Expanding Binomials

Name ________________________

Class ________________________

In this activity, a number pattern described in a paper written in 1653 by French mathematician Blaise Pascal will be used to simplify the process of expanding binomials.

On the left below, the first 7 rows of Pascal's Triangle are given. On the right, the same pattern is shown using combination notation. Letting n represent the row number, the top row is $n = 0$.

1. Complete the last row on the right.

1	${}_0C_0$
1 1	${}_1C_0$ ${}_1C_1$
1 2 1	${}_2C_0$ ${}_2C_1$ ${}_2C_2$
1 3 3 1	${}_3C_0$ ${}_3C_1$ ${}_3C_2$ ${}_3C_3$
1 4 6 4 1	${}_4C_0$ ${}_4C_1$ ${}_4C_2$ ${}_4C_3$ ${}_4C_4$
1 5 10 10 5 1	${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$
1 6 15 20 15 6 1	${}_6C_0$

Problem 1 – Exploring $(x + b)^n$

On the Home screen, use the **Expand** command from the Algebra (F2) menu to expand the following binomials.

2. $(x+1)^0 =$

$(x+1)^1 =$

$(x+1)^2 =$

$(x+1)^3 =$

3. What do you notice about the coefficients? The exponents? How do the expansions above seem to relate to Pascal's Triangle?

Expand the following. The letter *b* represents any integer value.

4. $(x+b)^0 =$

$(x+b)^1 =$

$(x+b)^2 =$

$(x+b)^3 =$

5. What effect does *b* have on the expanded binomial?

6. Rewrite $1 \cdot x^3 + 3 \cdot b \cdot x^2 + 3 \cdot b^2 \cdot x + 1 \cdot b^3$ using combination notation.

Problem 2 – Exploring $(ax + 1)^n$

7. Expand the following binomials. The letter *a* represents any integer value.

$(2x+1)^0 =$

$(2x+1)^1 =$

$(2x+1)^2 =$

$(2x+1)^3 =$

$(ax+1)^0 =$

$(ax+1)^1 =$

$(ax+1)^2 =$

$(ax+1)^3 =$

8. What effect does *a* have on the expanded binomial?

9. Write $(ax+1)^4$ in expanded form using Pascal's triangle. Do not use the calculator.

10. Rewrite $(ax+1)^4$ in expanded form using combination notation.

Problem 3 – Exploring $(ax + b)^n$

11. Expand the following binomials. Remember, *a* and *b* represent integer values.

$(3x+2)^0 =$	$(ax+b)^0 =$
$(3x+2)^1 =$	$(ax+b)^1 =$
$(3x+2)^2 =$	$(ax+b)^2 =$
$(3x+2)^3 =$	$(ax+b)^3 =$

12. What is the pattern involving *a* and *b* in $(ax+b)^n$?

13. Write expansion of the following binomials using combination notation. Remember that the first and last term have coefficients of 1.

$(ax+b)^0 =$

$(ax+b)^1 =$

$(ax+b)^2 =$

$(ax+b)^3 =$

14. The pattern established in this problem can be generalized as the Binomial Theorem. State the Binomial Theorem by writing the first two and last two terms of the expanded binomial $(ax+b)^n$ using combination notation.

$(ax+b)^n =$

Extra Problems

Use the Binomial Theorem to expand the following binomials.

1. $(6x + 1)^5$

2. $(x + 7)^6$

3. $(3x + 5)^4$

4. $(7x + 4)^8$