EXPLORATIONS™

Activity 11

Objectives

- Develop the idea of the derivative as a function
- Gather evidence toward some common derivative formulas
- Use numerical and graphical investigations to form conjectures

Materials

- TI-84 Plus / TI-83 Plus

Investigating the Derivatives of Some Common Functions

Introduction

Until now, you have found the instantaneous rate of change of a function at a point. This instantaneous rate of change is also the slope of the line tangent to the graph of the function at the point. This is called the derivative of the function at that point. In this activity, you will expand this concept by looking at an instantaneous rate of change function. An instantaneous rate of change function can be based on a function *f* by defining

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

where $f'(x)$ is the instantaneous rate of change function for *f*. This function could be called the derivative function of *f* but is more often called the *derivative* of *f*.

One of the many ways in which you can think of a derivative is as a function that uses *x* as an input and returns the slope of the line tangent to *f* at *x*. The derivative of a function is often another function with a formula that can be used and applied. You will investigate the derivatives of some common functions by approximating the instantaneous rate of change (using the symmetric difference quotient) at many inputs. You will also use the table and graphing capabilities of your graphing handheld.

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Exploration

Make sure that your graphing handheld is in **Radian** mode and in **Func** mode. Listed below are the steps used to investigate the derivative of *f*(*x*) = sin(*x*) with the table and the symmetric difference quotient.

- Press [Y=] and input **Y1** as **sin(X)**.

Build a virtual slope finder into **Y0**. This slope finder will use the symmetric difference quotient (with *h* = 0.001) to approximate the instantaneous rate of change of the function stored in **Y1**.

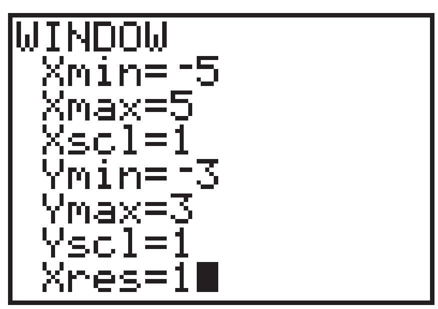
- Input **Y0** as **(Y1(X + 0.001) – Y1(X – 0.001))/0.002.**
- Set up the table as shown in the screen shot. (Note: ΔTbl = 0.1)
- View the table.

The first column (X) contains the input values, the second column (**Y1**) contains the output of *f*(*x*) = sin(*x*) at the corresponding input value, and the third column (**Y0**) contains an approximation of the derivative of *f*(*x*) = sin(*x*) at the corresponding input value. Now, find a common function that has outputs close to the values in the third column.

1. What is the maximum value of **Y0** in the table? What is the minimum value of **Y0** in the table?
2. Between what input values do the first three positive roots of **Y0** occur?
3. What common function do you predict to be $f'(x)$?

   Use your graphing handheld to generate a graph of *f* and the symmetric difference quotient for *f* (graph **Y1** and **Y0**).

   Suggested window settings are shown here.



4. Does the graph of the symmetric difference quotient for *f* look like the graph of the function that you answered in Question **3**? If not, what is your new prediction for $f'(x)$?

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To see how close your prediction for $f'(x)$ is to the symmetric difference quotient of *f*, store the function that is your prediction for the derivative of *f* into **Y2**, and press [2nd] [TABLE].

5. How close is your prediction of $f'(x)$ to the symmetric difference quotient? How many decimal places do they match for most entries?

The advantage of building a general slope finder in **Y0** based only on **Y1** is that the process can be applied to investigate the derivatives of other functions by merely changing **Y1**.

Input **Y1** as **cos(X)**, and press [2nd] [TABLE].

6. What is your prediction for $f'(x)$?

7. Explain your prediction.

Use your graphing handheld to generate a graph of *f* and the symmetric difference quotient for *f* (**Y1** and **Y0**).

8. Does the graph of the symmetric difference quotient for *f* look like the graph of the function that you predicted in Question **6**? If not, what is your new prediction for $f'(x)$?

Store the function that is your prediction for the derivative of *f* into **Y2**, and press [2nd] [TABLE].

9. How close is your prediction of $f'(x)$ to the symmetric difference quotient? How many decimal places do they match for most entries?

Input **Y1** as **ln(X)**, and make a table.

10. What is your prediction for $f'(x)$?
Hint: *Look at the* **X** *and* **Y0** *columns.*

11. Explain your prediction.

Generate a graph of *f* and the symmetric difference quotient for *f* (**Y1** and **Y0**).

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12. Does the graph of the symmetric difference quotient for *f* look like the graph of the function that you predicted in Question **10**? If not, what is your new prediction for $f'(x)$?

Store the function that is your prediction for the derivative of *f* into **Y2**, and press [2nd] [TABLE].

13. How close is your prediction of $f'(x)$ to the symmetric difference quotient? How many decimal places do they match for most entries?

Finally, input **Y1** as **e^X**, and make a table.

14. What is your prediction for $f'(x)$?

15. Briefly explain how you arrived at this prediction.

Use your graphing handheld to generate a graph of *f* and the symmetric difference quotient for *f* (**Y1** and **Y0**).

16. Does the graph of the symmetric difference quotient for *f* look like the graph of the function that you predicted in Question **14**? If not, what is your new prediction for $f'(x)$?

Store the function that is your prediction for the derivative of *f* into **Y2**, and press [2nd] [TABLE].

17. How close is your prediction of $f'(x)$ to the symmetric difference quotient? How many decimal places do they match for most entries?

18. Write a short paragraph summarizing what you have learned from this activity. Include all derivative formulas that you have conjectured.

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