Math TODAY™
Teacher Edition

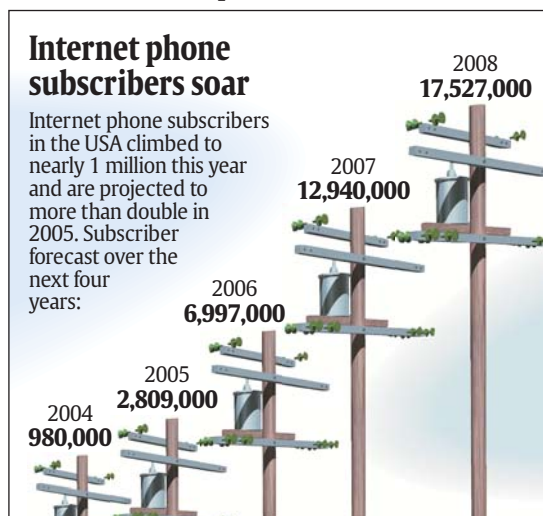**USA TODAY**

NO. 1 IN THE USA

Internet phone subscribers soar

By: Bob Tower

USA TODAY Snapshots™

Internet phone subscribers soar

Internet phone subscribers in the USA climbed to nearly 1 million this year and are projected to more than double in 2005. Subscriber forecast over the next four years:

2004 **980,000**
2005 **2,809,000**
2006 **6,997,000**
2007 **12,940,000**
2008 **17,527,000**

Source: Yankee Group   By April Umminger and Karl Gelles, USA TODAY

Activity Overview:

Students will create a scatter plot of the number of Internet subscribers during 2004 through 2008 using the USA TODAY Snapshot "Internet phone subscribers soar." This activity will allow students to explore real-life data that will be modeled by a logistic function. Students will have the opportunity to explore the behavior of this model. This activity will also allow students to explain what the instantaneous rate of change means in the context of a problem situation. The derivative will be used to explore when the number of Internet subscribers was increasing most rapidly during this time period.

Concepts:

- Evaluating, synthesizing and analyzing real-world data with a logistic model
- Instantaneous rate of change/derivative
- Symmetric difference quotient

Activity at a Glance:

- Grade level: 9-12
- Subject: Calculus
- Estimated time required: 20-30 minutes

Materials:

- TI-83 Plus family or TI-84 Plus family
- Overhead view screen calculator for instruction/demonstration
- Student handout
- Transparency
- USA TODAY newspapers (recommended)

Prerequisites:

Students should know:

- how to create a scatter plot with the calculator.
- how to use the regression capabilities of the calculator.
- how to use the nDeriv feature of the calculator and how to create a graph using this feature.

This activity works with

TI navigator
A Classroom Learning System

For more information, visit:
www.education.usatoday.com
www.education.ti.com/USAT

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Objectives:

Students will:
- model real-life data with a logistic function.
- use and interpret the average rate of change to approximate instantaneous rate of change.
- evaluate and interpret the meaning of the derivative.
- evaluate and interpret the meaning of the derivative at the inflection point of a graph.

Background:

Students will develop an understanding of the important concepts and relationships between the logistic function and its derivative. Students will analyze the data by creating a scatter plot and then use the regression capabilities of the calculator to determine the logistic model. Students will study the information available when patterns are presented as collections of numbers, graphs and symbolic representations. Internet telephone is a new type of worldwide communications service. Internet telephone technology generally uses a phone adapter to convert your voice from an analog signal to a digital signal. The digital signal is sent over a Broadband Internet connection. This process is sometimes referred to as Voice Over IP (VoIP) or Internet Telephony. These calls are transmitted over the Internet backbone instead of a traditional telephone network. However, the person you are calling or the person calling you does not have to be connected to the Internet. Often these services provide normal calling features such as caller ID, call waiting, etc., and are less expensive than other communications methods. More and more people are choosing this service because of the features offered and because there are no additional long distance fees.

Preparation:

- Provide one graphing calculator for each student.

- Each student should have a copy of the corresponding student activity sheet.

Data Source:

Yankee Group

National Council of Teachers of Mathematics (NCTM) Standards*:

Number and Operations Standard
- Compute fluently and make reasonable estimates.

Algebra Standard
- Understand patterns, relations, and functions.
- Use mathematical models to represent and understand quantitative relationships.
- Analyze change in various contexts.

Problem Solving Standard
- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and in other contexts.

Communication Standard
- Use the language of mathematics to express mathematical ideas precisely.

Connections Standard
- Recognize and apply mathematics in contexts outside of mathematics.

Representation Standard
- Use representations to model and interpret physical, social, and mathematical phenomena.

*Standards are listed with the permission of the National Council of Teachers of mathematics (NCTM), www.nctm.org. NCTM does not endorse the content or validity of these alignments.

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Classroom Management Tips:

- Students will have a better understanding of how to read the graphic and retrieve data if you use the transparency for a class discussion before the students start working.

- Students can work individually or in small groups on this activity. Working in groups is especially helpful as they learn the various features of the calculator.

- Allow students to talk about the "how" and "why" approach they used to find the solutions.

- Students may need to be reminded about how to use nDeriv with the calculator to create a graph of the derivative.

- Allow students time to discuss the data in the USA TODAY Snapshot "Internet phone subscribers soar" in small groups. Remind the students to plan their approach to solving the problem situation.

- This would be an excellent time to discuss the difference between the exponential function and logistic function.

Activity Extension:

- Instruct students to explore the limiting number of Internet subscribers as time approaches infinity. Ask students if the logistic model is an appropriate model for the year-Internet subscriber context.

- Ask students to create a table showing the values of the derivative for each of the years shown in the USA TODAY Snapshot. Then ask students to explain the behavior of the derivative over the time period shown.

- Suggest that students visit Internet businesses in your community and explore the data for your local area.

- Encourage students to use the United Nations Population Division site at www.un.org/esa/population/unpop.htm and explore the population of the United States or another region to determine if the population data would be modeled by a logistic function.

Curriculum Connections:

- Economics
- Business
- Biology
- Chemistry

Additional Resources:

- Student handout
- Transparency
- TI Technology Guide, for information on the following: TI-83 Plus family, TI-84 Plus family and List Editor
- TI-Navigator™ Basic Skills Guide for information on using the TI-Navigator Classroom Learning System

Teacher Notes:

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Assessment and Evaluation:

Activity 1:
Estimate and interpret the rate of change for 2005 using the data from 2004 and 2006.

A. | Year | Number of subscribers (thousands) | Average rate of change |
|---|---|---|
| 2004 | 980 | $\frac{6{,}997 - 980}{2006 - 2004} = \frac{6{,}017}{2} = 3008.5$ |
| 2006 | 6,997 | |

The value 3009 (4 significant digits) tells us that the average increase is about 3,009,000 Internet subscribers per year for this two-year period. This value can also be used as an estimate for the rate of change (increase) in Internet subscribers for 2005.

Activity 2:
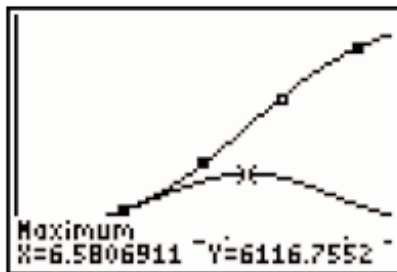**Find and interpret the instantaneous rate of change for 2005.**

A. By evaluating the derivative when x = 5, you can conclude that the rate of change (increase) in 2005 will be about 2,861,000 Internet subscribers per year.

```
nDeriv(Y₁,X,5)
          2860.8518
```

Activity 3:
For the years listed, when does the model indicate that the number of Internet subscribers is increasing most rapidly?

A. The graph of the derivative shows that the maximum value occurs approximately halfway through 2006. The increase in the Internet subscribers is about 6,117,000 at this time. The point where the logistic graph has its most rapid increase and the derivative graph has an extrema is the inflection point.

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Maximum
X=6.5806911  Y=6116.7552
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