

### **Objective**

• To investigate the properties of a rotation



# Rotations in the Plane

## Introduction

A **rotation** around a point is one of three types of rigid transformations. It is a transformation that turns a figure a certain number of degrees about a certain point. This activity explores the properties of rotations and the relationships between the original and image figures.

## Construction

#### Part I: Construct a rotation angle tool.

 $\bigcirc$   $\blacksquare$  Draw a small circle in one corner of the screen, and hide the radius point.

1. Open the **Drawing Tools Menu**, and then highlight **Circle**. Press ENTER.



 Move the cursor to the lower left corner of the screen and press ENTER to anchor the center of the circle. Move the cursor just far enough to create a small circle and press ENTER to anchor the radius point of the circle.



**3.** Open the **Display Tools Menu**, and then highlight **Hide/Show**. Press **ENTER**.



When using the Hide/Show tool:

- The pointer changes to an eraser when it is near an available object to hide.
- After you press ENTER, you will see the object displayed with dotted lines until the pointer is moved away.
- To show objects previously hidden, move the pointer to the location of the object. The pointer changes to a pen when it is close to a hidden object. Press ENTER to show the object.
- 4. Move the cursor to the radius point of the circle and press ENTER to hide it. Press CLEAR to exit the Hide/Show tool.

**Note:** Be sure to hide the radius point only, not the circle. If needed, press ENTER while the **Hide/Show** tool is active to show a previously hidden object.





Draw two radii of the circle to form a central angle.

5. Open the Drawing Tools Menu, and then highlight Segment. Press ENTER.



- 6. Move the cursor to highlight the circle, and press ENTER to anchor the endpoint of a segment on the circle.
- 7. Move the cursor to highlight the center of the circle, and press ENTER to anchor the other endpoint of the segment there.
- 8. Repeat this procedure to draw a second radius of the circle using the **Segment** tool.





Measure the central angle.

- Open the Display Tools Menu, and then highlight Measure. Press 
   To view the Measure Menu. Highlight Angle, and press ENTER.
- **10.** Angles are measured by selecting three points. Select the vertex point of the angle *second*.
  - Move the cursor to one radius point on the circle and press ENTER.
  - Move the cursor to the center of the circle (the vertex of the angle) and press ENTER.
  - Move the cursor to the other radius point on the circle and press ENTER.
  - Finally, move the measurement to a desired location and press ENTER to anchor it on the screen.

When finished, press **CLEAR** to exit the **Measure Angle** tool.





### Part II: Construct a triangle, and perform a rotation.

Draw a small scalene triangle on the right half of the screen.

1. Open the **Drawing Tools Menu**, and then highlight **Triangle**. Press ENTER.



2. Move the cursor to the lower left of the screen and press ENTER to anchor the first vertex of the triangle. Press ENTER two more times to anchor the other two vertices of the triangle.

**Note:** Make the triangle scalene to view more easily the properties of the rotation.

A Draw and label point P in the center of the screen.

- Open the Drawing Tools Menu, and then highlight Point. Press → to view the Point Menu. Highlight Point, and press ENTER.
- **4.** Move the cursor to the center of the screen, and press ENTER to anchor a point there.

5. Open the Display Tools Menu, and then highlight Alpha-Num. Press ENTER.









6. Move the cursor to the point. The point blinks when the cursor is close enough to the point to select it.



Press ENTER to create a label for this point.
 Press 3 to label the point P (note that [A-LOCK] is on), and then press ENTER to complete the label.



#### Rotate the triangle.

To use the Rotation tool, you must do the following:

- Select what is to be rotated.
- Select a point to be the center of rotation.
- Identify the angle of rotation by selecting three points that define an angle.

Select the rotation angle in the same way as an angle, side-vertex-side. The order in which these angle-defining points are selected controls the direction of the rotation, clockwise or counterclockwise.

8. Open the Transformation Tools Menu, and then highlight Rotation. Press ENTER.



9. Move the cursor to the triangle and press ENTER to select it.

**Note:** Be careful to select the whole triangle, not just one side. Wait for the triangle to blink, or press 2nd to switch between side only and full object.



**10.** Move the cursor to point *P* and press ENTER to select it.



- **11.** Move the cursor to one of the radius points on the circle and press ENTER to select it.
- 12. Move the cursor to the center of the circle and press ENTER to select it. Finally, move the cursor to the other point on the circle, and press ENTER to select it. A new triangle (the image) is created. Press CLEAR to exit the Rotation tool.



**Note:** Select the points you used to measure the angle in exactly the same order.

### Exploration

Investigate the relationship between the original triangle (the pre-image) and the new triangle (the image).

🚯 Drag a vertex of the pre-image.

1. Move the cursor to the desired vertex and press <u>ALPHA</u> to grab it.

**Note:** The pointer changes to an outlined arrow when it is near an available object.

2. Use the cursor keys to drag the vertex around the screen, and observe what happens to the image. When finished, press <u>CLEAR</u> to exit.



#### Drag a side of the pre-image.

- 3. Move the cursor to the desired side of the triangle. Wait until only the side is blinking (not the entire triangle) and press ALPHA to grab it.
- **4.** Use the cursor keys to drag the vertex around the screen, and observe what happens to the image. When finished, press <u>CLEAR</u>.



Drag (translate) the entire pre-image triangle.

- Move the cursor to the triangle and wait until the entire triangle is blinking. Press ALPHA to grab it.
- 6. Use the cursor keys to drag the vertex around the screen and observe what happens to the image. When finished, press <u>CLEAR</u>.



🚯 Drag the center of rotation.

7. Move the cursor to point *P* and press <u>ALPHA</u> to grab it. Use the cursor keys to drag the point around the screen, and observe what happens to the image. When finished, press <u>CLEAR</u>.



Drag one of the radius points around the circle to make the rotation dynamic.

 Move the cursor to one of the radius points and press <u>ALPHA</u> to grab it. Use the cursor keys to drag the point around the circle. Notice that the angle measurement will not exceed 180°, but the triangle will rotate in a complete circle around point *P*.



Measure the lengths of corresponding sides, angles, and areas of the pre-image and image.

 Open the Display Tools Menu, and then highlight Measure. Press → to view the Measure Menu. Highlight D. & Length. Press ENTER.



10. Move the cursor to the desired side of the triangle. Press ENTER to measure the side, and press ENTER again to anchor the measurement on the screen. Repeat this for the remaining length measurements of both the image and pre-image triangle. When finished, press CLEAR to exit the Distance & Length tool.

**Note:** Be careful to select the side of the triangle and not the whole triangle.

**Note:** As an alternative, measure the side length as a point-to-point distance. Select one endpoint, press ENTER, then select a second endpoint, and press ENTER again.

**11.** Use the **Measure Angle** tool to measure a corresponding angle of each triangle. Remember to press <u>ENTER</u> four times: once for the first side of the angle, then for the vertex of the angle, then for the other side of the angle, and a final time to anchor the measurement. When finished, press <u>CLEAR</u> to exit the tool.

**Note:** The graphing handheld application measures each angle to two decimal places, but only displays each measurement to one decimal place. Therefore, the displayed angle measurement may appear to be inaccurate.

12. Open the Display Tools Menu, and then highlight Measure. Press → to view the Measure Menu. Highlight Area. Press ENTER.







- **13.** Move the cursor to the desired triangle. Press ENTER to measure the area, and press ENTER again to anchor the measurement on the screen. Repeat this to measure the area of the image triangle. When finished, press CLEAR to exit the **Measure Area** tool.
- **14.** Press <u>ALPHA</u> to use to grab and drag a vertex or side of the original triangle and observe any changes in the measurements.
- Animate one of the points on the circle.
- **15.** Open the **File Tools Menu**, and then highlight **Animate**. Press **ENTER**.

**16.** Move the cursor to one of the radius points on the circle, and press ENTER to start the animation.

**Note:** The pointer changes to a double arrow when it is near an available object.

17. To stop the animation, press 2nd while the animated object is moving. The tool icon changes to 2000. Press ENTER to stop the animated object.





Undo Help





# **Data Collection and Analysis**

Name _			
Date			

### **Questions and Conjectures**

- **1.** Which properties of the original (pre-image) triangle are preserved by the rotation? Which properties are not?
- 2. What happens when the rotation angle is 0°?
- **3.** What did you observe when the center of rotation (point *P*) was *inside* the original triangle? What did you observe when the center of rotation is *on a vertex* of the original triangle?

# **Teacher Notes**



#### **Objective**

• To investigate the properties of a rotation



# Rotations in the Plane

Grabbing and dragging a radius point on the circle makes the rotation dynamic. It does not matter which point is grabbed. The direction of the rotation (clockwise or counterclockwise) is not determined by the direction of the dragging; it is determined by the order in which the vertices of the angle of rotation were chosen during its construction.

It is possible to animate *both* radius points on the circle at the same time. While one point is animated, move the cursor to the second point and press ENTER. The results might not be as expected. When both of the defining radius points of the rotation angle tool are animated, the rotation angle is held roughly constant. Both points are moving around the circle in the same direction at the same rate.

## Answers to Questions and Conjectures

**1.** Which properties of the original (pre-image) triangle are preserved by the rotation? Which properties are not?

A rotation preserves the following properties: length of sides, perimeter, area, angle measures, and orientation. Unlike translations, rotations do not preserve the slope of the corresponding sides of the pre-image and the image.

2. What happens when the rotation angle is 0°?

When the rotation angle is 0°, the image (the new triangle) is on top of the pre-image (the original triangle).

**3.** What did you observe when the center of rotation (point *P*) was *inside* the original triangle? What did you observe when the center of rotation is *on a vertex* of the original triangle?

When the center of rotation is inside the original triangle, the two triangles will overlap. When the center of rotation is on a vertex of the original triangle, the two triangles will be "attached" at that vertex.