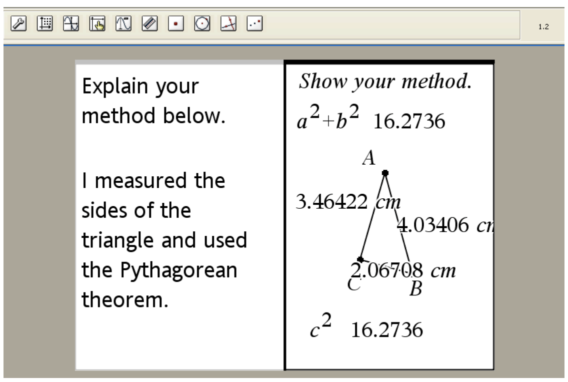
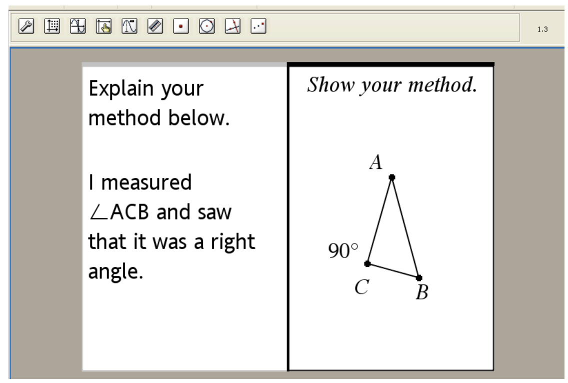
The Lunes of Hippocrates

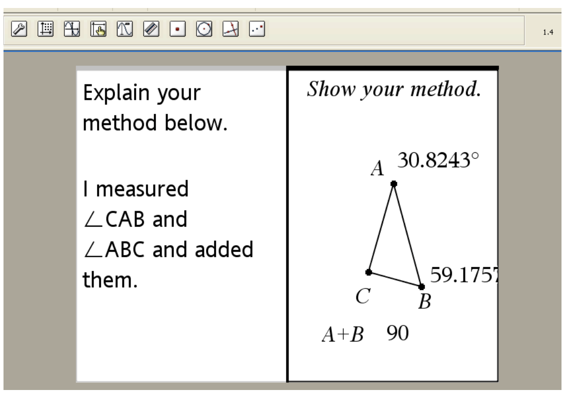
Objective: The students will discover the relationship between the area of the lunes and the area of the right triangle used to create the lunes.

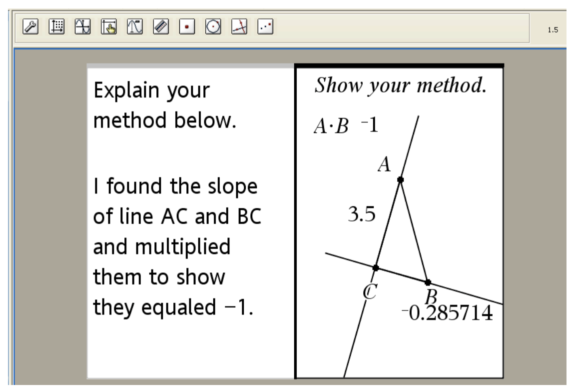
Send all the students the tns file *Lunes of Hippocrates*.

1. There are 4 ways your students could convince you that ΔABC is a right triangle.

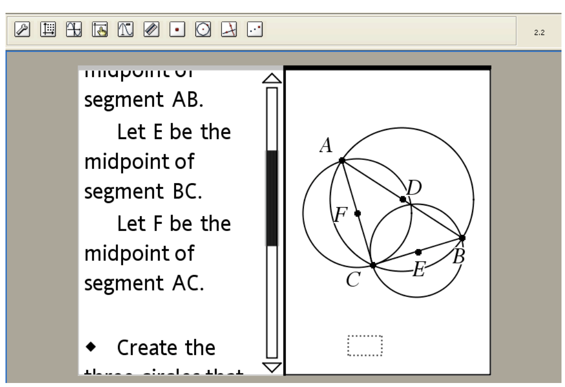




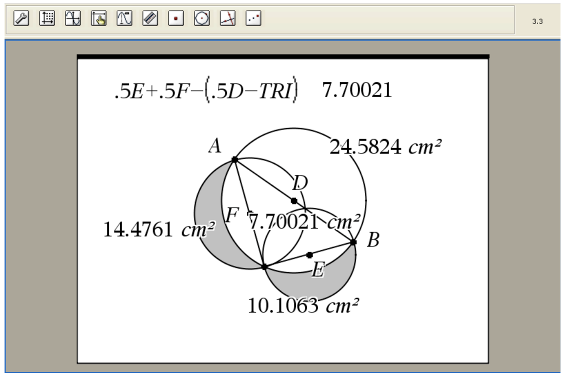




2. The figure the students create should look like this:



3. The students should discover that the area of the lunes is the same as the area of the triangle.



4. The relationship doesn't hold for other kinds of triangles.

5. Proof:

Area of circle D = $\pi\left(\frac{1}{2}AB\right)^2$ Area of circle E = $\pi\left(\frac{1}{2}CB\right)^2$ Area of circle F = $\pi\left(\frac{1}{2}AC\right)^2$

Area of triangle ABC = $\frac{1}{2}\bullet AC\bullet BC$

Area of the Lunes =

$$\frac{1}{2}\left(\pi\left(\frac{1}{2}AC\right)^2\right)+\frac{1}{2}\left(\pi\left(\frac{1}{2}CB\right)^2\right)-\left(\frac{1}{2}\left(\pi\left(\frac{1}{2}AB\right)^2\right)-\frac{1}{2}\bullet AC\bullet BC\right)$$

$$\frac{1}{2}\left(\pi\left(\frac{1}{4}AC^2\right)\right)+\frac{1}{2}\left(\pi\left(\frac{1}{4}CB^2\right)\right)-\left(\frac{1}{2}\left(\pi\left(\frac{1}{4}AB^2\right)\right)-\frac{1}{2}\bullet AC\bullet BC\right)$$

$$\left(\left(\frac{1}{8}\pi AC^2\right)+\left(\frac{1}{8}\pi CB^2\right)\right)-\left(\frac{1}{8}\pi AB^2-\frac{1}{2}\bullet AC\bullet BC\right)$$

$$\left(\frac{1}{8}\pi AC^2+\frac{1}{8}\pi CB^2\right)-\left(\frac{1}{8}\pi\left(AC^2+CB^2\right)-\frac{1}{2}\bullet AC\bullet BC\right)$$

$$\frac{1}{8}\pi AC^2+\frac{1}{8}\pi CB^2-\frac{1}{8}\pi AC^2-\frac{1}{8}\pi CB^2+\frac{1}{2}\bullet AC\bullet BC$$

$$\frac{1}{2}\bullet AC\bullet BC$$

6. Encourage your students to investigate other historical geometric figures.
The Wheel of Theodorus
The Lute of Pythagoras