How Long

Student Activity

7   8   **9   10**   11   12

TI-84CE   Investigation   Student   60 min

Problem

What is the average distance between two points selected randomly within a square with side lengths of 1 unit? To solve this question using analytic methods would require mathematical techniques well beyond high school mathematics courses; however this problem can be solved approximately using simulations, the greater the number of simulations the greater the accuracy of the result. The purpose of this activity is to illustrate how solutions to such problems can be reduced into much simpler versions using simulations.

Question: 1.

Based on the information above estimate the average length of a line drawn between two random points located inside a square with side lengths 1cm.

Question: 2.

To begin here are eight samples where two points A and B have been randomly generated so that they land inside the square. The square has been scaled up to 4cm × 4cm to make it easier to measure the length of line segment AB. Measure the length of the line AB and divide it by 4 to scale it back down, record the result for each square and then compute the average length.
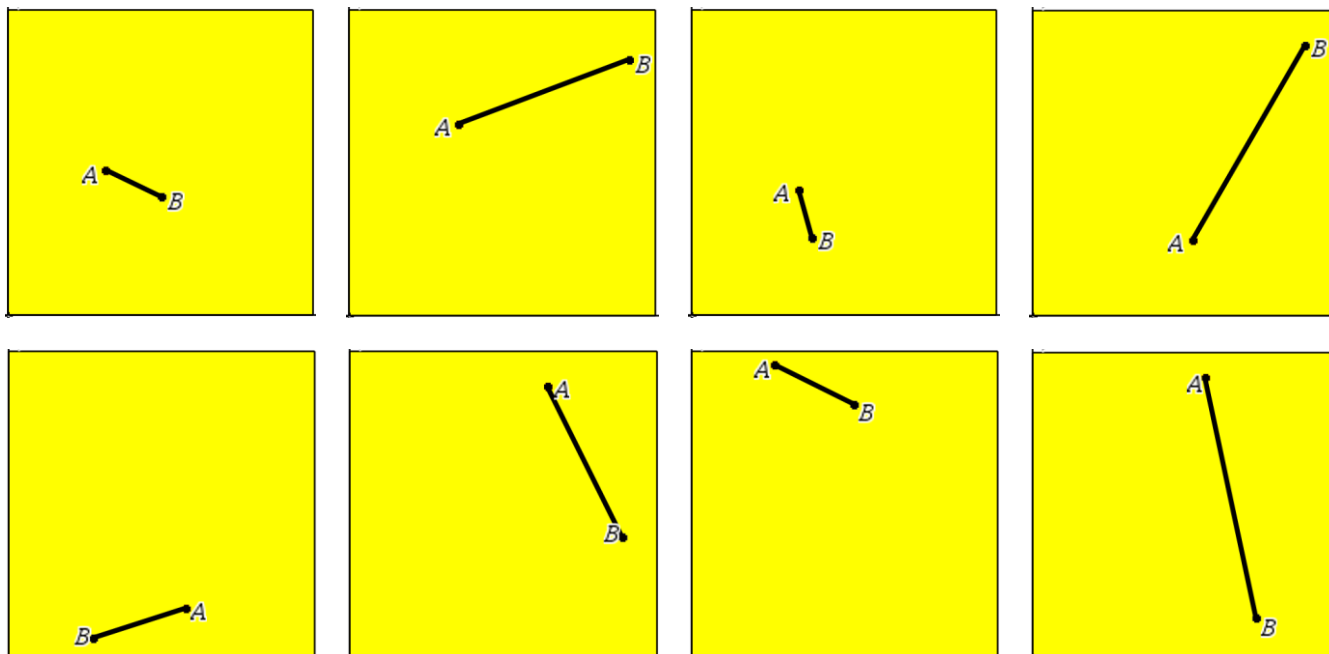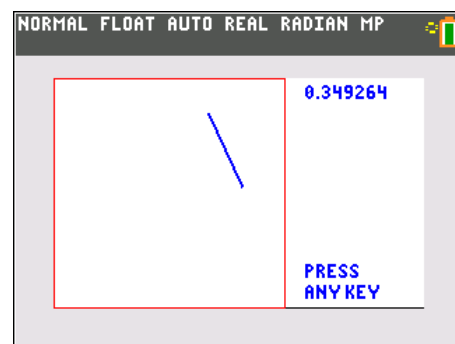

Question: 3.

The minimum length of a line segment produced using this method would be 0cm. What would be the longest possible length?

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TEXAS INSTRUMENTS

Using a Simulator

Run the program: **HowLong**

You will be prompted to select the number of lines you want to generate.

The program pauses between after each line is drawn and automatically computes the length.

```
NORMAL FLOAT AUTO REAL RADIAN MP

                          0.349264



                          PRESS
                          ANY KEY
```

Question: 4.

> Record 20 lengths and calculate the average of these lengths.

Creating Samples

The simulation on the calculator uses the Cartesian plane as a platform. Two points are plotted making it easy to produce random points. The rand() command produces a random number between 0 and 1, so two points can be generated by plotting x_1 = rand(), y_1 = rand() and x_2 = rand(), y_2 = rand(). The length of the line can then be calculated by computing the distance between the two points.
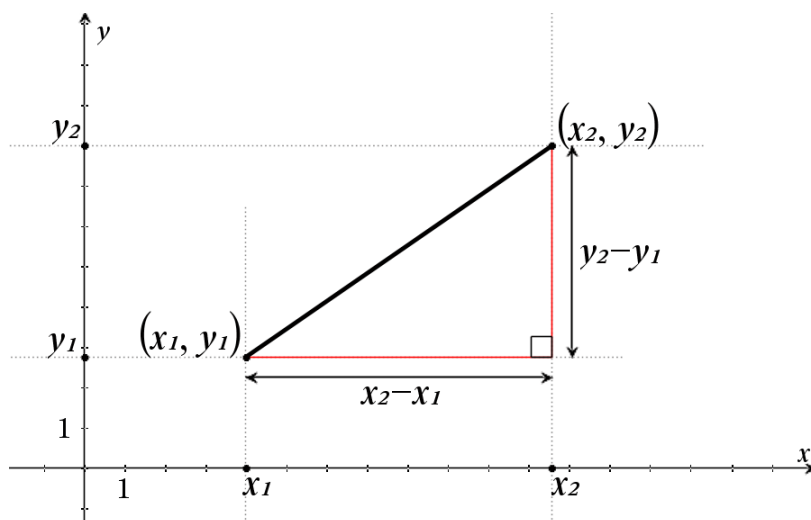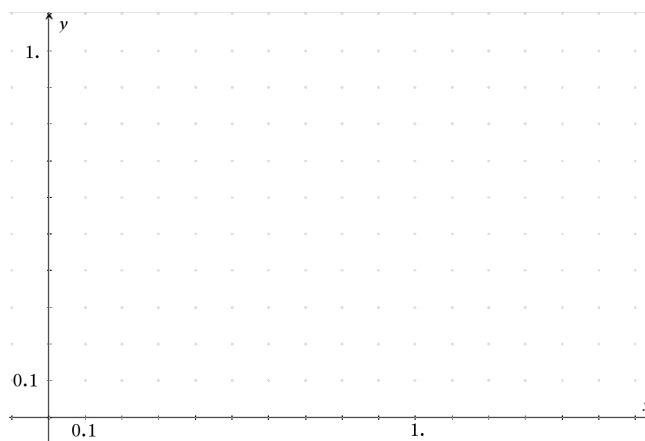
Question: 5.

> Use the diagram shown opposite and Pythagoras's theorem to derive a formula for the distance between two points.

Question: 6.

> Use the calculator to generate 4 random numbers corresponding to x_1, y_1, x_2, y_2.
> Record the points and plot them on the Cartesian plane.
> Use your formula to calculate the distance between the two points.

TEXAS INSTRUMENTS

Creating a LARGE sample

The random number generator can be used to generate large quantities of random numbers. Type the following command to generate 100 random numbers and store them in L_1.

rand(100)$\rightarrow L_1$

Repeat this process to store 100 random numbers in lists L_2, L_3 and L_4.

NOTE: The numbers shown opposite represent a sample only; your random numbers should be different.

In the Stat-List editor navigate to the top of List 5.

Assuming L_1 represents x_1; $L_2 = y_1$, $L_3 = x_2$ and $L_4 = y_2$, type your distance formula at the top of list 5.

The start of the formula is shown opposite.

NOTE: If you place your formula in " " (quotation marks) it will automatically update when values in the dependents lists are changed. This will however slow the calculator down a little as the calculator recomputes all 100 distances each time a list is changed.

Once the distances have been calculated in L_5, return to the home screen and use the LIST menu to calculate the mean of the values (lengths) stored in L_5.

This result is the mean of 100 random lines placed in a 1cm x 1cm box.

Question: 7.

Record the results of 10 sample means. Use this data to estimate the mean length of a line randomly placed inside a 1cm $\times$ 1cm square.

Question: 8.

Suppose the side lengths of the square are doubled in length, how would this affect the average length? Adjust all relevant formulas and record the mean for 10 samples. Use this data to determine the mean length of a line randomly generated inside a 2cm x 2cm square.

HINT: $2 \times$ rand() will produced random numbers between 0 and 2.

Question: 9.

Conduct an investigation into the average length of a line placed inside a rectangle with dimensions 1cm x 2cm.

Extension Investigation

Suppose two random points are placed inside a box measuring 1cm $\times$ 1cm $\times$ 1cm. What would be the average length of the line connecting these two points?

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TEXAS INSTRUMENTS