Closure Tables – ID: 8639

By Lynne Plettenberg

Time required
30 minutes

Activity Overview

In this activity, students create and complete closure tables to determine if the sets of whole numbers, integers, even numbers, and odd numbers are closed under the operations of addition, subtraction, multiplication, and division.

This activity provides a review of the definitions of whole number and integer as well as practice with integer operations. Students experience the properties of operations as concrete patterns by performing many operations and recording their answers in an organized way. Because of the large number of simple calculations involved, this activity is also a good way for students new to the handheld to gain confidence with its use and familiarity with the keypad layout.

Concepts

- *Closure of the set of integers under addition, subtraction, and multiplication.*
- *Determining the operations under which the sets of even and odd integers are closed.*

Teacher Preparation

This activity may be used to introduce the closure axioms and/or review integer operations. You may choose to use the activity in its entirety or break it up into separate activities by sets.

- *Students should be familiar with the definitions of whole number and integer prior to beginning this activity. Although it will reinforce related concepts, students should have experience with integer operations prior to this activity.*
- *It is very important that you thoroughly describe what it means for a set to be* ***closed*** *under an operation prior to the class completing the activity on their own. One way to do so is by completing and discussing Problem 1 together as a class.*
- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on page 6 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "8639" in the quick search box.***

Classroom Management

- *This activity is intended to be introduced in a* ***teacher-led****, whole class setting. Once it has been introduced, students can work* ***individually or in pairs*** *to complete the closure tables, following directions on the student worksheet.*
- *The student worksheet Alg1Act30_ClosureTables_worksheet_EN is intended to guide students through the main ideas of the activity and serves as a place for students to record their answers. Alternatively, you may wish to have the class record answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*
- *The closure table pages are vertically split: Lists & Spreadsheet on the left and Graphs & Geometry on the right. Calculations that drive the implementations are hidden in the spreadsheet, so as to expose only Column A. Make sure that students only change the numbers in Column A.*

TI-Nspire™ Applications

Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes

Algebra is a complex system built on a foundation of a few simple principles. Students are already familiar with many of these principles: basic properties like the identity property of addition form the structure that makes the rest of the algebra you will learn this year work. They are the "ground rules" for the field on which we play algebra.

The properties of operations are rules that describe how addition, subtraction, multiplication, and division work. Rules for different operations are different: addition is commutative and subtraction is not.

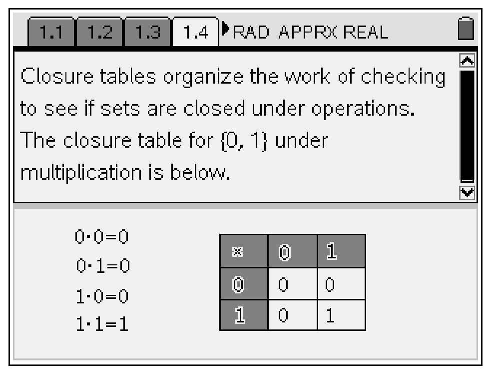
This lesson explores the closure properties of operations. These properties describe how an operation works over a set of numbers. A set is a group of numbers. Some examples of sets are {0, 1, 2, 3, 4}, all integers greater than 100, and the negative numbers.

We say a set is closed under a particular operation if you can choose any two numbers from the set, perform the operation with them, and the output still belongs to the set.

One way to demonstrate the idea of a closed set is to cut circles and rectangles from paper. Use two bags to represent sets. Explain to the class that only circles can go in the one bag and only rectangles can go in the other. Now pose the question: are these sets closed under the operation "cut in half"? Demonstrate that the set of rectangles is closed under "cut in half" and the set of circles is not.

Problem 1 – The set {0, 1}

Move through this problem slowly with students, taking time to be sure that they grasp the relationship between the four multiplication problems and the closure table.



Solutions

1. yes
2. no
3. no

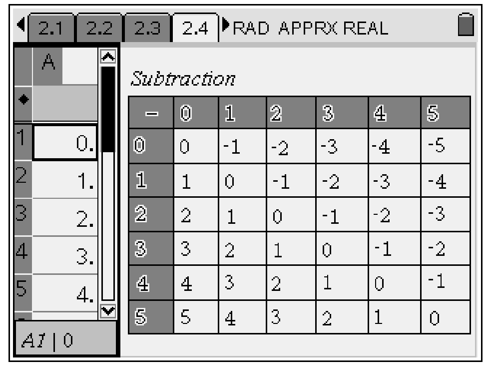
Note that closure under an operation depends on both the operation and the set. For example, the set of odd numbers is closed under multiplication but not subtraction.

Problem 2 – The whole numbers

Review the definition of whole number if necessary.

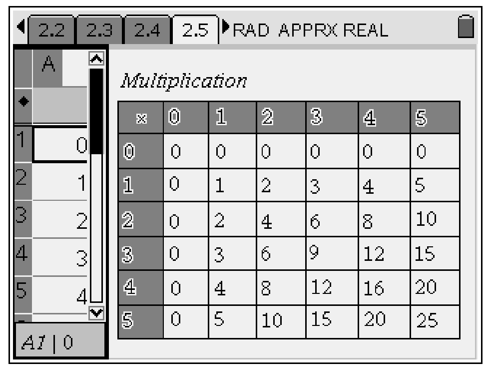
Solutions

1. There is a pattern along the diagonal of the table. Sums above and below the diagonal repeat.
2. The commutative property of addition
3. Yes
4. Yes
5. Answers may vary. Sample table shown at right.
6. There is a pattern along the diagonal of the table. The differences above the diagonal are the opposite of the differences below the diagonal.
7. No
8. No
9. Answers may vary. Sample tables shown at right.
10. There is a pattern along the diagonal of the table. Products above and below the diagonal repeat.
11. The commutative property of multiplication
12. Yes
13. No

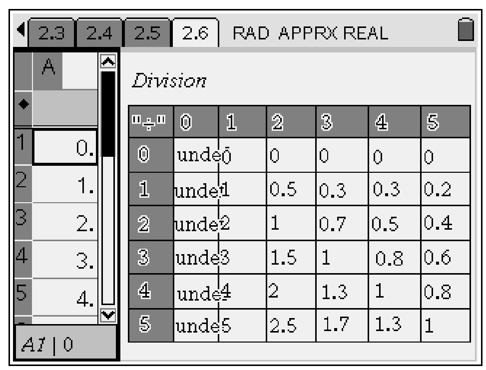


−	0	1	2	3	4	5
0	0	-1	-2	-3	-4	-5
1	1	0	-1	-2	-3	-4
2	2	1	0	-1	-2	-3
3	3	2	1	0	-1	-2
4	4	3	2	1	0	-1
5	5	4	3	2	1	0

Exercise 5



×	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	6	8	10
3	0	3	6	9	12	15
4	0	4	8	12	16	20
5	0	5	10	15	20	25



"÷"	0	1	2	3	4	5
0	unde	0	0	0	0	0
1	unde	1	0.5	0.3	0.3	0.2
2	unde	2	1	0.7	0.5	0.4
3	unde	3	1.5	1	0.8	0.6
4	unde	4	2	1.3	1	0.8
5	unde	5	2.5	1.7	1.3	1

Exercise 9

Problem 3 – The integers

Review the definition of the integers if necessary.

Solutions

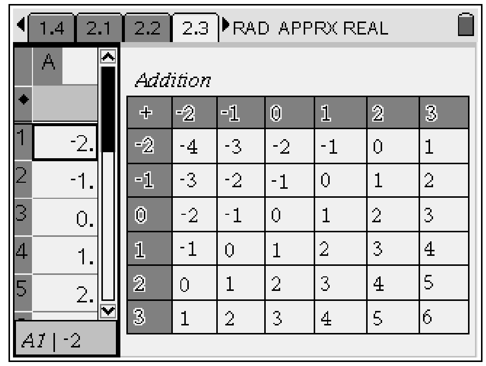
1. The negative numbers, such as –3 and –67.

2. Sample answer: –2, –1, 0, 1, 2, and 3

3-6. Answers may vary. Sample table shown at right and in Solution .tns file.

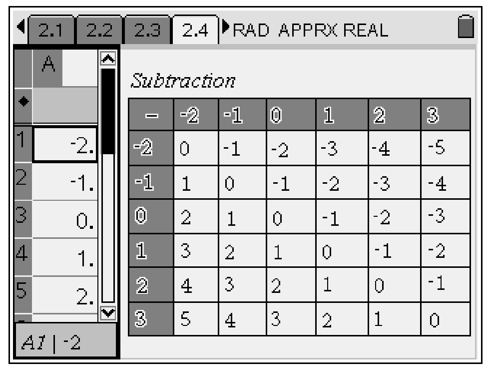
7. The products and quotients are positive when both factors are positive or both factors are negative. The products and quotients are negative when one factor is positive and the other is negative.

8. Addition, subtraction, and multiplication



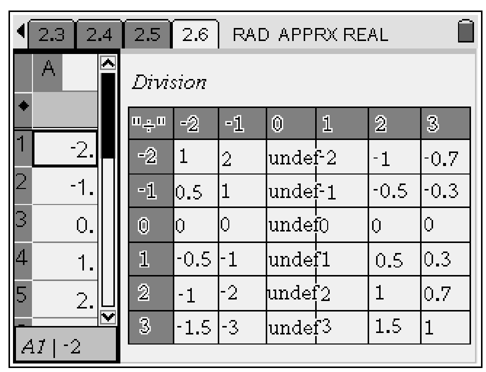
+	-2	-1	0	1	2	3
-2	-4	-3	-2	-1	0	1
-1	-3	-2	-1	0	1	2
0	-2	-1	0	1	2	3
1	-1	0	1	2	3	4
2	0	1	2	3	4	5
3	1	2	3	4	5	6

Exercise 3



−	-2	-1	0	1	2	3
-2	0	-1	-2	-3	-4	-5
-1	1	0	-1	-2	-3	-4
0	2	1	0	-1	-2	-3
1	3	2	1	0	-1	-2
2	4	3	2	1	0	-1
3	5	4	3	2	1	0

Exercise 4



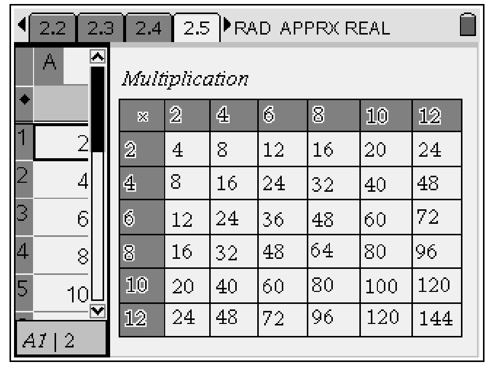
"÷"	-2	-1	0	1	2	3
-2	1	2	undef	-2	-1	-0.7
-1	0.5	1	undef	-1	-0.5	-0.3
0	0	0	undef	0	0	0
1	-0.5	-1	undef	1	0.5	0.3
2	-1	-2	undef	2	1	0.7
3	-1.5	-3	undef	3	1.5	1

Exercise 6

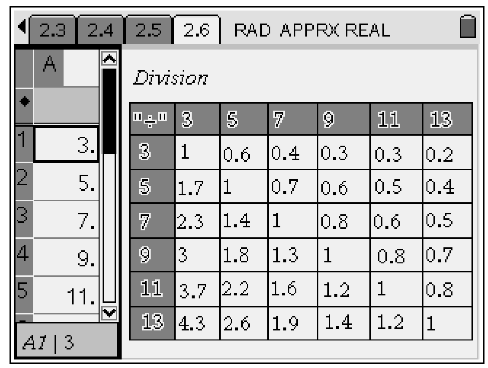
Problem 4 – The even and odd numbers

Solutions

1. Addition, subtraction, and multiplication
2. Multiplication



×	2	4	6	8	10	12
2	4	8	12	16	20	24
4	8	16	24	32	40	48
6	12	24	36	48	60	72
8	16	32	48	64	80	96
10	20	40	60	80	100	120
12	24	48	72	96	120	144



"÷"	3	5	7	9	11	13
3	1	0.6	0.4	0.3	0.3	0.2
5	1.7	1	0.7	0.6	0.5	0.4
7	2.3	1.4	1	0.8	0.6	0.5
9	3	1.8	1.3	1	0.8	0.7
11	3.7	2.2	1.6	1.2	1	0.8
13	4.3	2.6	1.9	1.4	1.2	1

3.

set	closed under +	closed under –	closed under ×	closed under ÷
{0, 1}	no	no	yes	no
whole numbers	yes	no	yes	no
integers	yes	yes	yes	no
even numbers	yes	yes	yes	no

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(Student)TI-Nspire File: *Alg1Act30_ClosureTables_EN.tns*