## Walk the Line: Straight Line Distance Graphs

When one quantity changes at a constant rate with respect to another, we say they are linearly related. Mathematically, we describe this relationship by defining a linear equation. In real-world applications, some quantities are linearly related and can be represented by using a straight-line graph.

In this activity, you will create straight-line, or constant-speed, distance versus time plots using a Motion Detector, and then develop linear equations to describe these plots mathematically.


## OBJECTIVES

- Record distance versus time data for a person walking at a uniform rate.
- Analyze the data to extract slope and intercept information.
- Interpret the slope and intercept information for physical meaning.


## MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator
EasyData application
CBR 2 or Go! Motion and direct calculator cable or Motion Detector and data-collection interface

## PROCEDURE

1. Set up the Motion Detector and calculator.
a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
b. Turn on the calculator and make sure it is on the home screen.

Connect it to the Motion Detector. (This may require the use of a data-collection interface.)
2. Position the Motion Detector on a table or chair so that the head is pointing horizontally out into an open area where you can walk. There should be no chairs or tables nearby.
3. Set up EasyData for data collection.
a. Start the EasyData application, if it is not already running.
b. Select (File) from the Main screen, and then select New to reset the application.
4. Stand about a meter from the Motion Detector. When you are ready to collect data, select Start from the Main screen. Walk away from the Motion Detector at a slow and steady pace. You will have five seconds to collect data.
5. When data collection is complete, a graph of distance versus time will be displayed. Examine the graph. It should show a nearly linearly increasing function with no spikes or flat regions. If you need to repeat data collection, select $\sqrt{\text { Main }}$ and repeat Step 4.
6. Once you are satisfied with the graph, select Main to return to the Main screen. Exit EasyData by selecting Quit from the Main screen and then selecting $\widetilde{\mathrm{OK}}$.

## ANALYSIS

1. Redisplay the graph outside of EasyData.
a. Press 2 nd [STAT PLOT].
b. Press Enter to select Plot1 and press Enter again to select On.
c. Press 200 m .
d. Press $\Theta$ until ZoomStat is highlighted; press $\mathbb{E N T E R}$ to display a graph with the $x$ and $y$ ranges set to fill the screen with data.
e. Press trace to determine the coordinates of a point on the graph using the cursor keys.
2. The slope-intercept form of a linear equation is $y=m x+b$, where $m$ is the slope of the line and $b$ is the $y$-intercept value. The independent variable is $x$, which represents time, and $y$ is the dependent variable, which represents distance in this activity. Trace across the graph to the left edge to read the $y$-intercept. Record this value as $b$ in the Data Table on the Data Collection and Analysis sheet.
3. One way to determine the slope of the distance versus time graph is to guess a value and then check it by viewing a graph of the line with your data. To do this, enter an equation into the calculator, and then enter a value for the $y$-intercept and store it as variable B.
a. Press $r=$.
b. Press ClEAR to remove any existing equation.
c. Enter the equation $\mathrm{M}^{*} \mathrm{X}+\mathrm{B}$ in the Y 1 field.
d. Press $\mathbb{C}$ until the icon to the left of $Y_{1}$ is blinking. Press ENTER until a bold diagonal line is shown in order to display your model with a thick line.
e. Press 2nd [QUIT] to return to the home screen.
f. Enter your value for the $y$-intercept and then press $\rightarrow$ STOD $\mathbb{E N T E R}$ to store the value in the variable $B$.
4. Now set a value for the slope $m$, and then look at the resulting graph. To obtain a good fit, you will need to try several values for the slope. Use the steps below to store different values to the variable $M$. Start with $M=1$. Experiment until you find one that provides a good fit.
a. Enter a value for the slope $m$ and press sToD $M$ ENTER to store the value in the variable M.
b. Press $\overbrace{\text { GRAPH }}$ to see the data with the model graph superimposed.
c. Press 2 [QUIT] to return to the home screen.
5. Record the optimized value for the slope in the Data Table on the Data Collection and Analysis sheet. Use the values of the slope and intercept to write the equation of the line that best fits the distance versus time data.
6. Another way to determine the slope of a line to fit your data is to use two well-separated data points. Use the cursor keys to move along the data points. Choose two points $\left(x_{1}, y_{1}\right)$ and $\left(x_{2}\right.$, $y_{2}$ ) that are not close to each other and record them in the Data Table on the Data Collection and Analysis sheet.
7. Use the points in the table to compute the slope, $m$, of the distance versus time graph.

$$
m=\frac{y_{2}-y_{1}}{x_{2}-x_{1}}=
$$

$\Rightarrow$ Calculate the slope and answer Question 1 on the Data Collection and Analysis sheet.
8. You can also use the calculator to automatically determine an optimized slope and intercept.
a. Press STAT and use the cursor keys to highlight CALC.
b. Press the number adjacent to LinReg(ax+b) to copy the command to the home screen.
c. Press 2 2nd $[\mathrm{L} 1] \leadsto$ 2nd $\rightarrow \mathrm{L} 6] \leadsto$ to enter the lists containing your data.
d. Press vars and use the cursor keys to highlight Y-VARS.
e. Select Function by pressing ENTER.
f. Press ENTER to copy $\mathrm{Y}_{1}$ to the expression.

On the home screen, you will now see the entry LinReg(ax+b) L1, L6, Y1. This command will perform a linear regression using the $x$-values in L 1 as and the $y$-values in L6. The resulting regression line will be stored in equation variable $\mathrm{Y}_{1}$.
g. Press ENTER to perform the linear regression. Use the parameters $a$ and $b$ to write the equation of the calculator's best-fit line, and record it in the Data Table.
h. Press Graph to see the graph.
$\Rightarrow$ Answer Questions 2-5 on the Data Collection and Analysis sheet.
$\qquad$
Date

## DATA TABLE

| $y$-intercept $\boldsymbol{b}$ |  |
| :---: | :--- |
| optimized slope $\boldsymbol{m}$ |  |
| optimized line equation |  |
| $x_{1}, y_{1}$ |  |
| $x_{2}, y_{2}$ |  |
| regression line equation |  |

## QUESTIONS

1. How does this value compare with the slope you found by trial and error?
2. How do the values of the slope and intercept as determined by the calculator compare to your earlier values? Would you expect them to be exactly the same?
3. Slope is defined as change in $y$-values divided by change in $x$-values. Complete the following statement about slope for the linear data set you collected.

In this activity, slope represents a change in $\qquad$ divided by a change in $\qquad$ -
4. Based on this statement, what are the units of measurement for slope in this activity?
5. The $y$-intercept can be interpreted as the starting position or the starting distance from the Motion Detector. What does the slope represent physically?
Hint: Consider the units of measurement for the slope you described in the previous question.

