Chapter 6: Biological Models with Differential Equations

1. For exponential regression of 1790-1860 census data, the 1870 prediction is 41,952,888 (actual 39,818,449) and the 1880 prediction is 56,356,480 (actual 50,155,783). For logistic regression of the same data, the 1870 prediction is 41,625,575 and the 1880 prediction is 54,538,241. Both models over-estimate the actual population, which should not be surprising because the U. S. Civil War had approximately 620,000 casualties and disrupted normal reproduction patterns.

2. Using only 1950-1990 census data, the exponential regression prediction for 2020 is 368,523,538 (compared to the exponential regression using the complete census data which was 691,581,863), and the logistic regression prediction for 2020 is 298,049,393 (compared to the logistic regression using the complete census data which was 321,528,956). In addition, the carrying capacity of this new logistic model is only 368,975,136. Comparing to the results of Example 1, you might conclude that some of the "constants" assumed in the differential equations of the models have, in fact, been slowly changing.

3. Initial values $0 < R0 < 2$ rapidly decline to zero, while initial values $2 < R0$ tend to a limiting value of 10. Initial conditions of exactly $R0 = 2$ or 10 give a constant solution.

4. As this parameter is made smaller than 0.2, the loops flatten out more horizontally and go closer to the x = Q1 axis. The stationary point that is within all of the other loops moves down as well. For 0.05 this stationary point is (4/5, 5/70), for 0.1 it is (4/5, 1/7), for 0.15 it is (4/5, 15/70), for 0.2 it is (4/5, 2/7), and for 0.3 it is (4/5, 3/7).

5. Solutions to this system slowly spiral around and inward, approaching the stationary point (4/5, 12/70).

6. Neither population "wins" or "loses" as solutions tend to the stationary point P = 136/11 and Q = 32/11.

7. The effect is for the population S = Q1 to decline more slowly and for the peak in the infected population I = Q2 to be lower and take longer to occur. Everyone still gets sick and then recovers, but it takes longer for this to happen.

Hint: *Don't forget to change k in both the equations for Q'1 and Q'2.*