Exploring Quadratics in Factored Form: Solutions

The quadratics we have explored so far have all been in the form $y = ax^2 + bx + c$. Recall what an equation in this form tells us about its graph.

$y = ax^2 + bx + c$
The "a" value determines: ***the direction of the opening. If a is positive the parabola open up. If a is negative the parabola opens down. The value of a also affects the steepness of the graph.***
The "c" value determines: ***the y-intercept of the graph.***

1. Using this information fill in the chart below. Use your TI-Nspire calculator to check your conclusions.

Equation	Direction of the Opening of the parabola	Point where the parabola crosses the y axis
a) $y = x^2 - 8x + 15$	***up***	(0,15)
b) $y = x^2 + x - 6$	***up***	(0,−6)
c) $y = -x^2 + 4x$	***down***	(0,0)
d) $y = 2x^2+4x + 2$	***up***	(0,2)
e) $y = -3x^2 + 3$	***down***	(0,3)
f) $y = 2x^2 + 12x + 10$	***up***	(0,10)

2. You will be filling in the table below with information you have collected on your calculator. Work through the first example with your teacher and then repeat the same steps for the remainder of the examples.

Equation in form $y = x^2 + bx + c$	Equation in the form $y = (x - r)(x - s)$	x-intercepts	Vertex
a) $y = x^2 - 8x + 15$	$y = (x - 3)(x - 5)$	(3,0)(5,0)	(4,−1)
b) $y = x^2 + x - 6$	$y = (x - 2)(x + 3)$	(2,0)(−3,0)	(−0.5,−6.2)
c) $y = x^2 - 4x - 5$	$y = (x - 5)(x + 1)$	(5,0)(−1,0)	(2,−9)
d) $y = x^2 - 3x$	$y = x(x - 3)$	(0,0)(3,0)	(2,−2)
e) $y = x^2 - 9$	$y = (x - 3)(x + 3)$	(3,0)(−3,0)	(0,−9)
f) $y = x^2 + 6x + 9$	$y = (x + 3)(x + 3)$	(−3,0)	(−3,0)
g) $y = x^2 + 7x + 12$	$y = (x + 5)(x + 2)$	(−5,0)(−2,0)	(−3.5,−2.3)

3. Can you draw a conclusion about the relationship between the factored form of a quadratic and it's x-intercepts? Record it in the box below. Include an example.

$y = (x - r)(x - s)$
The x-intercepts of $y = (x - r)(x - s)$ ***are (r,0) and (s,0)*** ***e.g. For the relation*** $y = (x - 2)(x + 5)$ ***, the intercepts are (2,0) and (-5,0)***

4. a) From your chart in #2, can you see a relationship between the x-intercepts and the x-value of the vertex of the parabola?

The x-value of the vertex is halfway between the x-intercepts of the graph.

b) How could the x-value of the vertex be used to find the y-value of the vertex?

To find the y-value of the vertex substitute the x-value of the vertex into the equation.

c) Use $y = x^2 - 8x + 15$ as an example. The x-intercepts are 3 and 5. How can we use this information to find the vertex of the parabola?

If the x-intercepts are 3 and 5 then then the x-value of the vertex is 4. ***(*** $x_{vertex} = \frac{3+5}{2}$ ***)***

Substitute x=4 into the equation

$y_{vertex} = (x - 1)(x - 5)$

$y_{vertex} = (4 - 1)(4 - 5)$

$y_{vertex} = (3)(-1)$

$y_{vertex} = -3$

therefore the vertex of the parabola is $(4,-3)$

Checking For Understanding: Solutions

Once you have completed the activity sheet Exploring Quadratics in Factored Form, answer the following questions. You may use your TI-Nspire handheld to help you.

1. Given the quadratic $y = x^2 - 11x + 10$. Answer the following questions and provide an explanation.

a) Does the relation have a maximum or minimum point?

The relation has a minimum point because the "a" value is positive so the parabola opens up.

b) What is the y-intercept?

The y-intercept is at (0,10). The y-intercept is the same as the "c" value in the equation.

c) What are the zeroes(x-intercepts) for the relation?

The x-intercepts of the relation are at (1,0) and (10,0). You can see this by using the calculator to factor the expression to $y = (x - 10)(x - 1)$ ***or you can graph the relation and trace to find the points.***

2. Describe how you could find the zeroes(x-intercepts) of the quadratic relation $y = x^2 + 2x - 15$ without graphing the relation.

If we factor the expression we can rewrite the relation in factored form as $y = (x + 5)(x - 3)$ ***and conclude from there that the x-intercepts are at (-5,0) and (3,0).***

3. The vertex of a quadratic relation is at $(-2,5)$. Does the relation have a maximum or minimum value? Explain.

We cannot tell from the information provided if the vertex is a maximum or minimum.

4. Which of the following equations does the parabola shown represent?

Since the x-intercepts shown on the graph are at –2 and 3 the equation must be the one shown in b)

$y = (x + 2)(x - 3)$