Exploring Geometric Sequences

ID: 11007

Time required
45 minutes

Activity Overview

In this activity, students will explore geometric series. They will consider the effect of the value for the common ratio and first term using sliders. Students will graphically and numerically analyze geometric series using graphs and spreadsheets. They will also consider the derivation of the sum of a finite geometric series and use it to solve several problems while comparing their answer to those found using sigma notation.

Topic: Sequences and Series

- *Explore geometric sequences*
- *Sum a geometric sequence*

Teacher Preparation and Notes

- *This activity serves as a nice introduction to geometric series. Students will need the ExploringGeomSequences.tns file on their TI-Nspire handheld. The accompanying worksheet can add depth to the questions and helpful key press explanation. An extension would be to solve infinite geometric series that converge using sigma notation and the limit of the partial sum formula.*
- *To use the sliders, students can either grab and drag the slider or grab it and press left or right on the TouchPad to change the number.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11007" in the keyword search box.***

Associated Materials

- *ExploringGeomSequences_Student.doc*
- *ExploringGeomSequences.tns*
- *ExploringGeomSequences_Soln.tns*

Suggested Related Activities

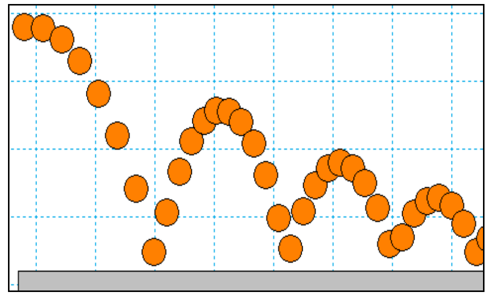
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Geometric Sequence & Series (TI-Nspire technology) — 8674*
- *Exploring Infinite Series (TI-84 Plus family) — 4374*
- *Serious Series (TI-89 Titanium) — 3466*
- *Applications of Finite and Infinite Series (TI-89 Titanium) — 3086*
- *Sequence & Series Introductory Quiz (TI-84 Plus family) — 10486*

Example of Geometric Sequence

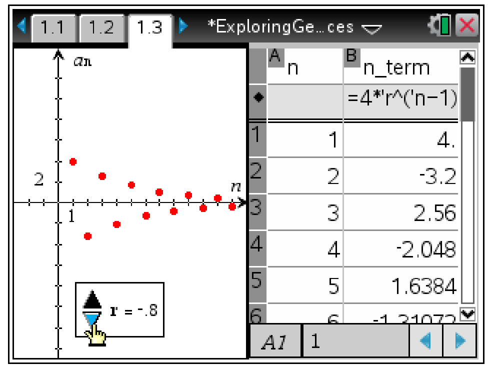
Students are shown the path of a ball that is bouncing. Show that the common ratio of the heights is approximately the same.

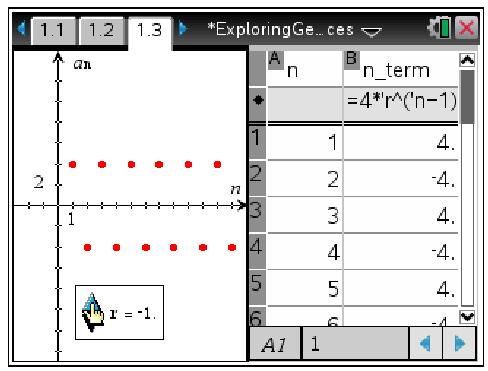
$\frac{2.8}{4.0} = 0.7 \qquad \frac{2.0}{2.8} \approx 0.71 \qquad \frac{1.4}{2.0} = 0.7$



Changing the Common Ratio

On page 1.2 of this activity, students explore geometric sequences graphically and numerically by varying the value of *r*, the common ratio. Clicking on the up or down arrow of the minimized slider while viewing a split screen of a graph and a spreadsheet makes this a dynamic activity.





Inquiry questions

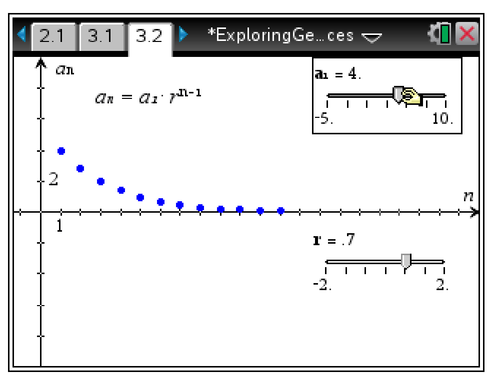
- Why do you think the *r*-value is called the common ratio? [*Answer: The ratios of consecutive terms are the same; they have that in common.*]
- What would happen if you added all the terms of this sequence? For what common ratio conditions do you think the sum will diverge (get larger, and not converge to some number)?

TI-Nspire Navigator Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

Changing the Initial Value and Common Ratio

On page 3.2, two sliders change the graph of the series $a_n = a_1 \cdot r^{n-1}$. Students are to grab and move both sliders and observe the effects. They should be sure to try negative and positive and values between –1 and 1 for both variables.



Inquiry question

- Which variable seems to have a more profound effect on the sequence? Explain.

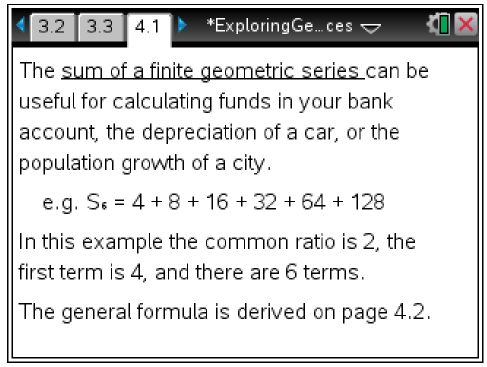
Deriving and Applying the Partial Sum Formula

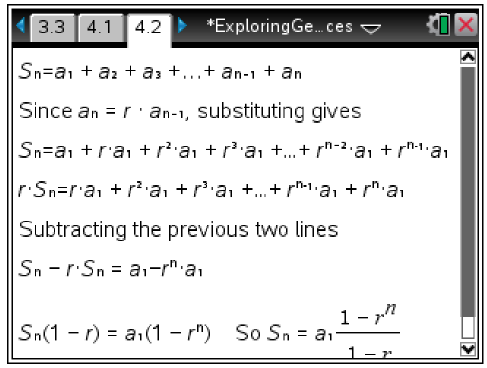
Students are shown the derivation of the formula for the sum of a finite geometric series on page 4.2. You may need to explain in detail the substitution in the third line.

For example, $a_2 = r \cdot a_1$, so then $a_3 = r \cdot a_2 = r \cdot (r \cdot a_1) = r^2 \cdot a_1$. In the fourth line, r is multiplied by both sides, changing r^{n-1} to r^n.

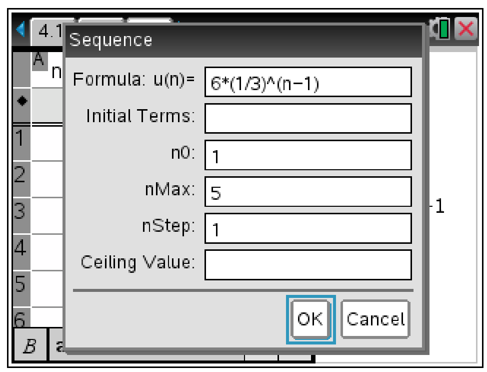
<u>Inquiry questions:</u>

- If you cut a piece of paper in half, then cut one of the halves in half and repeated the process, what is the sum of all these fractional pieces? [*Answer: one whole*]
- However, if you gave your teacher one penny on the first day of class, 2 on the next, then 4, 8, 16, 32, … would this number converge?
- How would the formula for a finite geometric sequence look if r is a fractional number and n is increasing?





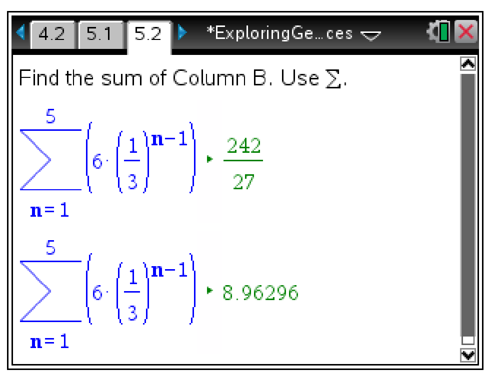
After looking at the derivation, students will generate the first five terms of $6\left(\frac{1}{3}\right)^{n-1}$ using the **Generate Sequence** command in spreadsheet found on page 5.1.



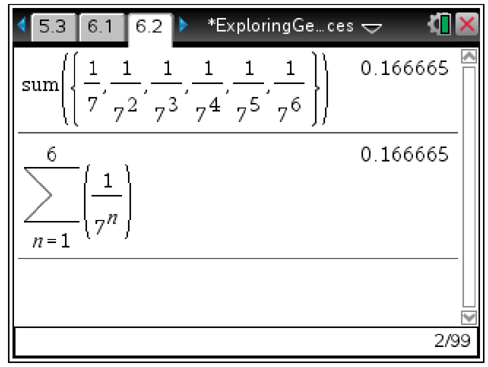
To complete the dialog box, students should enter the information should at the right. Note that *Initial Terms* and *Ceiling Value* should be left blank.

Then they will find the sum using the **sum** command, **sigma notation**, and the **partial sum formula** on pages 5.2 and 5.3.

On the spreadsheet, the title of Column B is **a**. Students can use the letter *a* to refer to the terms in this column.



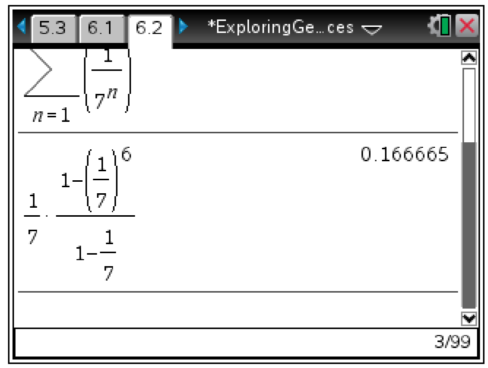
Apply What Was Learned



Here, the students are to find the sum of a finite geometric series using various methods: formula, sigma notation, and sum of a list. Note: when using **sum()**, what is inside the parentheses must be a list. Therefore the sequence needs curly brackets around the terms, as pictured to the right.

In Question 10, students will see the value in using the formula instead of trying to correctly type 25 terms.

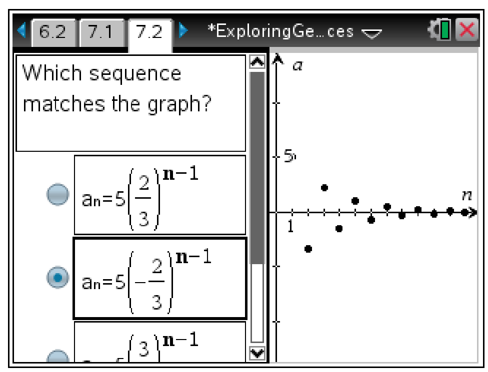
The sum is approximately 56.486.



The Question 11 series may be difficult for some students to identify the components so that they can use the formula. Remind them that a_1 is the first term and the common ratio is found by dividing successive terms, minus signs included.

So $a_n = 64\left(-\frac{1}{2}\right)^{n-1}$ and the sum is approximately 42.668.

Conclusion



Students have the opportunity to check what they learned at the beginning of the activity. ctrl + ▲ will reveal if they got the answer right or wrong. ctrl + ▼ will to return to the question.

Note that each question has 4 answer choices. Students will need to scroll down to see the last choice.

TI-Nspire Navigator Opportunities

Note 1
Problems 1–3, *Quick Poll*

You may choose to use *Quick Poll* to assess student understanding. The worksheet questions and inquiry questions can be used as a guide for possible questions to ask.