Similar Figures

## Math Objectives

- Students will identify corresponding sides and corresponding angles.
- Students will understand and use the definition of similar figures.
- Students will look for and make use of structure (CCSS Mathematical Practice).


## Vocabulary

- ratios
- corresponding sides
- corresponding angles
- congruent
- similar figures


## About the Lesson

- Students will drag vertices of rectangles and triangles and observe what happens to ratios of pairs of sides within each figure.
- As a result, students will:
- Apply the definition of similarity to identify similar figures.
- Make connections between ratios within figures to ratios between figures.
- Confront two common misconceptions: defining similar figures as those with the same shape but different size and identifying all rectangles as similar.


## TI-Nspire ${ }^{\text {TM }}$ Navigator ${ }^{\text {TM }}$ System

- Send out the Similar_Figures.tns file.
- Monitor student progress using Class Capture.
- Use Live Presenter to spotlight student answers.


## Activity Materials

- Compatible TI Technologies: TI-Nspire ${ }^{\text {TM }}$ CX Handhelds,


TI-Nspire ${ }^{\text {TM }}$ Apps for iPad®, ㄹ TI-Nspire ${ }^{\text {TM }}$ Software

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Grab and drag the open circles as directed. Watch the ratios to see when they become equal.

## Tech Tips:

- This activity includes screen captures taken from the TINspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire App. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calcul ators/pd/US/OnlineLearning/Tutorials


## Lesson Materials:

## Student Activity

- Similar_Figures_Student.pdf
- Similar_Figures_Student.doc

TI-Nspire document

- Similar_Figures.tns


## Discussion Points and Possible Answers

Tech Tip: : If students experience difficulty dragging a point, make sure they have not selected more than one point. Press [eso to release points. Check to make sure that they have moved the cursor (arrow) until it becomes a hand (S) getting ready to grab the point. Also, be sure that the word point appears. Then select ctril to grab the point and close the hand (§). When finished moving the point, select esc to release the point.

Tech Tip: To drag a point, have studnets move their finger over the point and then drag it along the screen.

## Move to page 1.2.

1. Drag the two open circles on the bottom rectangle.
a. What happens to the figures?

Answer: One stays the same and the other changes. The shape of the rectangle changes.


Teacher Tip: Students need to realize that both dimensions of the rectangle on the bottom can be changed. Each open circle controls one of the dimensions.
b. What do the numbers in the ratios represent?

Answer: They represent the measures of the sides of the rectangles. The 2 and 6 are the lengths of the sides of the top fixed rectangle. The numerator of the other ratio is always the measure of the shorter side of the bottom rectangle. The denominator is always the measure of the longer side of the bottom rectangle. See the tip below.

Teacher Tip: Students should realize that the given ratios represent the shorter side to the longer side and not the ratio of the vertical measure to the horizontal measure. Students need to drag the circles enough to realize this.

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Teacher Notes
Math Nspired
2. Drag the two open circles until the ratios are equal but the rectangles are not congruent. Describe how the bottom rectangle is related to the top rectangle.

Answer: The two rectangles have the same shape. The lengths of the sides of the bottom rectangle can be found by multiplying the lengths of the sides of the top rectangle by the same number.

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Show a Class Capture of students' work so that students can see different options for making the ratios equal.
3. Drag the open circles on the bottom rectangle to find three other rectangles whose side lengths have a ratio equal to $\frac{2}{6}$. Record the side lengths and ratios in the table below.

|  | Short Side | Long Side | Ratio |
| :---: | :---: | :---: | :---: |
| Top Rectangle | 2 | 6 | $2: 6$ |
| Rectangle 1 | $\mathbf{3}$ | $\mathbf{9}$ | $\mathbf{3}: 9$ |
| Rectangle 2 | $\mathbf{2}$ | $\mathbf{6}$ | $\mathbf{2}: 6$ |
| Rectangle 3 | $\mathbf{1}$ | $\mathbf{3}$ | $\mathbf{1}: 3$ |

Teacher Tip: Students may not find these in the order listed in this table, and they may find others. Be sure they explore both stretching and shrinking the bottom rectangle as they look for the equivalent ratios.
4. Two figures are similar if and only if the ratios of all pairs of corresponding sides are equal and all corresponding angles are congruent. Are any of the rectangles above similar? Tell how you know.
Answer: All of these rectangles are similar. The corresponding angles are congruent because they all measure $90^{\circ}$. The short sides correspond to each other and the long sides correspond to each other. The ratios of the corresponding sides are equal.

Teacher Tip: The above answer is the ideal answer. Students should observe that the angles are congruent because the figures are rectangles. However, transferring from ratios within a figure (ratio of shorter side to longer side) to ratios between two figures (ratios of corresponding sides) is less obvious and may require more questioning from the teacher.

For example: Do the numbers in each ratio come from the same rectangle or different rectangles? Are the ratios on the screen ratios of corresponding sides? What are the ratios of the corresponding sides?
5. Susie drew a rectangle with side $A B$ equal to 12 units and side $B D$ equal to 45 . Find the dimensions of two rectangles that are similar to this one.

Sample Answer: There are many correct answers; for example: 24 units by 90 units, 4 units by 15 units.

Teacher Tip: Look for student answers that are not both whole numbers (e.g., 12 units by 22.5 units). It would be good to bring these to the attention of the entire class.
6. Are all rectangles similar? Why or why not?

Answer: No. Many times when dragging the open circles, the ratios were not equal.

Send the following Always/Sometimes/Never Quick Poll to students:
Two rectangles are $\qquad$ similar.

Answer: Two rectangles are sometimes similar.

## Move to page 2.1.

7. How would you know if the two triangles are similar?

Answer: All the corresponding angles are congruent and the ratios formed by corresponding sides are equal.

8. Drag the open circles at points $I$ and $L$.
a. What happens to the figures?

Answer: The triangle on the right changes; the triangle on the left stays the same.
b. What happens to the angle measures?

Answer: The angle measures in the triangle on the left stay the same. The measure of $\angle l$ and $\angle L$ change.
c. What happens to the numbers in the ratios?

Answer: One ratio stays the same and the other two change.
9. Drag the open circles at points $/$ and $L$ until all three ratios are equal (but not equal to 1 ).
a. What do the numbers in the ratios represent?

Answer: The numbers in the ratios represent the lengths of the corresponding sides of the triangles.
b. What is the same about the two triangles?

Answer: They have the same shape. The side measures of one can be found by multiplying the side measures of the other by the same number. The angle measures are the same.
10. a. Are these two triangles similar? Explain.

Answer: Yes, the corresponding angles are congruent, and the ratios of the corresponding sides are equal.

Teacher Tip: Unlike the rectangles, the ratios on the screen are the ratios of the corresponding sides. Help students make the connection between ratios within these figures and ratios between 2 similar figures (ratios of corresponding sides). If the ratios of 2 pairs of corresponding sides are $3: 6=4: 8$, then the ratios within the figures are equal, i.e., $\frac{3}{6}=\frac{4}{8}$.
b. Use page 2.1 in your tns file to create another triangle similar to the given triangle. Sketch and label the two triangles. Explain why the two triangles are similar.

Sample Answer: Answers will vary depending on what pair of similar figures students find. They will need to explain that the corresponding angles are congruent and the ratios of corresponding sides are all equal.

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Show a Class Capture of students' work so that students can see different pairs of similar triangles.

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11. Are all triangles similar?

Answer: No. When dragging the open circles, you see lots of pairs of triangles where the ratios are not equal. In the pairs when the ratios are not equal, the angles are not equal.

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Send the following Always/Sometimes/Never Quick Poll to students:
Two triangles are $\qquad$ similar.

Answer: Two triangles are sometimes similar.
12. Some people say the definition of two similar figures could be: Two similar figures are two figures that have the same shape and different size. Is this a good definition? Explain your reasoning.

Answer: If you think of a rectangle or triangle as a shape, this definition could be misleading because you have found that not all rectangles are similar and not all triangles are similar. Another problem with this definition is that figures that are the same size (and shape) are similar (and congruent). If you think same shape means corresponding angles are congruent and the ratios of corresponding side are equal, then the definition is OK.

Teacher Tip: From very early in their education, students have answered questions like "What shape is this figure?" with words like triangle and rectangle. The first definition of similarity they hear is "same shape different size." The combination of these two "previous" understandings leads to the misconceptions that this activity is designed to overcome.

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students:

- Transfer between ratios within figures to ratios between figures (corresponding sides).
- Identify similar figures.
- Justify that figures are similar using a more rigorous definition than "same shape, different size."

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## Assessment



Answer: Triangle $C$ is similar to triangle $R$. Corresponding sides are proportional.

