

## Closure Tables – ID: 8639

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Time required  
30 minutes

## Activity Overview

*In this activity, students create and complete closure tables to determine if the sets of whole numbers, integers, even numbers, and odd numbers are closed under the operations of addition, subtraction, multiplication, and division.*

*This activity provides a review of the definitions of whole number and integer as well as practice with integer operations. Students experience the properties of operations as concrete patterns by performing many operations and recording their answers in an organized way. Because of the large number of simple calculations involved, this activity is also a good way for students new to the handheld to gain confidence with its use and familiarity with the keypad layout.*

## Concepts

- *Closure of the set of integers under addition, subtraction, and multiplication.*
- *Determining the operations under which the sets of even and odd integers are closed.*

## Teacher Preparation

*This activity may be used to introduce the closure axioms and/or review integer operations. You may choose to use the activity in its entirety or break it up into separate activities by sets.*

- *Students should be familiar with the definitions of whole number and integer prior to beginning this activity. Although it will reinforce related concepts, students should have experience with integer operations prior to this activity.*
- *It is very important that you thoroughly describe what it means for a set to be **closed** under an operation prior to the class completing the activity on their own. One way to do so is by completing and discussing Problem 1 together as a class.*
- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on page 6 for a preview of the student TI-Nspire document (.tns file).*
- **To download the student .tns file and student worksheet, go to [education.ti.com/exchange](http://education.ti.com/exchange) and enter “8639” in the quick search box.**

## Classroom Management

- *This activity is intended to be introduced in a **teacher-led**, whole class setting. Once it has been introduced, students can work **individually or in pairs** to complete the closure tables, following directions on the student worksheet.*
- *The student worksheet Alg1Act30\_ClosureTables\_worksheet\_EN is intended to guide students through the main ideas of the activity and serves as a place for students to record their answers. Alternatively, you may wish to have the class record answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *The closure table pages are vertically split: Lists & Spreadsheet on the left and Graphs & Geometry on the right. Calculations that drive the implementations are hidden in the spreadsheet, so as to expose only Column A. Make sure that students only change the numbers in Column A.*

## TI-Nspire™ Applications

*Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes*

Algebra is a complex system built on a foundation of a few simple principles. Students are already familiar with many of these principles: basic properties like the identity property of addition form the structure that makes the rest of the algebra you will learn this year work. They are the “ground rules” for the field on which we play algebra.

The properties of operations are rules that describe how addition, subtraction, multiplication, and division work. Rules for different operations are different: addition is commutative and subtraction is not.

This lesson explores the closure properties of operations. These properties describe how an operation works over a set of numbers. A set is a group of numbers. Some examples of sets are  $\{0, 1, 2, 3, 4\}$ , all integers greater than 100, and the negative numbers.

We say a set is closed under a particular operation if you can choose any two numbers from the set, perform the operation with them, and the output still belongs to the set.

One way to demonstrate the idea of a closed set is to cut circles and rectangles from paper. Use two bags to represent sets. Explain to the class that only circles can go in the one bag and only rectangles can go in the other. Now pose the question: are these sets closed under the operation “cut in half”? Demonstrate that the set of rectangles is closed under “cut in half” and the set of circles is not.

### Problem 1 – The set $\{0, 1\}$

Move through this problem slowly with students, taking time to be sure that they grasp the relationship between the four multiplication problems and the closure table.

#### **Solutions**

1. yes
2. no
3. no

Note that closure under an operation depends on both the operation and the set. For example, the set of odd numbers is closed under multiplication but not subtraction.

The calculator screen displays a window with a title bar containing tabs for 1.1, 1.2, 1.3, and 1.4, and a mode indicator for RAD, APPRX, and REAL. The main text area contains the following text:

Closure tables organize the work of checking to see if sets are closed under operations. The closure table for  $\{0, 1\}$  under multiplication is below.

To the left of the table are four multiplication problems:

$$\begin{array}{l} 0 \cdot 0 = 0 \\ 0 \cdot 1 = 0 \\ 1 \cdot 0 = 0 \\ 1 \cdot 1 = 1 \end{array}$$

The closure table is a 3x3 grid with the following values:

$\times$	0	1
0	0	0
1	0	1

**Problem 2 – The whole numbers**

Review the definition of whole number if necessary.

**Solutions**

1. There is a pattern along the diagonal of the table. Sums above and below the diagonal repeat.
2. The commutative property of addition
3. Yes
4. Yes
5. Answers may vary. Sample table shown at right.
6. There is a pattern along the diagonal of the table. The differences above the diagonal are the opposite of the differences below the diagonal.
7. No
8. No
9. Answers may vary. Sample tables shown at right.
10. There is a pattern along the diagonal of the table. Products above and below the diagonal repeat.
11. The commutative property of multiplication
12. Yes
13. No

The calculator screen displays a subtraction table titled "Subtraction". The table has a grid with rows and columns labeled 0 through 5. The values in the cells represent the result of subtracting the column number from the row number.

-	0	1	2	3	4	5
0	0	-1	-2	-3	-4	-5
1	1	0	-1	-2	-3	-4
2	2	1	0	-1	-2	-3
3	3	2	1	0	-1	-2
4	4	3	2	1	0	-1
5	5	4	3	2	1	0

Exercise 5

The calculator screen displays a multiplication table titled "Multiplication". The table has a grid with rows and columns labeled 0 through 5. The values in the cells represent the product of the row number and the column number.

×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25

Exercise 9

The calculator screen displays a division table titled "Division". The table has a grid with rows and columns labeled 0 through 5. The values in the cells represent the result of dividing the row number by the column number.

$\frac{n}{m}$	0	1	2	3	4	5
0	unde0	0	0	0	0	0
1	unde1	0.5	0.3	0.3	0.2	
2	unde2	1	0.7	0.5	0.4	
3	unde3	1.5	1	0.8	0.6	
4	unde4	2	1.3	1	0.8	
5	unde5	2.5	1.7	1.3	1	

**Problem 3 – The integers**

Review the definition of the integers if necessary.

**Solutions**

1. The negative numbers, such as  $-3$  and  $-67$ .
2. Sample answer:  $-2, -1, 0, 1, 2,$  and  $3$
- 3-6. Answers may vary. Sample table shown at right and in Solution .tns file.
7. The products and quotients are positive when both factors are positive or both factors are negative. The products and quotients are negative when one factor is positive and the other is negative.
8. Addition, subtraction, and multiplication

The calculator screen displays a table titled "Addition". The table has 8 rows and 8 columns. The first row is the header row with values: +, -2, -1, 0, 1, 2, 3. The subsequent rows are labeled 1 through 3 on the left side. The values in the table are as follows:

	+	-2	-1	0	1	2	3
-2	-2	-4	-3	-2	-1	0	1
-1	-1	-3	-2	-1	0	1	2
0	0	-2	-1	0	1	2	3
1	1	-1	0	1	2	3	4
2	2	0	1	2	3	4	5
3	3	1	2	3	4	5	6

Exercise 3

The calculator screen displays a table titled "Subtraction". The table has 8 rows and 8 columns. The first row is the header row with values: -, -2, -1, 0, 1, 2, 3. The subsequent rows are labeled 1 through 3 on the left side. The values in the table are as follows:

	-	-2	-1	0	1	2	3
-2	-2	0	-1	-2	-3	-4	-5
-1	-1	1	0	-1	-2	-3	-4
0	0	2	1	0	-1	-2	-3
1	1	3	2	1	0	-1	-2
2	2 <td>4</td> <td>3</td> <td>2</td> <td>1</td> <td>0</td> <td>-1</td>	4	3	2	1	0	-1
3	3 <td>5</td> <td>4</td> <td>3</td> <td>2</td> <td>1</td> <td>0</td>	5	4	3	2	1	0

Exercise 4

The calculator screen displays a table titled "Division". The table has 8 rows and 8 columns. The first row is the header row with values:  $\frac{n}{m}$ , -2, -1, 0, 1, 2, 3. The subsequent rows are labeled -2, -1, 0, 1, 2, 3 on the left side. The values in the table are as follows:

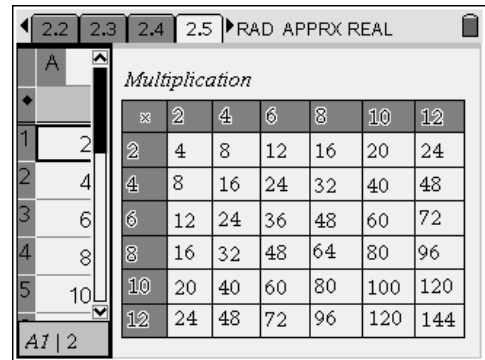
$\frac{n}{m}$	-2	-1	0	1	2	3
-2	1	2	undef	2	-1	-0.7
-1	0.5	1	undef	1	-0.5	-0.3
0	0	0	undef	0	0	0
1	-0.5	-1	undef	1	0.5	0.3
2	-1	-2	undef	2	1	0.7
3	-1.5	-3	undef	3	1.5	1

Exercise 6

**Problem 4 – The even and odd numbers**

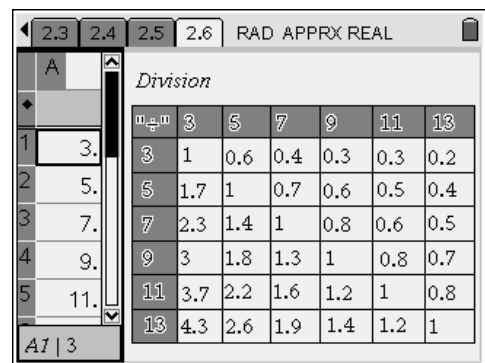
**Solutions**

1. Addition, subtraction, and multiplication
2. Multiplication



A screenshot of the TI-nspire calculator interface showing a multiplication table. The table has columns labeled 2, 4, 6, 8, 10, 12 and rows labeled 2, 4, 6, 8, 10, 12. The values are the products of the row and column numbers. The interface also shows a scrollable list on the left with values 2, 4, 6, 8, 10 and a status bar at the bottom indicating 'A1 | 2'.

$\times$	2	4	6	8	10	12
2	4	8	12	16	20	24
4	8	16	24	32	40	48
6	12	24	36	48	60	72
8	16	32	48	64	80	96
10	20	40	60	80	100	120
12	24	48	72	96	120	144



A screenshot of the TI-nspire calculator interface showing a division table. The table has columns labeled 3, 5, 7, 9, 11, 13 and rows labeled 3, 5, 7, 9, 11, 13. The values are the results of dividing the row number by the column number. The interface also shows a scrollable list on the left with values 3, 5, 7, 9, 11 and a status bar at the bottom indicating 'A1 | 3'.

$\frac{n}{m}$	3	5	7	9	11	13
3	1	0.6	0.4	0.3	0.3	0.2
5	1.7	1	0.7	0.6	0.5	0.4
7	2.3	1.4	1	0.8	0.6	0.5
9	3	1.8	1.3	1	0.8	0.7
11	3.7	2.2	1.6	1.2	1	0.8
13	4.3	2.6	1.9	1.4	1.2	1

3.

set	closed under +	closed under -	closed under $\times$	closed under $\div$
$\{0, 1\}$	no	no	yes	no
whole numbers	yes	no	yes	no
integers	yes	yes	yes	no
even numbers	yes	yes	yes	no

## Closure Tables – ID: 8639

(Student)TI-Nspire File: *Alg1Act30\_ClosureTables\_EN.tns*

1.1 1.2 1.3 1.4 RAD APPRX REAL

### CLOSURE TABLES

**Algebra 1**

Which sets are closed  
under which operations?

1.1 1.2 1.3 1.4 RAD APPRX REAL

Is the set  $\{0, 1\}$  closed under multiplication?  
Make all the multiplication problems you can  
using two numbers of this set. You can use a  
number twice.

0 · 0 = 0

1/99

1.1 1.2 1.3 1.4 RAD APPRX REAL

There are 4 possible multiplication problems.

$0 \cdot 0 = 0$   
 $1 \cdot 0 = 0$   
 $0 \cdot 1 = 0$   
 $1 \cdot 1 = 1$

Look at the answers. Do they all belong to the  
set  $\{0, 1\}$ ?

1.1 1.2 1.3 1.4 RAD APPRX REAL

Closure tables organize the work of checking  
to see if sets are closed under operations.  
The closure table for  $\{0, 1\}$  under  
multiplication is below.

$0 \cdot 0 = 0$   
 $0 \cdot 1 = 0$   
 $1 \cdot 0 = 0$   
 $1 \cdot 1 = 1$

×	0	1
0	0	0
1	0	1

1.2 1.3 1.4 2.1 RAD APPRX REAL

The screens on pages 2.3–2.6 contain blank  
closure tables for the operations addition,  
subtraction, multiplication, and division.

To use the tables to test for closure, input up  
to 6 examples from the set you are testing  
into Column A of the spreadsheet on the left.  
Then examine the results in the table.

1.3 1.4 2.1 2.2 RAD APPRX REAL

Use the closure tables on screens 2.3–2.6 to  
find under which operations  $(+, -, \cdot, \div)$  each  
of these sets are closed.

1. Whole numbers:
2. Integers:
3. Even numbers:
4. Odd numbers:

1.4 2.1 2.2 2.3 RAD APPRX REAL

**Addition**

+	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	6
2	2	3	4	5	6	7
3	3	4	5	6	7	8
4	4	5	6	7	8	9
5	5	6	7	8	9	10

2.1 2.2 2.3 2.4 RAD APPRX REAL

**Subtraction**

-	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

2.2 2.3 2.4 2.5 RAD APPRX REAL

**Multiplication**

×	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

2.3 2.4 2.5 2.6 RAD APPRX REAL

**Division**

÷	0	0	0	0	0	0
0	undef	undef	undef	undef	undef	undef
0	undef	undef	undef	undef	undef	undef
0	undef	undef	undef	undef	undef	undef
0	undef	undef	undef	undef	undef	undef
0	undef	undef	undef	undef	undef	undef
0	undef	undef	undef	undef	undef	undef