Introduction to the Central Limit Theorem

ID: 9892

Time required
50 minutes

Activity Overview

In this activity, students discover the Central Limit Theorem by simulating rolls of two, four, and seven number cubes via the random number generator. They find sample means and standard deviations on spreadsheets and construct histograms of the sample distributions. By working in small groups and sharing data, the simulations are not as time consuming as they would otherwise be.

Topic: Sampling

- *Central Limit Theorem*
- *Mean and standard deviation;* $\frac{\sigma}{\sqrt{n}}$.

Teacher Preparation

- *Explain to students that rolling two number cubes is akin to selecting two members from a population of six, with replacement. Rolling four number cubes is akin to selecting four members, and so on.*
- *Ideally, there should be at least five groups of three students each. Groups can exist with two only students if needed. At a minimum, there should be three groups, so that students can compile at least 30 sample means (10 from each group). Alternatively, each group can generate 15–20 sample means instead of 10 as listed in the documents and on the worksheet.*
- *Give each group a unique number (it can be any number of digits). They will use this number to set their RandSeed, ensuring different random numbers are generated for each group.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9892" in the keyword search box.***

Associated Materials

- *IntroCLT_Student.doc*
- *IntroCLT.tns*
- *IntroCLT_Soln.tns*

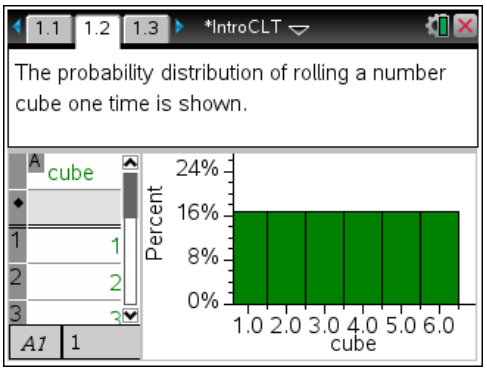
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Central Limit Theorem (TI-Nspire technology) — 9998*
- *The Central Limit Theorem & Sampling Distribution (TI-Nspire technology) — 9641*

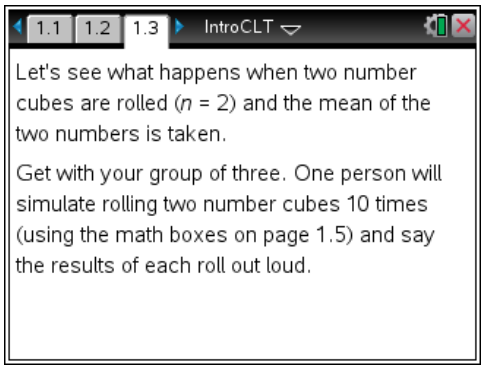
Problem 1 – Rolling Two Number Cubes

Have students look at the probability distribution of rolling a number cube one time on page 1.2. Ask how they would describe the distribution (uniform, nonnormal).



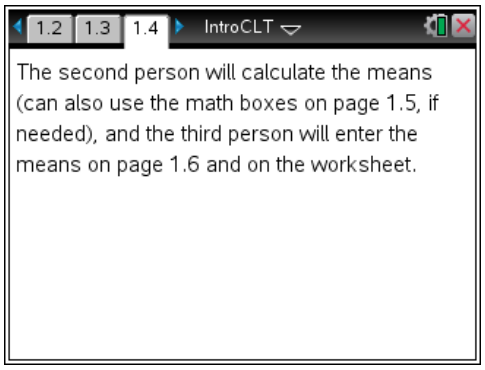
Students will use simulation to study sampling distributions of rolling two, four, and seven number cubes. Mostly they will work independently. Once they get started, walk around the room to assist and answer questions as needed.

Break students into groups of three. If there must be groups of two, that will be okay, but three is ideal. Each student should have a handheld and a student worksheet.



Pages 1.3 and 1.4 explain the roles of the group members. There will be a total of three sets of simulations, so students can switch roles throughout the activity if they wish.

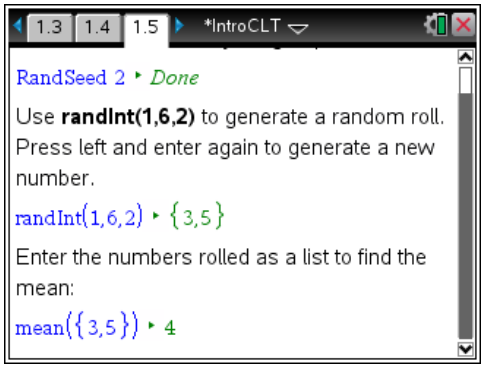
For Problem 1, when $n = 2$, the mean can be found mentally. The role of the person who calculates the means will become more important in Problems 2 and 3.



Before students begin the simulation, have each student set the **RandSeed** to their group number or another number (such as last 4 digits of a phone number) if group numbers are not assigned.

Let students perform the simulation of rolling two number cubes, finding the means, and recording them in the spreadsheet and on the worksheet. (Listing them on the worksheet will make it easier to exchange information later.)

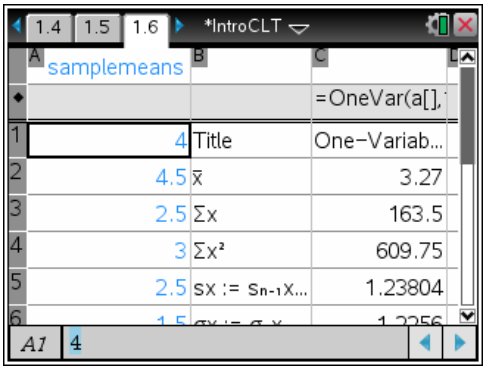
Note: Because of the roles of the group members, no one student will be using every page of the TI-Nspire document.



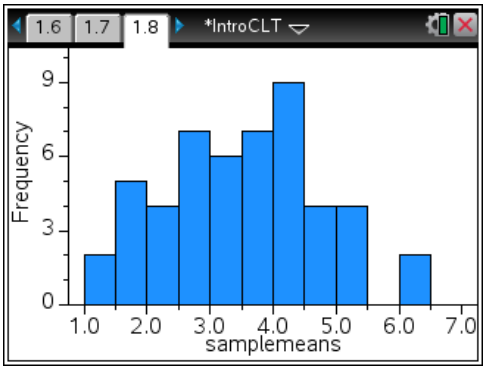
TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 1 at the end of this lesson.

Students will be prompted to find the mean and standard deviation of their 10 sample means in the spreadsheet. If needed, explain that they can do this by selecting **MENU > Statistics > Stat Calculations > One-Variable Statistics**.

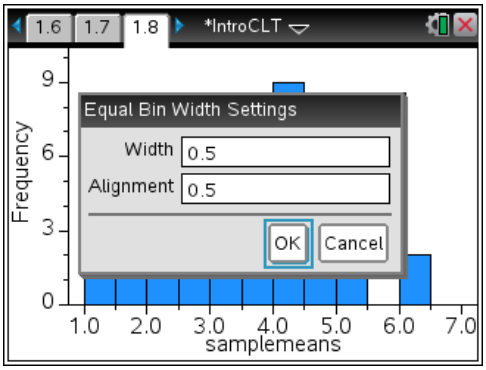


Students will also be prompted to create a histogram. Once students click in the center of the lower part of the screen on page 1.8 and choose **samplemeans**, a dot plot will appear. They can change this to a histogram by selecting **MENU > Plot Type > Histogram**.



For consistency and comparison with later histograms, have students adjust the axes so they can see horizontal values up through 6. They can grab and drag either axis to be able to see all the bars as well as the tops of the bars. They can also move the cursor between bars until the dark horizontal arrows appear, press click (the *x*-value appears), grab the bar, and then change the width of the bars using the TouchPad.

Students can also adjust the histogram by selecting **Bin Settings** from the **Plot Properties** menu.



Page 1.9 explains that one member should visit four other groups to copy their simulated sample means (from that group's worksheet) into their spreadsheet. This is quicker than performing 40 more simulations.

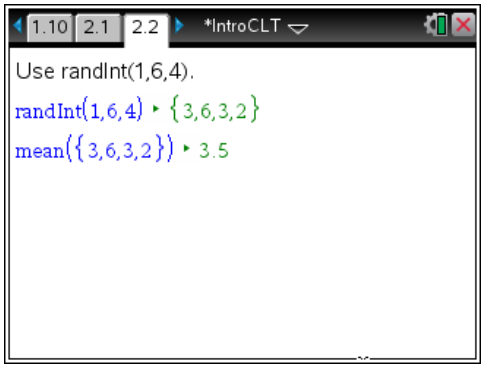
The mean and standard deviation will automatically update in the spreadsheet, as will the histogram. (Note: Students may need to be reminded to adjust the vertical axis.) Once students are back in their groups, all members should copy the mean and standard deviation of the 50 sample means onto their worksheet. They should also make a rough sketch of the histogram.

Problem 2 – Rolling Four Number Cubes

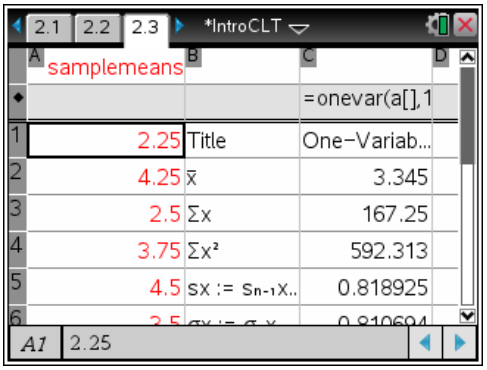
In Problem 2, students are asked to repeat the process, but now for rolling four number cubes.

The student responsible for calculating the mean of the four numbers can use either the Math Box on page 2.2 or the *Scratchpad*.

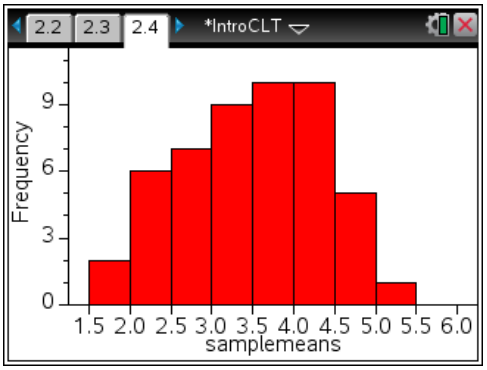
The student performing the simulations is also using page 2.2, but on their own handheld, while the recorder is entering the means on page 2.3.



Again, one member of each group should collect data from four other groups so that each group has 50 sample means. They will compute the mean and standard deviation and construct the histogram.

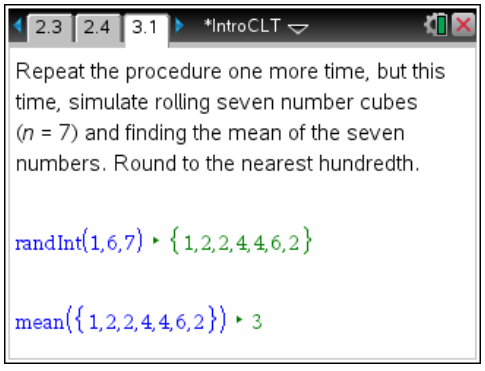


Remind students to adjust their histogram to display the horizontal axis values up through 6. (This will make it more obvious that the standard deviation is decreasing as *n* is increasing.)

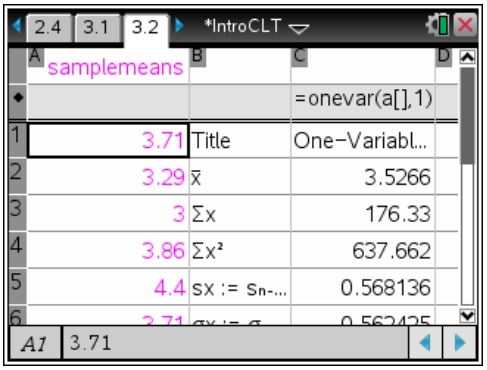


Problem 3 – Rolling Seven Number Cubes

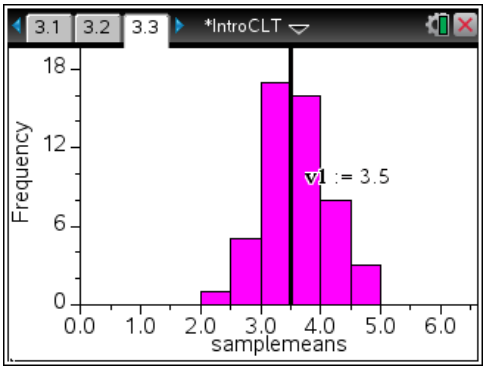
Students should require little help to work through Problem 3.



Once students finish with Problem 3, they can begin to answer the questions in Problem 4.



The histograms among the groups will not be exactly alike, but they should all be becoming more compact as the standard deviation decreases. They should also have a somewhat normal distribution, centered around 3.5, the population mean.



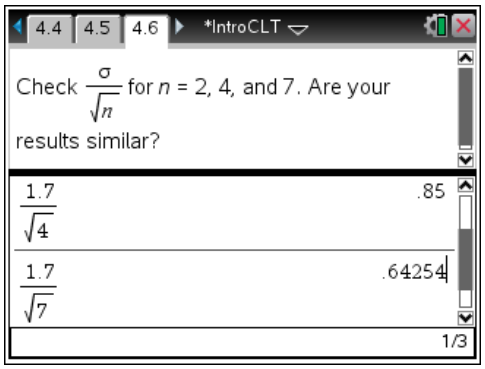
Problem 4 – Bringing It All Together

Once all of the groups are finished, discuss the answers to the questions in Problem 4 and officially introduce the Central Limit Theorem (CLT).

Point out that the distribution of sample means becomes normal as *n* increases, regardless of the shape of the distribution of the population.

Here, it was uniform but, it could also be normal, skewed, bimodal, or have no pattern.

Also, sample means should be getting closer to μ, and sample standard deviations closer to $\frac{\sigma}{\sqrt{n}}$.



TI-Nspire™ Navigator™ Opportunities

Note 1

Problem 1–3: *Class Capture*

As an alternative to having students swap groups to share data, you can use Class Capture to display data from the Lists & Spreadsheets pages.