Polar Functions

by

Mary Ann Connors

Department of Mathematics
Westfield State College
Westfield, MA 01086

Textbook Correlation: Key Topic

- Polar Functions
- Applications of Integration

NCTM Principles and Standards:

- Process Standard
    - Representation
    - Connections

A *polar curve* is the graph of an equation $r = f(\theta)$ where (r,θ) are standard polar coordinates of a point P. The relationship between the Cartesian coordinates (x, y) and polar coordinates (r,θ) of a point P are given by the equations $x = r \cdot \cos(\theta)$ and $y = r \cdot \sin(\theta)$.

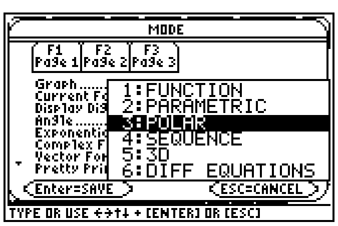
Exercise 1.

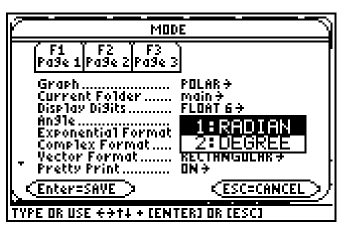
Plot the graph of $r = \sin 5\theta$ on the TI-89 (TI-92 Plus).

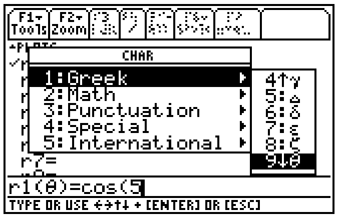
Solution:

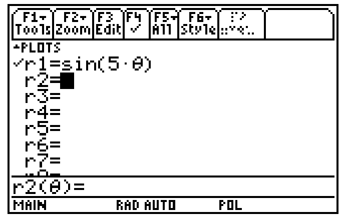
Any equation of the form $y = a \cdot \cos(n\theta)$ or $y = a \cdot \sin(n\theta)$ where a is an arbitrary real number and n is an arbitrary positive integer is called a *rose curve*. If n is odd, there are exactly n leaves (loops); if n is even, there are exactly $2n$ leaves.

Press the **MODE** key. Select **3:POLAR** for Graph. Select **1:RADIAN** for Angle. Enter the equation in the **Y=** Editor. Press **2$^{\text{nd}}$** + **(CHAR)** and select **1:Greek** and **9:**θ.

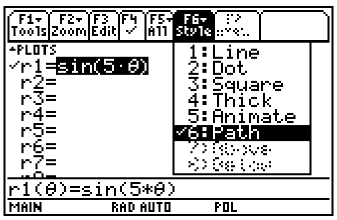


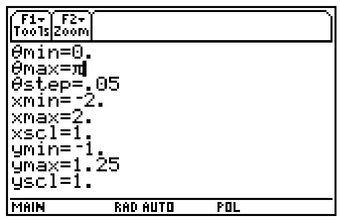


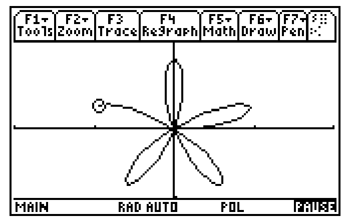


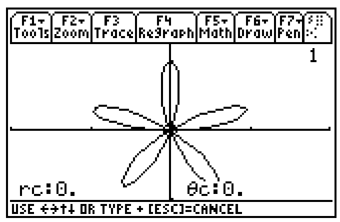


To see the five-leaved rose traced out, highlight the equation and select **6:Path** under **F6 Style**. Set the appropriate values for window variables. Note that the graph is completely traced out as θ varies from 0 to π. Letting θ vary from 0 to 2π retraces the graph.

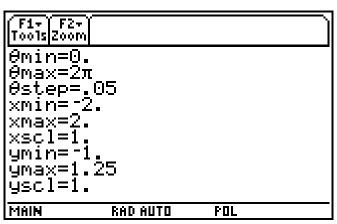


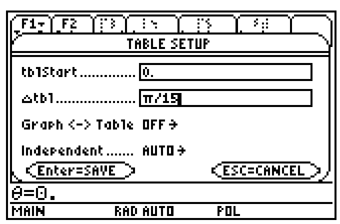


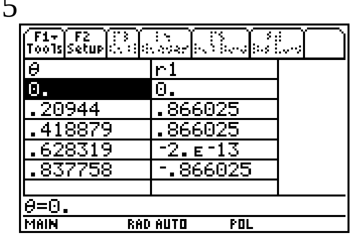




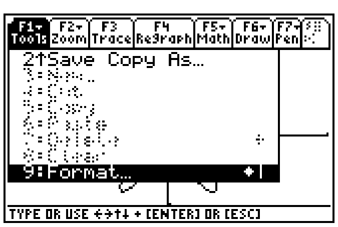
To observe the retracing of the graph change the θ **max** variable in the window to 2π. In the TbSet (Table Set Window) set **tblStart** to 0 and Δ **tbl** to $\dfrac{\pi}{15}$. Look at the table of values.

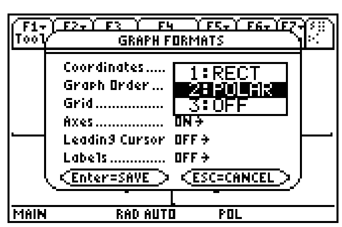




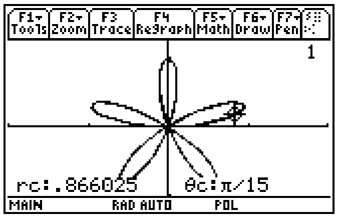


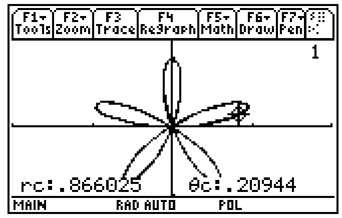
In the **GRAPH** window press **F1 Tools**. Select **9:Format** and **2:Polar** for coordinates.

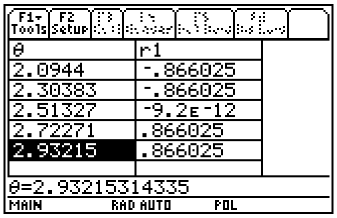




Press **F3 Trace** on the Graph Screen to see the points.

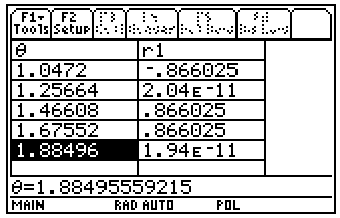






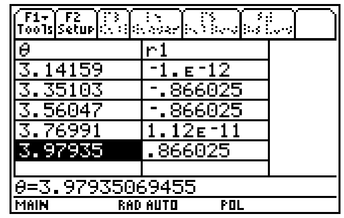
θ	r1
2.0944	-.866025
2.30383	-.866025
2.51327	-9.2E-12
2.72271	.866025
2.93215	.866025

θ=2.93215314335



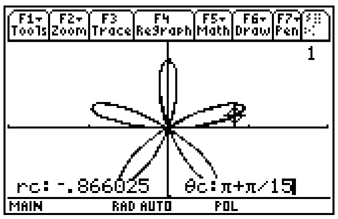
θ	r1
1.0472	-.866025
1.25664	2.04E-11
1.46608	.866025
1.67552	.866025
1.88496	1.94E-11

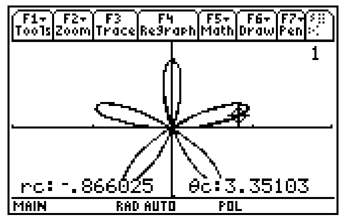
θ=1.88495559215



θ	r1
3.14159	-1.E-12
3.35103	-.866025
3.56047	-.866025
3.76991	1.12E-11
3.97935	.866025

θ=3.97935069455





Recall that the points $(\frac{\pi}{15},.866025)$ and $(\frac{16\pi}{15},-.866025)$ are the same point. On the table these points are given as $(.20944,.866025)$ and $(3.35103,-.866025)$

Exercise 2.

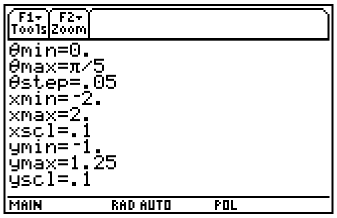
Find the area enclosed by one loop of the five-leaved rose.

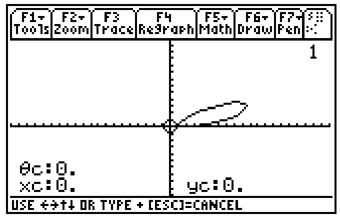
Solution:

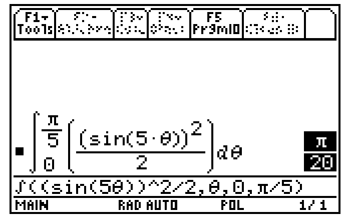
The area A of a region bounded by the graph of a continuous nonnegative function $r = f(\theta)$ and $\theta = a$ and $\theta = b$ where $0 \le b - a \le 2\pi$ is given by

A = $\int_a^b \frac{1}{2} f(\theta)^2 \, d\theta$.

The area of one loop of the five-leaved rose pictured in the graph below can be calculated on the **HOME** screen as illustrated below.





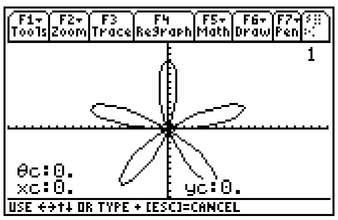


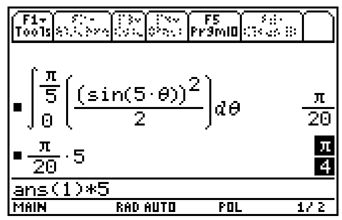
Answer: The area of one loop of the five-leaved rose is $\dfrac{\pi}{20}$ square units.

Exercise 3.

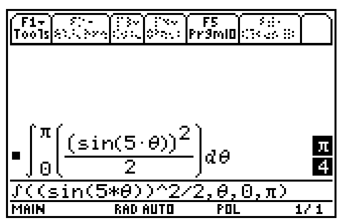
Find the total area enclosed by the five-leaved rose.

Solution:
The total area enclosed by the five-leaved rose pictured in the graph below can be calculated on the **HOME** screen as illustrated below. One method entails multipling the area enclosed by one loop by 5.





An alternative method calculates the area of all five leaves in one operation as depicted below.



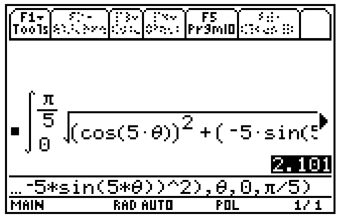
Answer: The total area enclosed by the five-leaved rose is $\dfrac{\pi}{4}$ square units.

Exercise 4.

Find the length around the edge of one loop of the five-leaved rose.

Solution:
The length L of a polar curve $r = f(\theta)$ where $a \le \theta \le b$ is given by
L = $\int_a^b \sqrt{f(\theta)^2 + f'(\theta)^2}\, d\theta$.

Calculate the integral on the **HOME** screen as depicted below.



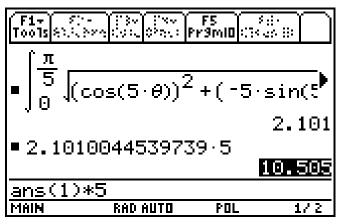
Answer: The length around the edge of one loop of the five-leaved rose is 2.101 units.

Exercise 5:

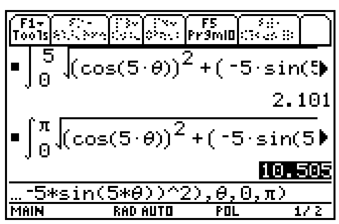
Find the total length around the edges of the five-leaved rose.

Solution:

Calculate the integral on the **HOME** screen as depicted below. One method entails multipling the length around one loop by 5.



An alternative method calculates the length around all five leaves in one operation as portrayed below.



Answer: The total length around the edges of the five-leaved rose is 10.505 units.