Eccentricity: Polar Equations of Conics

By Janice Mitchener 15 minutes

Activity Overview

This activity will give students a series of polar equations of conics to discover a pattern of the eccentricity of each type of conic.

Teacher Preparation

This lesson can be used as an introduction to eccentricity and the concept does not need to be introduced beforehand. The concept of polar equations and polar graphs should be discussed prior to this lesson.

Classroom Management

This activity could be teacher led or done independently by the students.

Applications

Graphs & Geometry, Notes

Step by Step Instructions

1) In order to graph in polar mode, press (menu)		1.1 1.2 1.3 1.4 RAD AUTO REAL	
Graph Type #3 Polar. Recall that		^{1.42} . y	
\bigcirc will remove the equation line for better		^{0.5} axes	
viewing of the graph. Remind students to put	-	-5	
() around the θ or the equation will not be		-1.42	
graphed.		۲.++2 ۶	
		Graph in polar mode: $r = \frac{1}{8 + 2\sin(\theta)}$	
2) Students can type responses in the answer	_	1.1 1.2 1.3 1.4 RAD AUTO REAL	
section of the document. Arrow down to see		Question	
the entire answer section		Compare the equation graphed in the previous screen to the equation	
		$r = \frac{ep}{1 + esin\theta}$, what is the eccentricity?	
		Identify the conic.	
	Ē	Answer 🛛 👻 🗸	
3) Again, change to polar mode before		1.1 1.2 1.3 1.4 RAD AUTO REAL	
3) Again, change to polar mode before graphing.	Γ	1.1 1.2 1.3 1.4 RAD AUTO REAL	
3) Again, change to polar mode before graphing.		1.1 1.2 1.3 1.4 PRAD AUTO REAL	
3) Again, change to polar mode before graphing.	1	1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 -20 2 20	
3) Again, change to polar mode before graphing.	11	1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL	
3) Again, change to polar mode before graphing.		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 -20 2 20 $\Rightarrow \equiv f_2(x) =$ a a a	
3) Again, change to polar mode before graphing.		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 -20 2 20 $\Rightarrow \equiv f_1(x) =$ x x x Graph in polar mode: $r = \frac{8}{2+4cos\theta}$ x x	
 3) Again, change to polar mode before graphing. 4) In this screen, type in csc(θ) in the input 		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 - -20 2 20 $\Rightarrow \mathbb{E} f_{2}^{T}(x) =$ \Rightarrow \Rightarrow Graph in polar mode: $r = \frac{8}{2 + 4cos\theta}$ \Rightarrow I.3 1.4 1.5 1.6	
 3) Again, change to polar mode before graphing. 4) In this screen, type in csc(θ) in the input line. This calculator recognizes csc(θ). There 		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 -20 2 20 $\Rightarrow \equiv f?(x) =$ \Rightarrow \Rightarrow \Rightarrow \Rightarrow Graph in polar mode: $r = \frac{8}{2+4cos\theta}$ \Rightarrow \Rightarrow \Rightarrow 1.3 1.4 1.5 1.6 PRAD AUTO REAL 14.75 γ γ ϕ ϕ	
 3) Again, change to polar mode before graphing. 4) In this screen, type in csc(θ) in the input line. This calculator recognizes csc(θ). There is no need to change to sine in this screen. 		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 - -20 2 20 $\Rightarrow \equiv f_2(x) =$ a Graph in polar mode: $r = \frac{8}{2+4cos\theta}$ a 1.3 1.4 1.5 1.6 PRAD AUTO REAL a a 1.3 1.4 1.5 γ 5 b γ 5 γ 5	
 3) Again, change to polar mode before graphing. 4) In this screen, type in csc(θ) in the input line. This calculator recognizes csc(θ). There is no need to change to sine in this screen. 		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 -20 2 20 $\Rightarrow \equiv f_2(\chi) =$ $\Rightarrow = \frac{8}{2+4cos\theta}$ $\Rightarrow = \frac{8}{2+4cos\theta}$ $\Rightarrow = \frac{14.75}{14.75}$ γ 5 -20 2 20 $\Rightarrow = \frac{14.75}{2+4cos\theta}$ $\Rightarrow = \frac{14.75}{2}$ 2 $= 20$ 2 20 2 20 $\Rightarrow = \frac{14.75}{2}$	
 3) Again, change to polar mode before graphing. 4) In this screen, type in csc(θ) in the input line. This calculator recognizes csc(θ). There is no need to change to sine in this screen. 		1.1 1.2 1.3 1.4 PRAD AUTO REAL 14.75 γ 5 -20 2 20 $\Rightarrow \equiv f_1(x) =$ x x x Graph in polar mode: $r = \frac{8}{2+4cos\theta}$ x x 1.3 1.4 1.5 1.6 PRAD AUTO REAL 14.75 γ 5 x <	

5) This is where the student can transfer the equation in terms of sine. To get the fraction, press (err) and then the division sign.		Question What does $r = \frac{3csc\theta}{csc\theta-1}$ simplify to in terms of sine? Answer
 6) This calculator might have all the signs reversed compared to what the student might have. This would be a good teaching moment. In this screen, press (and tab) to toggle between the different parts of the screen 	[1.5 1.6 1.7 1.8 RAD AUTO REAL Use the calculator below to check your solution.
7) The remaining screens are self-explanatory.		

Eccentricity: Polar Equations of Conics (student) TI-Nspire files *eccentricity of polar equation of conics.tns*

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Eccentricity	1.42 y	Question
of Polar Graphs	0.5 axes	Compare the equation graphed in the previous screen to the equation $r = \frac{\epsilon p}{1 + esin\theta}, \text{ what is the eccentricity?}$
of Conics	Graph in polar mode: $r = \frac{6}{8 + 2\sin(\theta)}$	Identify the conic.
1.1 1.2 1.3 1.4 RAD AUTO REAL	1.2 1.3 1.4 1.5 ▶ RAD AUTO REAL □	1.3 1.4 1.5 1.6 ▶ RAD AUTO REAL □
$14.75 \qquad \gamma$ 5 $-20 \qquad 2 \qquad 20$ $\implies \boxed{r}(x) = \qquad \texttt{R}$ Graph in polar mode: $r = \frac{8}{2+4\cos\theta}$	QuestionCompare the equation graphed in the previous screen to the equation $r = \frac{ep}{1 + ecos\theta}$, what is the eccentricity?Identify the conic.Answer \checkmark	14.75 y 5 -20 2 2 20 $\Rightarrow \equiv f_{1}(x) =$ a Graph in polar mode: $r = \frac{3csc\theta}{csc\theta - 1}$
I.4 1.5 1.6 1.7 RAD AUTO REAL Question Image: Constraint of the second state of the s	1.5 1.6 1.7 1.8 PRAD AUTO REAL Use the calculator below to check your solution.	Image: The second system Image: The second system Question Image: The second system Compare the equation you just found in terms of sine to the equation removes the equation $r = \frac{ep}{1+esin\theta}$, what is the eccentricity? Identify the conic. Answer Image: The second system

1.7 1.8 1.9 1.10 ▶RAD AUTO REAL	
Question	^
How can you identify the conic considering the eccentricity of a polar equation with the focus at the pole?	
Answer 🛛 🛛 🛛 🕹	

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Question
Where is the focus located when using the polar equations for conics that was used in this lesson?
Answer 🛛 🛛 🕹

1.10 1.11 1.12 1.13 RAD AUTO REAL

The following page is for you to test your hypothesis with more examples. Remember to graph in polar mode.