Expecting a Win

Student Worksheet

7 8 9 10 **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

Many board games require players to land precisely on the last square of the board in order to complete the game. This investigation involves the simplest of board games. Players take it in turns to roll a die, each time advancing toward the final square. Players must finish precisely on the last square in order to finish the game. So far the game sounds simple and fair. There is however one catch! Players can nominate the number of 'sides' on their die. As the die is virtual (simulated by the calculator), players can nominate any quantity between 1 and 20. There are 20 squares on the board, so exactly 20 squares must be advanced and the first player to do so is the winner!

Instructions

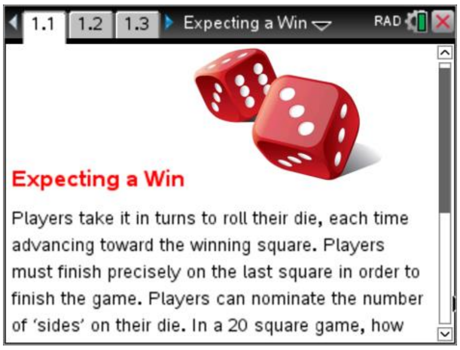
Open the TI-nspire file "Expecting a Win".

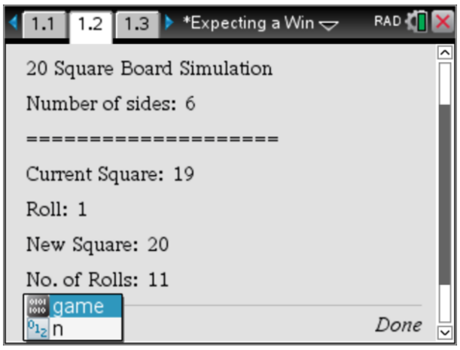
Page 1.2 includes a game simulation where you can simulate a single game at a time. Game updates are displayed on screen as:

- Current Square
- Dice Roll
- New Square
- Number of rolls

To run the game, select "Game" from the variable menu.

Explore a number of games using a different number of sides each time. Carefully observe the "Current Square" and the dice roll.





Question: 1.

Discuss the advantages and disadvantages of using a 20 sided die to play the game compared with a 4 sided.

Question: 2.

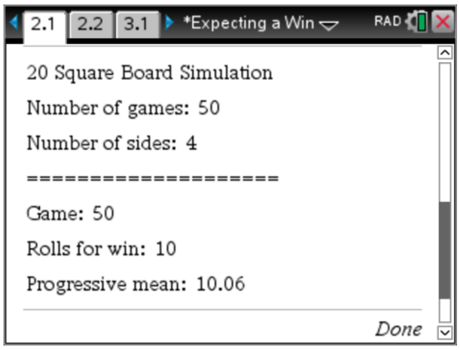
How many sides do you think would give you the maximum chance of winning?

Author: P. Fox

Navigate to page 2.1.

Run the GameSim program. This program plays the requested number of games using the corresponding number of sides input for the die. Updates displayed on screen:

- Game Number
- Rolls to Win
- Progressive Mean



Question: 3.

Play 50 games with a 4 sided die. Use a dot plot (Page 2.2) to display the distribution of results.

A record of the simulations can be made by storing the most recent results into a different variable. The variable "R" is the list where the original results are stored for each individual game. To transfer the results to R1:

R1: = R

Additional data sets can be named: R2, R3, R4 … OR … R10 for 10 sided, R4 for 4 sided.

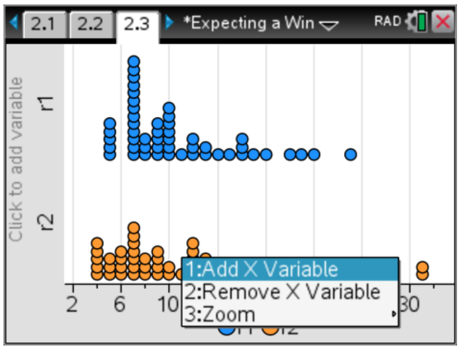
Question: 4.

Play 50 games with a 10 sided die. Use a dot plot to display the distribution of results.

In the example shown opposite two data sets have been saved as R1 and R2 where R1 = 50 games with a 6 sided die and R2 = 50 games with a 9 sided die.

To add a second dot plot press:

Menu > **Plot Properties** > **Add X Variable**



Question: 5.

Discuss differences between the distribution of results for the 10 sided die and the 4 sided.

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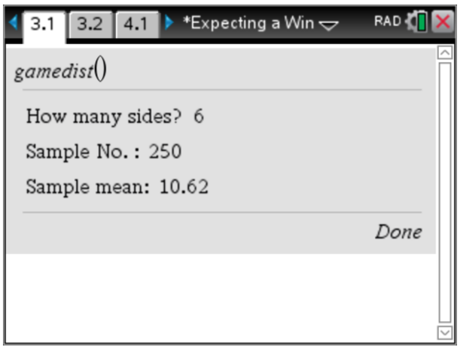
Navigate to Page 3.1

The GameDist program in this problem generates the requested number of samples, each consisting of the specified number of games, and corresponding quantity of sides on the die.

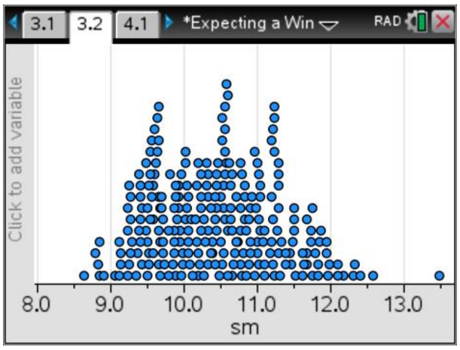


> Large combinations of simulations such as 500 samples each consisting of 100 games will most likely involve approximately half a million rolls of a die. Choose your quantities wisely to avoid lengthy simulations!

Once the sample means have been generated, they can be graphed as a dot plot.

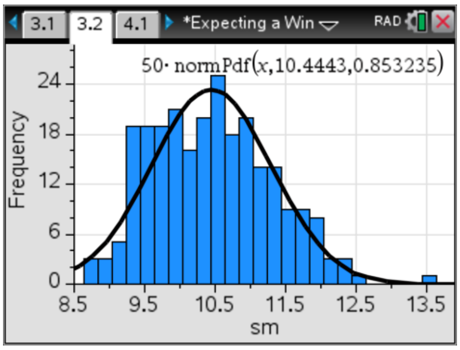
The variable 'sm' contains a list of the sample means calculated from each sample.

The plot shown represents 250 sample means where each sample consisted of 25 games using a 6 sided die.



Changing the plot type to a histogram activates the **Show Normal** option in the **Analyse** menu.

The mean: 10.44 and standard deviation: 0.853 are displayed for this distribution of sample means.



Question: 6.

Generate 200 samples of 25 games for an 8 sided die.

a) Graph the results using a dot plot and describe the approximate shape of the distribution of the sample means. (Sample means are stored in a variable: sm)

b) Change the graph type to a histogram and include an approximate Normal Distribution curve. Discuss and compare the differences (if any) between the mean and standard deviation of your data with the example shown above.

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Question: 7.

Determine an appropriate number of samples and games and complete the table below including the computed mean and standard deviation[1] of each sampling distribution.

Number of Sides	2	4	6	8	10	12	14	16	18
Dist. Mean: $\mu_{\bar{x}}$									
Dist. Standard Deviation. $\sigma_{\bar{x}}$									

Question: 8.

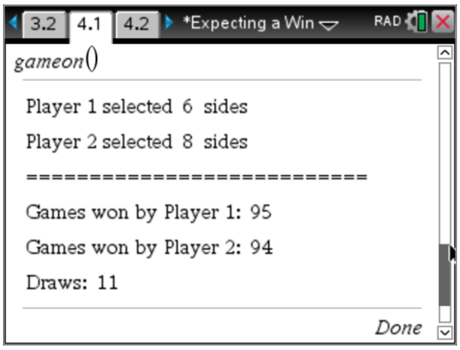
Which die would you chose to give you the best opportunity of winning?

Navigate to page 4.1.

Now is your chance to simulate games where players can determine how many sides they would like to have on their die.

The GameOn program simulates the desired number of games using the selected number of sides for each player. Try simulating some games using your table as a guide.

In the example shown, 200 games were simulated with player 1 selecting a 6 sided die and player 2 selecting an 8 sided die. Note that a draw is counted when both players take exactly the same number of rolls to reach the finish.



Question: 9.

Using the GameOn program, play 200 games where Player 1 has a 4 sided die and Player 2 has a 8 sided die. Who would you predict to win most often?
[Try this simulation a couple of times or try playing a larger quantity of games.]

Note: Data for each Player 1 and Player 2 are stored in T_1 and T_2 respectively.

[1] Standard Deviation of Sampling Distribution – This is also referred to as the "Standard Error".

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Question: 10.

Review your exploration so far. Discuss the outcomes to date and continue your exploration as necessary to determine the optimal number of sides for a dice in this game consisting of a 20 square board.

Answer: Answers will vary. Students should however recognise that the mean is not the ideal statistical measure for determining which die is best. Recommendations and future explorations should use the mode or median noting that for small samples the mode will be too volatile.

Teacher Notes:

Further exploration for this activity could include a smaller board making it possible to calculate theoretical values much easier.

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