EXPLORATIONS™

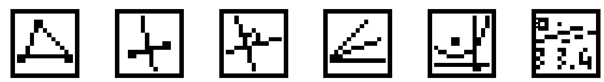
Activity 20

Properties of the Centers of a Triangle

Objectives

- To investigate the relationships among the four centers of a triangle
- To investigate a special property of the circumcenter and orthocenter of a triangle
- To use the Cabri® Jr. **Locus** Tool

Cabri® Jr. Tools



Introduction

In this activity, you will investigate some properties among the four centers of a triangle that you studied in Activity 19, Centers of a Triangle*.* You will also use two of these centers, the circumcenter and the orthocenter, to investigate an interesting relationship that occurs in a triangle.

This activity makes use of the following definitions:

Centroid — the point at which the medians of a triangle intersect.

Circumcenter — the point at which the perpendicular bisectors of a triangle intersect.

Orthocenter — the point at which the altitudes of a triangle intersect.

Incenter — the point at which the angle bisectors of a triangle intersect.

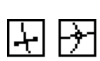
Part I: Properties among the Centers of a Triangle

Construction

Construct a triangle and its four centers.

 Draw an acute scalene triangle in the center of the screen.

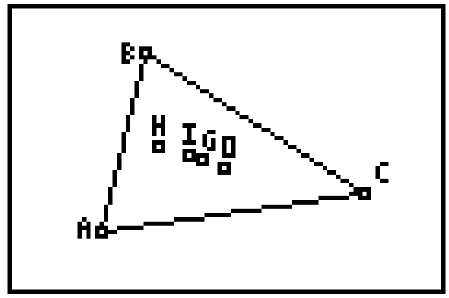
 Construct the centroid (*G*) of the triangle. If necessary, refer to Activity 19 – Part I. Hide the lines and points used to construct the centroid.

 Construct the circumcenter (*O*) of the triangle. If necessary, refer to Activity 19 – Part II. Hide the lines used to construct the circumcenter.

Construct the orthocenter (*H*) of the triangle. If necessary, refer to Activity 19 – Part III. Hide the lines used to construct the orthocenter.

Construct the incenter (*I*) of the triangle. If necessary, refer to Activity 19 – Part IV. Hide the lines used to construct the incenter.

Save your figure.



Exploration

Drag the vertices and sides of the triangle and observe any relationships that appear to be true for the centers of a triangle.

Questions and Conjectures

Make conjectures about the relationship among the four centers of a triangle. Be prepared to demonstrate.

Part II: An Interesting Property of the Circumcenter and the Orthocenter

Construction

Adjust the construction to further investigate the circumcircle and orthocenter.

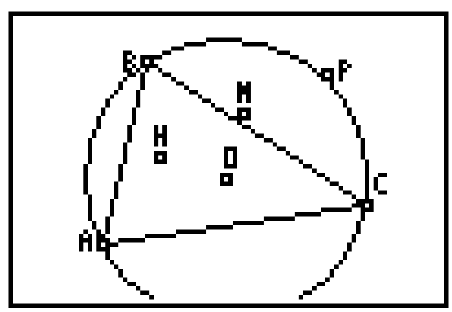
Continue using the previous construction.

Clear or hide the centroid (*G*) and the incenter (*I*).

Construct the circumcircle of your triangle using the circumcenter (*O*) as the center and a vertex of the triangle as the radius point of the circle.

Construct a point *P* on the circumcircle.

Construct a point *M* that is the midpoint between point *P* and the orthocenter (*H*).



Exploration

Drag and/or animate point *P* and observe the apparent path of point *M*.

Use the **Locus** tool to create the locus of points defined by the different locations of point *M* as point *P* moves around the circumcircle.

- Select point *M* (the object generating the locus).
- Select point *P* (the controlling point for the previous object).

Use the **Display** tool to change the locus display from the box to the dot. Verify that the locus shows possible locations for point *M* by dragging or animating point *P*.

Drag a vertex or a side of the triangle and observe the properties of the locus as you change the triangle. Verify that the midpoint between the circumcenter and the orthocenter is the center of the locus.

Questions and Conjectures

1. The locus you generated is known as the nine-point circle (or *Feuerbach Circle*). Describe and be prepared to demonstrate that the following nine points are in the locus: the three midpoints of the sides of the triangle, the three points where the altitudes intersect the sides (or extensions to the sides) of the triangle, the three midpoints of the segments connecting the vertices and the orthocenter (*H*).

2. Make a conjecture about the radius and area of the nine-point circle compared to the circumcircle. Verify your conjecture using the **Dilation** tool. Describe the properties of the dilation you used (object, center, and dilation factor) and be prepared to demonstrate.

Teacher Notes

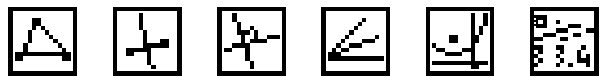
Activity 20

Properties of the Centers of a Triangle

Objectives

- To investigate the relationships among the four centers of a triangle
- To investigate a special property of the circumcenter and orthocenter of a triangle
- To use the Cabri® Jr. **Locus** Tool

Cabri® Jr. Tools



Additional Information

When locating the centers, have the students use only two defining lines.

When constructing the circumcircle, be certain that the radius point of the circle is one of the vertices of the triangle so that the circle is attached to the triangle. If you do not attach the circle to one of the vertices, the circle will not move with the triangle as the vertices are dragged on the screen.

Have students observe the location of point *M* as point *P* moves around the circle. Having students use the **Animate** tool allows them to view the motion of point *M* for as long as they wish. Many students will not guess that the locus of point *M* is a circle. Often they think the path is elliptical because the center of the rotation is not the center of the circumcircle.

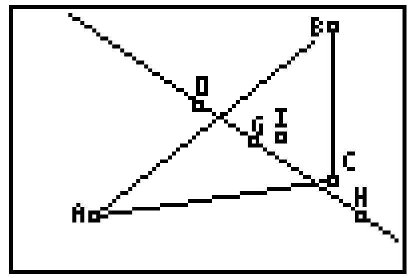
While the Cabri Jr. application provides a more than adequate investigative environment to study these properties, some students or teachers may wish to transfer Cabri Jr. figures to a computer running Cabri Geometry™ II Plus software and continue their explorations in the more robust computer environment. See your Cabri Geometry II Plus guidebook for directions on this procedure.

Part I: Properties among the Centers of a Triangle

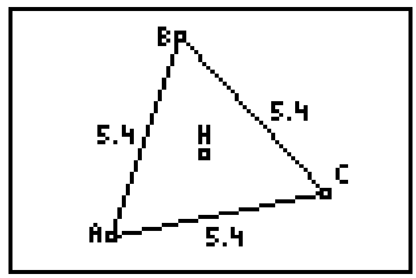
Answers to Questions and Conjectures

Make conjectures about the relationship among the four centers of a triangle. Be prepared to demonstrate.

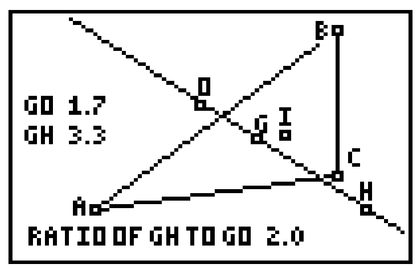
The centroid (*G*), circumcenter (*O*), and the orthocenter (*H*) are always collinear. The incenter, *I*, will also fall on this line (called the *Euler line*) when the triangle is isosceles.



All of these points coincide when the triangle is equilateral.



The centroid, circumcenter, and orthocenter form the Euler segment that is a part of the Euler line. The centroid divides the Euler segment in the ratio of 2 to 1 in the same way it divides the medians of the triangle in the 2 to 1 ratio.

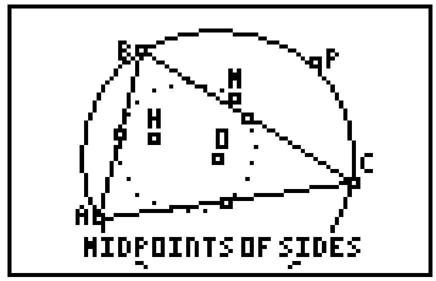


Part II: An Interesting Property of the Circumcenter and the Orthocenter

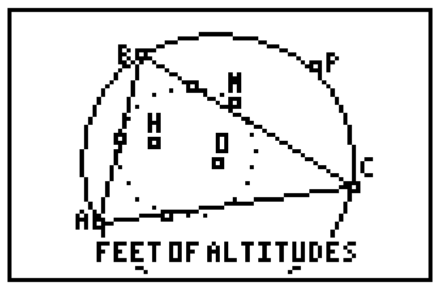
Answers to Questions and Conjectures

1. The locus you generated is known as the nine-point circle (or *Feuerbach Circle*). Describe and be prepared to demonstrate that the following nine points are in the locus: the three midpoints of the sides of the triangle, the three points where the altitudes intersect the sides (or extensions to the sides) of the triangle, the three midpoints of the segments connecting the vertices and the orthocenter (*H*).

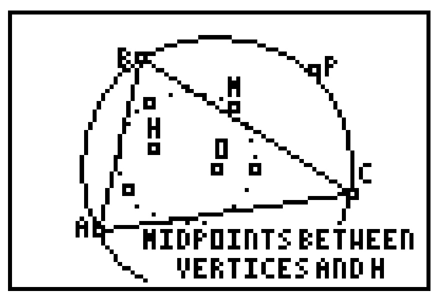
   Construct the midpoints of the sides of the triangle. The three midpoints of $\triangle ABC$ are on the circular locus of point *M*.



   Construct (or show again using the **Hide/ Show** tool) the three altitudes of $\triangle ABC$ and locate the points where these lines cross the sides of the triangle. These points are known as the feet of the altitudes.

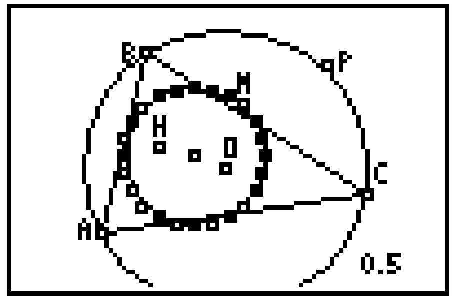


   The final three points that define the nine-point circle are the midpoints of the segments connecting the orthocenter *H* and the vertices of $\triangle ABC$. These points must be on the circle since point *M* that defines the nine-point circle is the midpoint of the segment from the orthocenter to the circumcircle. Since the circumcircle passes through the vertices of $\triangle ABC$, point *P* coincides with the vertices as it moves on the circumcircle. Therefore, point *M* is necessarily the midpoint between each of the vertices and the orthocenter as the definition states.



   It can be shown that each of these nine points are equidistant from the midpoint between points *O* and *H*.

2. Make a conjecture about the radius and area of the nine-point circle compared to the circumcircle. Verify your conjecture using the **Dilation** tool. Describe the properties of the dilation you used (object, center and dilation factor) and be prepared to demonstrate.



Since *M* is a midpoint, the radius of the nine-point circle is one half the radius of the circumcircle, the circular path of *P*. Since the radii are in ratio of 2 to 1, the circumferences share the same ratio and the areas are in the ratio of 4 to 1.

One possible dilation would be the dilation of the circumcircle through the orthocenter using a dilation factor of 0.5. Another would be a dilation of the nine-point circle through the orthocenter using a dilation factor of 2.

When one end of a segment moves on a circular path and the other end is fixed, every point on the segment except the fixed endpoint traces out a circular path. The same is true for any shaped pathway. The size (area, perimeter, and radius) of the traced figure is directly proportional to the distance from the fixed rotation center to the tracing point. This construction is one way to create a dilation. This is the relationship between points *P*, *H*, and *M* in this figure. Point *P* travels on a circular path, point *H* is the fixed end of $\overline{HP}$, and point *M* is the midpoint of the segment.