we can use the definition of the derivative and the result above with the function $f(x) = e^x$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x e^h - e^x}{h} = \lim_{h \to 0} \frac{e^x \left(e^h - 1\right)}{h} = e^x$$

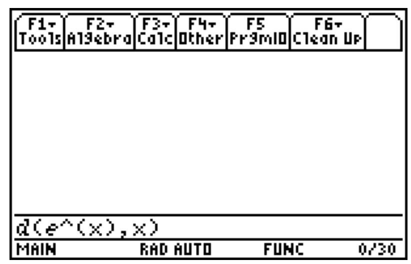
At a specific point such as $x = a$, we can use the **limit** command to find the derivative of $f(x) = e^x$ at $x = a$. What is the result?



$$\lim_{h \to 0} \frac{e^{(a+h)} - e^a}{h} =$$

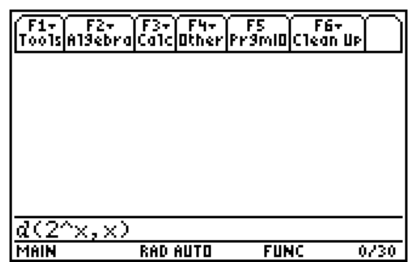
Now try the derivative command for the exponential function $f(x) = e^x$.

What is your answer?



Problem 2 – The Derivative of *f*(*x*) = *a*^{x}

What happens if we use a different base?

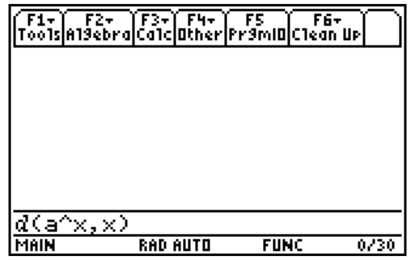


Use the derivative command for the following functions. What were the results? Do you notice a pattern?

$f(x) = 2^x$ $\quad$ $f'(x)$ = __________

$g(x) = 3^x$ $\quad$ $g'(x)$ = __________

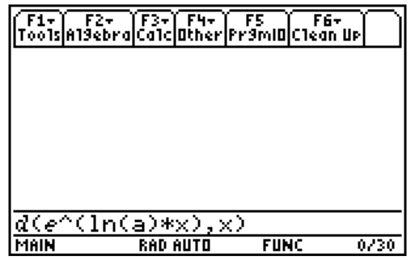
What do you think that the derivative of the function $f(x) = a^x$ will be?



Why do you think this result happened?

Look at $a = e^{ln(a)}$ and rewrite as $y = a^x = e^{(\ln(a)x)}$.

Using the chain rule, take the derivative of this function.



Now find the derivative of the following functions with the chain rule:

$f(x) = e^{\left(x^2\right)}$

$g(x) = e^{7x + 3}$

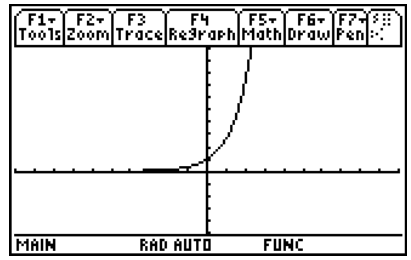
$h(x) = 2^{5x}$

Why does 32^x appear in the last problem on your calculator?

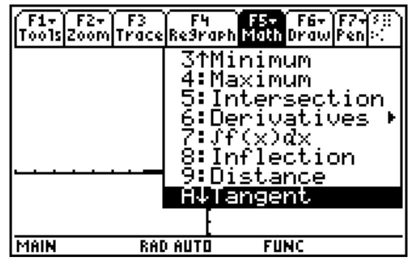
Problem 3 – Slope of the Exponential Function

Graph the function $f(x) = e^x$

Trace the graph and find a point close to $x = 1$. List the coordinates. ___________



Draw the tangent to the graph at that point. Write its equation below.



What is the relationship between the *y*-coordinate and the slope?

Since the derivative is the slope of the tangent line, we expect to see the *y*-coordinate and the slope to be identical for the function $f(x) = e^x$.