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## Investigating the Relationship between the Sides of Right Triangles and Oblique Triangles with TI-Nspire Handheld

1. What does the Pythagorean Theorem tell us about the relationship of sides $a, b$, and $c$ of right $\triangle \mathrm{ABC}$ ? State the formula as you know it.

$$
a^{2}+b^{2}-c^{2}
$$

2. What is the value of $a^{2}+b^{2}-c^{2}$ for a right triangle? $\qquad$

3. Open file "Triangle Investigation" on TI-Nspire and do problem \#1.
4. If $a$ and $b$ stay constant and $\angle \mathrm{C}$ becomes an acute angle, predict whether $a^{2}+b^{2}-c^{2}$ is positive, negative, or zero.
$\qquad$
Explain why.
Since c is becoming smaller, we will be subtracting a smaller amount than
 we did for the right triangle.
5. If $a$ and $b$ stay constant and $\angle \mathrm{C}$ becomes an obtuse angle, predict whether $a^{2}+b^{2}-c^{2}$ is positive, negative, or zero.

Explain why.
Since c is becoming larger, we will be subtracting a larger amount than we did for the right triangle.

6. Based on your answers above, if $\angle \mathrm{C}$ varies from $0^{\circ}$ to $180^{\circ}$, describe the behavior of $a^{2}+b^{2}-c^{2}$.
It will be positive at $0^{\circ}$, decreasing to zero at $90^{\circ}$, then becoming more negative as you approach $180^{\circ}$.
7. Do problem \#2 from file "Triangle Investigation" on TI-Nspire to check your thinking.
8. On page 2.2 of file "Triangle Investigation", side $a$ and side $b$ are constant: $a=3 \mathrm{~cm}$ and $b=4 \mathrm{~cm}$.
Side $c$ and $\angle \mathrm{C}$ vary. Collect some data from your drawing, and enter it in the chart on the right.
9. If you graph $\angle \mathrm{C}$ on the x -axis and $a^{2}+b^{2}-c^{2}$ on the y -axis, describe what you know about the shape of the graph from your observations.
There is a negative association so it will be decreasing, but it will not be linear because the rate of change is not constant.

| $\angle \mathrm{C}$ | $a^{2}+b^{2}-c^{2}$ |
| :--- | :--- |
| $0^{\circ}$ | 24 |
| $30^{\circ}$ | 20.8 |
| $60^{\circ}$ | 12 |
| $90^{\circ}$ | 0 |
| $120^{\circ}$ | -12 |
| $150^{\circ}$ | -20.8 |
| $180^{\circ}$ | -24 |

10. Let's look at the relationship in the table from \#8 a little further. We'll collect more data in a spreadsheet and graph that data. Go to problem \#3 in the file "Triangle Investigation", and follow the directions carefully. You will be graphing $\mathrm{m} \angle \mathrm{C}$ on the x -axis and the algebraic expression $a^{2}+b^{2}-c^{2}$ on the y -axis.
11. Does the shape of the graph look like you thought it would? $\qquad$
12. What type of functions would fit this data? $\qquad$ $\sin (x)$ or $\cos (x)$
13. Return to page 3.5 in the file "Triangle Investigation". To enter your function guess, you will need to show the function entry line. To do this press menn, then 2:View, and 6:Show Entry Line. Press

Guess $\mathrm{f} 1(\mathrm{x})=\ldots \quad \cos (\mathrm{x})$
14. What features of your function are correct?

It has the right shape and the x -intercept is correct.
15. What features of your function need adjustment?

The altitude or vertical stretch needs to be much greater. The table suggests it will be 24 .
16. Edit the function to better fit the data. (If the entry line now says $f 2(x)=$, up arrow to return to $\mathrm{f} 1(\mathrm{x})=$.)

What is your final function?

$$
\mathrm{f} 1(\mathrm{x})=\quad 24 \cos (\mathrm{x})
$$

17. Side a and side b of the triangle were constants in this investigation: side $\mathrm{a}=3$ and side $\mathrm{b}=4$. How does the constant in your function relate to these constant sides?

It is twice the product of the constant sides: $2 \times 3 \times 4=24$
18. Generalize your hypothesis, and complete this equation for all triangles.

$$
a^{2}+b^{2}-c^{2}=-2 \operatorname{abcos}(\mathrm{C})
$$

19. Solve the equation for $c^{2}$.

$$
c^{2}=a^{2}+b^{2}-2 a b \cos (C)
$$

Extension:
In the extension, side b and $\mathrm{m} \angle \mathrm{C}$ are constant, and side a and side c will vary. Go to problem \#4 in the file "Triangle Investigation" and follow directions carefully.

What is the shape of the graph?
linear
Write an equation to fit the data \& enter it in $\mathrm{f} 1(\mathrm{x})=$
$\qquad$ (Refer to \#13 to show function entry line.)
Explain the shape of the graph in relation to the triangle. $y=a^{2}+b^{2}-c^{2}=2 a b \cos (C)$ a is constant, $\cos (\mathrm{C})$ is constant, $\& \mathrm{~b}$ is variable. It is a line with $\mathrm{m}=2 \operatorname{acos}(\mathrm{C})=2 \mathrm{x} 4 \mathrm{x} \cos \left(80^{\circ}\right)$.

