Activity 7
Evaluating Rational Functions

Answers to Instructions: Part A

2. $f(x) = \dfrac{2 \cdot (x - 2) \cdot (x + 2)}{(x - 4) \cdot (x + 4)}$

3. Zeros of the numerator = 2 and −2.
   Zeros of the denominator = 4 and −4.

4. The y-intercept is 1/2.

5. (a) The value of the function increases without bound near x = 4.
   (b) The value of the function decreases without bound near $x = -4$.
   (c) $x = 4$ and $x = -4$ are the vertical asymptotes.

6. $f(x)$ approaches = 2;
   $y = 2$ is the horizontal asymptote.

Teacher Information *(Continued)*

Activity 7
Evaluating Rational Functions
(Continued)

Answers to Instructions: Part B

1. Factored form of $f(x) = \dfrac{2 \cdot (x - 1)}{(x + 1)}$

   The zero of the numerator is 1, and the zero of the denominator is −1.

   The y-intercept is −2.

   The behavior of $f(x)$ near the zero of the denominator is that it increases without bound to positive or negative infinity. Thus, the vertical asymptote is $x = -1$.

   The behavior of $f(x)$ as x approaches positive or negative infinity is that it approaches 2. Thus, $y = 2$ is the horizontal asymptote.

2. Factored form of $f(x) = \dfrac{5 \cdot (x - 2)}{(x - 5) \cdot (x + 2)}$

   The zero of the numerator is 2 and the zeros of denominator are 5 and −2.

   The y-intercept is 1.

   The behavior of $f(x)$ near the zeros of the denominator is that it increases without bound to positive or negative infinity. Thus, the vertical asymptotes are $x = 5$ and $x = -2$.

   The behavior of $f(x)$ as x approaches positive or negative infinity is that it approaches 0. Thus, $y = 0$ is the horizontal asymptote.

Activity 7
Evaluating Rational Functions
(Continued)

Answers to Instructions: Part B

3. Factored form of $f(x) = \dfrac{-2 \cdot (x - 2) \cdot (x + 2)}{(x^2 + 4)}$

   The zeros of the numerator are 2 and −2 and the denominator has no real zeros. The y-intercept is 2.

   Because there are no zeros of the denominator, there isn't any behavior to investigate. Thus, there aren't any vertical asymptotes.

   The behavior of $f(x)$ as x approaches positive or negative infinity is that it approaches −2. Thus, $y = -2$ is the horizontal asymptote.

4. Factored form of $f(x) = \dfrac{(x - 3) \cdot (x + 1)}{3 \cdot (x + 2)}$

   The zeros of the numerator are 3 and −1. The zero of the denominator is −2.

   The y-intercept is $\dfrac{-1}{2}$.

   The behavior of $f(x)$ near the zero of the denominator is that it increases without bound to positive or negative infinity. Thus, the vertical asymptote is $x = -2$.

   The behavior of $f(x)$ as x approaches positive or negative infinity is that it approaches positive or negative infinity. Thus, there is no horizontal asymptote.

5. Factored form of $f(x) = \dfrac{3 \cdot (x^2 - 3)}{x \cdot (x - 4) \cdot (x + 6)}$

   The zeros of the numerator are $-\sqrt{3}$ and $\sqrt{3}$. The zeros of the denominator are 0, 4 and −6.

   The behavior of $f(x)$ near the zeros of the denominator is that it increases without bound to positive or negative infinity. Thus, the vertical asymptotes are $x = 0$, $x = 4$ and $x = -6$.

   The y-intercept is undefined.

   The behavior of $f(x)$ as x approaches positive or negative infinity is that it approaches 0. Thus, $y = 0$ is the horizontal asymptote.

Activity 7
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(Continued)

Answers to Questions

1. All except number 3 have vertical asymptotes since the denominators had zeros. The absolute value of the function near the zeros is infinitely large.

2. The degree of the denominator is larger than the degree of the numerator.

3. The degree of the numerator is larger than the degree of the numerator.

4. The degrees of the numerator and the denominator are the same. The ratio of the leading coefficients is the same as the horizontal asymptote.

5. $y = 0$.
   There isn't one.
   y = the quotient of the leading coefficients.