NUMB3RS Activity: Spiraling Out
Episode: "Spree, Part II – Daughters"

Topic: Polar Spirals **Grade Level:** 11 - 12
Objective: Students will explore the graphs of different polar spirals
Time: 15 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator

Introduction

In "Spree, Part II," the FBI is chasing a criminal on the run. Don believes the criminal is within a certain bounded area in Los Angeles. Charlie compares the situation to a famous problem known as the Trawler Problem. This problem involves a fast boat chasing a slow boat until the slow one disappears into a fog bank. (In Charlie's case, the FBI is the fast boat, and the fog bank is the boundary of the dragnet.) The Trawler Problem assumes that the slow boat enters the fog bank, turns at a particular angle, and then continually heads in that direction.

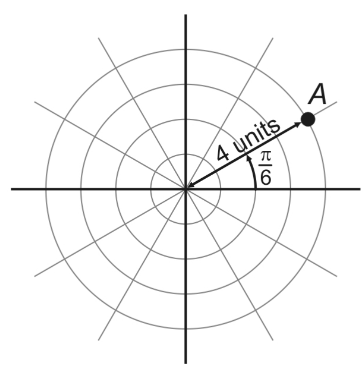
The surprising solution to the Trawler Problem is for the fast boat to proceed to the point where the boats would have met if the slow boat had made a 180° turn and headed back towards the fast boat. (Note that the slow boat would never actually do this, but it is a necessary assumption for the solution of the problem.) From that point, the fast boat should spiral out logarithmically. The boats will then intersect before the fast boat completes one full turn.

In most curricula, when students study the graphs of polar equations, the focus is on circles, lemniscates, limaçons, and rose curves. This activity focuses on another group of polar graphs called spirals, and culminated with graphing the logarithmic spiral that Charlie mentions. The intent of this activity is not to teach polar graphing, but to introduce students to polar graphs they may not have seen in a traditional textbook.

Discuss with Students

Students should be familiar with polar graphing. It is the intent of this activity to supplement students' prior knowledge of polar graphing with new graphs such as logarithmic spirals.

Remind students that angles can be measured in radians (180° = π, 90° = $\pi/2$, 60° = $\pi/3$, etc.). Also, remind students that plotting a point in polar notation involves a radius and directional angle. For example, point $A(4, \pi/6)$ is shown at the right.



In this activity, all window settings (e.g., Xmin, Xmax, etc.) will be described using the following notation: x:[min, max], y:[min, max], θ:[min, max].

Student Page Answers:
1. *The spacing between loops in the Archimedes spiral remains constant.* **2.** *They are inverses of each other if the pole is the center of the inversion.* **3.** *The greater the value of a, the greater the rate at which the spiral rotates from the center. Negative values of a reflect the graph over the x-axis.*

Patrick Flynn, Olathe East High School, Olathe, KS

Name: ____________________________________ Date: _________________

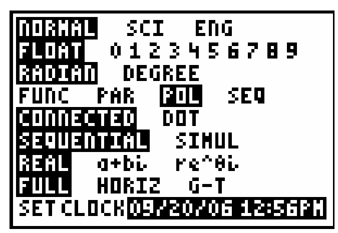
NUMB3RS Activity: Spiraling Out

In "Spree, Part II," the FBI is chasing a criminal on the run. Don believes the criminal is within a certain bounded area in Los Angeles. Charlie compares the situation to a famous problem known as the Trawler Problem. This problem involves a fast boat chasing a slow boat until the slow one disappears into a fog bank. (In Charlie's case, the FBI is the fast boat, and the fog bank is the boundary of the dragnet.) The Trawler Problem assumes that the slow boat enters the fog bank, turns at a particular angle, and then continually heads in that direction.

The surprising solution to the Trawler Problem is for the fast boat to proceed to the point where the boats would have met if the slow boat had made a 180° turn and headed back towards the fast boat. (Note that the slow boat would never actually do this, but it is a necessary assumption for the solution of the problem.) From that point, the fast boat should spiral out logarithmically. The boats will then intersect before the fast boat completes one full turn.

What exactly do these logarithmic spirals look like and how are they graphed? One of the best ways to graph spirals is to use polar graphs. Remember that in polar notation, a point (r, θ) is defined in terms of its distance from the center r and the counterclockwise angle θ (in radians), measured from the positive x-axis. The equations of graphs can also be written in polar form. For example, the equation for a circle of radius 5 in the Cartesian system is $x^2 + y^2 = 25$ while in polar form it is $r = 5$.

To begin, set your graphing calculator to polar and radian modes by pressing [MODE] and then selecting **POL** and **RADIAN**.



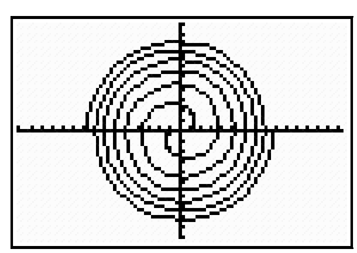
Now you will explore the many types of spiral graphs in the polar world. For each spiral, the standard equation is given along with a specific example. However, do not let the equation limit your exploration – try other values for the coefficients and observe how they affect the graphs.

Note: Each spiral should be graphed independently of the others.

Fermat's Spiral: $r = \pm\sqrt{a^2\theta}$, where a is a constant

On your graphing calculator use {–1, 1} in place of the $\pm$ sign.

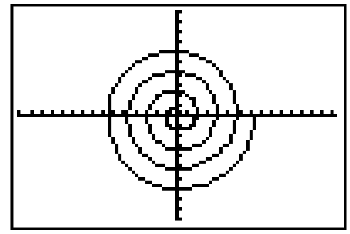
Window settings: Press [ZOOM] and select **5:ZSquare**. For θ, use $[0, 6\pi]$.



$r = \pm\sqrt{4\theta}$

Patrick Flynn, Olathe East High School, Olathe, KS

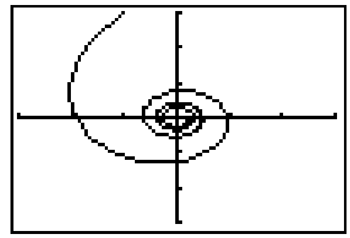
Spiral of Archimedes: $r = a\theta$, where *a* is a constant


$r = 0.3\theta$

Window settings: Use **5:ZSquare** and θ:[0, 8π]

1. How is the Spiral of Archimedes different from Fermat's?

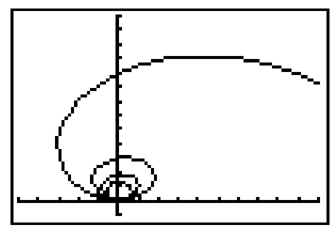
Hyperbolic Spiral: $r = \frac{a}{\theta}$, where *a* is a constant


$r = \frac{6}{\theta}$

Window settings: *x*:[–3, 3], *y*:[–3, 3], and θ:[0, 8π]

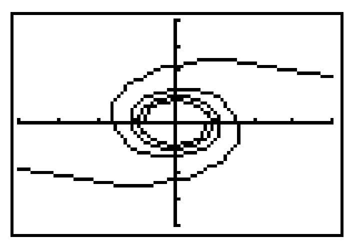
2. How is the hyperbolic spiral related to the Spiral of Archimedes?

Cochleoid: $r = \frac{a\sin(\theta)}{\theta}$, where *a* is a constant


$r = \frac{14\sin(\theta)}{\theta}$

Window settings: *x*:[–5, 10], *y*:[–1, 13], and θ:[0, 4π]

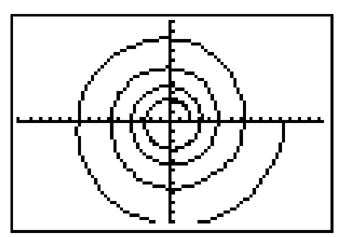
Lituus: $r = \pm\sqrt{\frac{a^2}{\theta}}$, where *a* is a constant


$r = \pm\sqrt{\frac{8}{\theta}}$

Again, on your graphing calculator, use {–1, 1} in place of the ± sign.

Window settings: *x*:[–4, 4], *y*:[–4, 4], and θ:[0, 4π]

Logarithmic Spiral: $r = ab^{c\theta}$, where *a* is a constant, *c* > 0, and *b* > 1


$r = 2(2)^{0.1\theta}$

Window settings: Use **5:ZSquare** and θ:[0, 8π]

3. What effect does the coefficient *a* have for all of these spirals?

Patrick Flynn, Olathe East High School, Olathe, KS

The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research

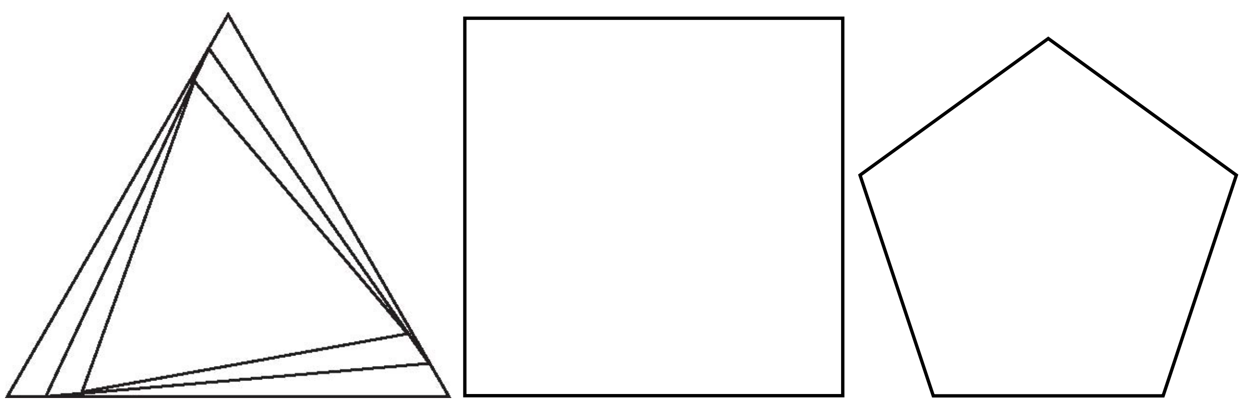
Extensions

Activity: Whirling Around

Introduction

Whirls are figures constructed by spiraling a sequence of regular polygons (each slightly smaller than the first). When the vertices are connected, they form logarithmic spirals emanating from the center of the polygon.

To create your own whirls in the regular polygons below, measure $\frac{1}{2}$ centimeter counterclockwise from each vertex of the polygon and connect these "new" vertices. The whirl on the triangle has been started. Continue this with each new set of vertices. Notice that the number of vertices determines the number of logarithmic spirals.



Additional Resources

See how your whirls compare to those at the Web site
http://mathworld.wolfram.com/Whirl.html

Learn how to walk logarithmic spirals by reading the *NUMB3RS* activity "The Four Bug Problem: Step On No Pets." To download this activity, go to **http://education.ti.com/exchange** and search for "7423."

The Four Bug Problem is also known as the "Mice Problem." An animation of this for various regular polygons can be found at the Web site
http://mathworld.wolfram.com/MiceProblem.html

Patrick Flynn, Olathe East High School, Olathe, KS