EXPLORATIONS

CHAPTER 16

When using the TI-83 Plus or TI-84 Plus calculators you access ***Finance*** by pressing the APPS key.

Black-Scholes Option Pricing Model

The Black-Scholes formula has been used to estimate the fair value of a call option. A call option gives its holder the right to purchase an asset at a predetermined price and time. A European option can only be exercised on the last day of its time period, while an American option can be exercised at any time before it expires. Although Black and Scholes (Fischer Black and Myron Scholes, "The Pricing of Options and Corporate Liabilities" *Journal of Political Economy* 81, no. 3 (May/June 1973):637-54) developed their valuation formula for European options, it has been used in situations where the market price of an option differs substantially from its fair value. A call option that sells for less than the value from the Black-Scholes formula indicates a purchase should be made, while a price greater than the value from the Black-Scholes formula indicates that the option is overpriced.

The Black-Scholes formula, with the variable names changed from the traditional to one letter variables to accommodate input into the calculator, is given below.

$$V = P * \text{normalcdf}(A) - \frac{E}{e^{R*T}} * \text{normalcdf}(B)$$

$$A = \frac{\ln(P/E) + (R + .5*S^2)*T}{S*\sqrt{T}}, \quad B = \frac{\ln(P/E) + (R - .5*S^2)*T}{S*\sqrt{T}}$$

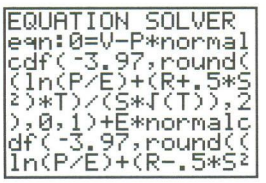
normalcdf(A)	=	cumulative normal probability density function (the probability that a normally distributed random variable will be less than or equal to A)
V	=	value of the option
E	=	exercise price of the option
T	=	time to maturity in years
P	=	current price of the underlying asset
S^2	=	instantaneous variance of the rate of return on the stock
R	=	risk-free interest rate (continuously compounded)

Example:
Calculate the Black-Scholes value for the following situation. A call option expires in three months and has an exercise price of $40 (that is, T = 3/12 = .25 and E = 40). The current price is $36, the risk of the common stock is 50%, and the risk-free rate is 5% (P = 36, S = .5 and R = .05).

Method 1:

The formula can be evaluated by using the Solver option found in the MATH menu from the [MATH] key *(4A)*†. Remember that the equation must have zero on the left and all the other variables on the right hand side of the equation. The formula is evaluated in one step, with the formulas for A and B incorporated into the equation.

(Figure 1)



1. Press the [MATH] key *(4A)* and choose **0:Solver** from the MATH menu.
2. Press [▲] until EQUATION SOLVER appears at the top of the screen.
3. If necessary, press [CLEAR] to delete any equations form the Solver.
4. Input the formula in the equation solver.

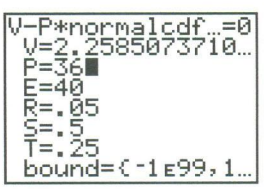
0 = V - P * normalcdf(-3.97,round((ln(P/E) + (R + .5 * S²) * T)/(S * √(T)),2),0,1) + E * normalcdf(-3.97,round((ln(P/E) + (R - .5 * S²) * T)/(S * √(T)),2),0,1) / e ^(R * T)

(Figures 1 and 2)

(Figure 2)

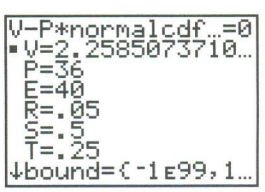
Note: **normalcdf(** can be found by pressing [2nd] [DISTR] *(4D)*. **round(** is in the NUM menu of the [MATH] key *(4A)*.

(Figure 3)



5. Enter the values as shown for P, E, R, S, and T. (Figure 3)

(Figure 4)



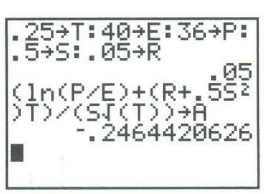
6. Move the cursor to V and press [ALPHA] [SOLVE] *(10E)*. (Figure 4)

The Black-Scholes value is 2.259.

Method 2:

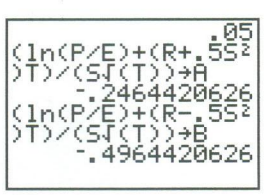
Another method is to enter the formula and the values on the Home Screen, and then evaluate A and B and use their calculated values in the formula.

(Figure 5)



1. On the Home Screen, input the values for the variables using the [STO▶] key. Use a colon ([ALPHA] [:] *(10C)*) to separate each of the inputs. This makes it easier to change the variables for a different situation.
2. Input the formula for A. (Figure 5)
3. Recall the formula by pressing [2nd] [ENTRY] *(10E)*.
4. Change the addition sign between R and .5 to a subtraction sign. Store the new formula in B. (Figure 6)

(Figure 6)



5. Press [2nd] [DISTR] *(4D)* and choose **2:normalcdf(** from the DISTR submenu. (Figure 7)

(Figure 7)

† **Refer to the section on Key Arrangement in Chapter 1 for an explanation of the key locator codes used in this manual.**

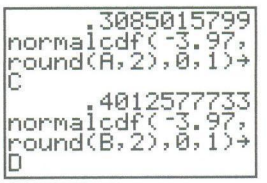
The syntax for this command is **normalcdf**(lowerbound, upperbound, mean, standard deviation). The defaults are 0 for the mean and 1 for the standard deviation.

6. Round A and B to two decimal places by using the **round(** command. (Figure 8)

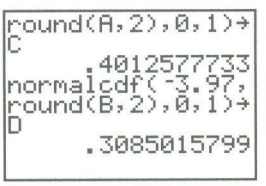
(Figure 8)

7. Store the **normalcdf** for A in C.
8. Repeat the process for B, but store the **normalcdf** in D. (Figures 9 and 10)

(Figure 9)

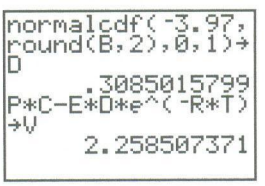


(Figure 10)



To calculate the value V, use **P * C - E * D * e^{-R*T}**. (Figure 11)

(Figure 11)



The price of $2.26 indicates that if the call option is $1, one should consider buying; however, if the call option is selling for $4.50, it is overpriced.

The equations below are the heart of the options pricing models.

Y_1 = X*normalcdf(-1E11,(ln(X/E)+RT+.5T*S²)/(S√(T)))-(E/e^(R*T))normalcdf(-1E11,(ln(X/E)+RT-.5TS²)/(S√(T)))

(Figure 12)

Y_2 = (X-E)(X≥E)

In these equations, X is the stock price and Y_2 gives the actual value of the option. When the X is less than the exercise price E of the option, the option is worthless. When X is greater than the exercise price, the value increases as the stock price increases.

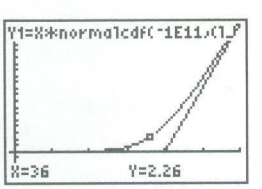
(Figure 13)

The equation in Y_1 gives the Black-Scholes option price and indicates a fair price for the option. Graphing these functions in an appropriate window gives a visualization. Note that values of the variables, E, T, P, S, and R must be stored in the variables on the Home Screen. These values were stored in the previous exercise.

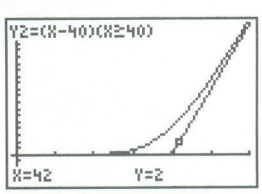
(Figure 14)

9. Press the [Y=] key *(1A)*.
10. Enter the first equation in Y_1. (Figure 12)
11. Enter the second equation in Y_2. (Figure 13)
12. Press the [WINDOW] key *(1B)* and enter the window parameters as shown. (Figure 14)
13. Press [GRAPH] *(1E)*. Press [TRACE] *(1D)* to examine different values for X and Y. (Figures 15 and 16)

(Figure 15)



(Figure 16)



Method 3:

The program listed below will automate the calculations of the Black-Scholes Option Pricing Model (BSOPM). First, enter the code for the program.

1. Press the PRGM key *(4C)* and choose NEW.
2. Enter the name BSOPM, and then enter the commands.

Black-Scholes Program (BSOPM.83P)

```
Prompt E,T,P,S,R
(ln(P/E)+RT+.5*S² *T)/(S*√(T))→A
(ln(P/E)+(R-.5*S² )*T)/(S*√(T))→B
P*normalcdf(-1E11,A)-
(E/e^(R*T))*normalcdf(-1E11,B)→V
Disp "VALUE=",V
"X*normalcdf(-1E11,(ln(X/E)+RT+.5T*S² )/(S√(T)))-
(E/e^(R*T))normalcdf(-1E11,(ln(X/E)+RT-
.5TS² )/(S√(T)))"→Y1
"(X-E)(X≥E)"→Y2
```

This program calculates and displays V. It also sets up Y_1 to find the value of a call determined by BSOPM as a function of the current price (X).

3. Before running the program, set the parameters for an appropriate window. (Figure 17)
4. To run the program, press the PRGM key *(4C)* and choose **BSOPM** from the EXEC menu. (Figure 18)
5. Press the ENTER key. This will paste **prgmBSOPM** on the Home Screen. (Figure 19)
6. Press the ENTER key again. The program will prompt you for values to be entered and calculate the result. (Figures 20 through 22)

(Figure 17)

```
WINDOW
 Xmin=-1
 Xmax=60
 Xscl=10
 Ymin=-4
 Ymax=20
 Yscl=1
 Xres=1
```

(Figure 18)

```
EXEC EDIT NEW
1:BSOPM
2:LIGHT1
3:LIGHT2
```

(Figure 19)

```
prgmBSOPM
```

(Figure 20)

```
prgmBSOPM
E=?
```

(Figure 21)

```
prgmBSOPM
E=?40
T=?.25
P=?36
S=?.5
R=?.05
```

(Figure 22)

```
T=?.25
P=?36
S=?.5
R=?.05
VALUE=
                2.26
                Done
```