Rendezvous For Two

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will
- write an equation of an ellipse in standard form;
- calculate the lengths of semi-major and semi-minor axes;
- calculate the distance of the foci;
- calculate eccentricity; and
- write an equation of a circle.

Prerequisites

Students should have prior knowledge of the standard form of a circle and of an ellipse. They should know how to calculate the lengths of the major and minor axes, the distance from the center of the ellipse to the foci and the eccentricity. They should also be able to graph a circle and an ellipse.

Background

This problem is part of a series that applies mathematical principles in NASA's human spaceflight.

The International Space Station (ISS) is an internationally developed research facility and is the largest human-built satellite in Earth's orbit. It travels around Earth once every 91 minutes at an altitude of approximately 250 miles (400 km). It is an incredible and complex engineering endeavor that has been assembled while in orbit.

Construction of the ISS began in November 1998, when Russia placed the Zarya module in orbit. With the exception of the Russian Zvezda module and Zarya module, all other modules were delivered by a NASA space shuttle. Each module was installed by ISS and space shuttle crewmembers during spacewalks and using a robotic arm. The space shuttle is the only space vehicle capable of carrying the large modules that make up the ISS.

When the space shuttle launches from NASA Kennedy Space Center, it must launch within a certain time frame (called a launch window) in order to successfully dock with the ISS. Launch windows are calculated so that the space shuttle will reach an orbit that is slightly lower than the ISS, but in the same orbital plane. The space shuttle travels slightly faster in its lower orbit, and thus "catches up" to the ISS, while making small orbit corrections to

Key Concepts
Circles, ellipses

Problem Duration
60 minutes

Technology
Computer with projector, movie player, TI-Nspire™ handheld

Materials
- *Rendezvous for Two* Student Edition
- *Real World: Calculating Shuttle Launch Windows* video

Skills
Conics

NCTM Standards
- Algebra
- Geometry
- Data Analysis
- Problem Solving
- Connections

raise its orbit and align the vehicles. The space shuttle then docks with the ISS to resupply, exchange crew members, and deliver hardware—such as a new ISS module, solar panels, or hardware for experiments.

During the rendezvous and docking maneuver sequence, teams from the ISS and Space Shuttle Mission Control Centers work closely together to ensure a safe and successful docking. The ISS Trajectory Operations Officer (TOPO) and space shuttle Flight Dynamics Officer (FDO) flight controllers work together throughout the entire maneuver sequence. The TOPO flight controller works hand in hand with other ISS flight controllers to determine and maintain the orbital position of the ISS. The FDO flight controller is responsible for matching that orbit by determining maneuvers for the space shuttle. Docking two fast-moving vehicles in space is a very delicate and potentially dangerous task, so the shuttle will perform up to ten maneuvers during the rendezvous sequence to align the orbits and maintain a safe trajectory while approaching the ISS.

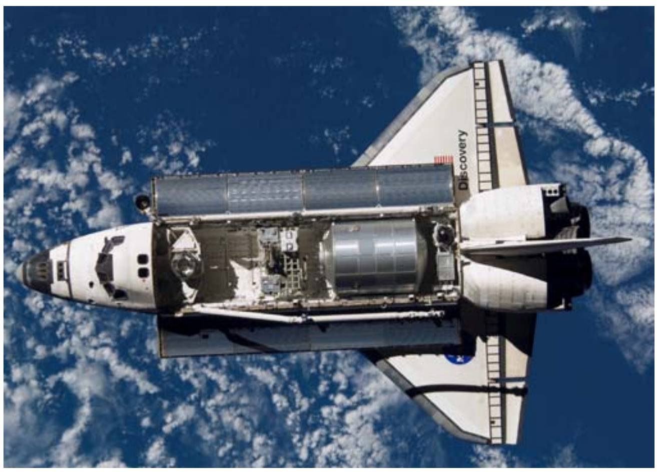

Figure 1: Space Shuttle Discovery, with payload doors open, is viewed from the ISS during rendezvous.

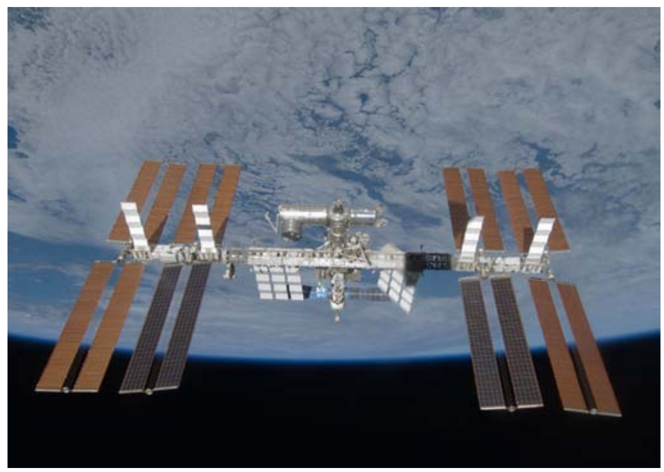

Figure 2: The ISS orbiting the Earth as observed by Space Shuttle Discovery on March 26, 2009

NCTM Principles and Standards

Algebra
- Interpret representations of two variables
- Use symbolic algebra to represent and explain mathematical relationships

Geometry
- Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric systems
- Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates
- Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture

Problem Solving
- Solve problems that arise in mathematics and in other contexts

Connections
- Recognize and apply mathematics in contexts outside of mathematics

Representations

- Select, apply, and translate among mathematical representations to solve problems
- Use representations to model and interpret physical, social and mathematical phenomena

Lesson Development

Following are the phases of the 5-E's Instructional Model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

The questions in this activity are embedded in the TI-Nspire file: *Rendezvous*. Some screenshots have been provided in the solution key to show what the students will see on their handhelds.

1 – Engage (15 minutes)

- Play the video, *Real World: Calculating Shuttle Launch Windows*. (approximately *7 minutes*) The video can be played or downloaded from this link: http://www.nasa.gov/audience/foreducators/nasaeclips/search.html?terms=&category=0100

  To find the video after clicking on the link, type video title in the search box or scroll down the page to find the title.
- Have students read the Background section aloud to the class. Stop occasionally to check for understanding.

2 – Explore (15 minutes)

- With students in groups of two to four, distribute the TI-Nspire file: *Rendezvous.*
- Have students read the problem set-up on page 1.2 and answer the questions on pages 1.3-1.6.

3 – Explain (10 minutes)

- Allow students to remain in their groups.
- Have students read the directions and answer questions on pages 1.7-1.9.

4 – Extend (15 minutes)

- Allow students to remain in their groups.
- Have students read the directions and answer questions on pages 1.10-1.13.

5 – Evaluate (5 minutes)

- Allow students to remain in their groups.
- Have students answer the questions on pages 1.14 -1.15.

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Solution Key

Problem

The space shuttle is preparing to launch, rendezvous, and dock with the ISS. Both FDO and TOPO flight controllers have worked together to predict the trajectory of the space shuttle and to match the orbit of the ISS, once the space shuttle reaches low Earth orbit. The space shuttle is launched to a lower orbit so that it circles the Earth faster and can "catch up" to the ISS.

Figure 3 depicts the coplanar, circular orbits of the space shuttle and the ISS. The space shuttle adjusts its orbit to an ellipse, in order to transfer between the two orbits and rendezvous with the ISS.

Note to Teacher: Use presentation technology to show Figure 3 to your students or print copies of the pdf document. The image is difficult to see on the handheld.

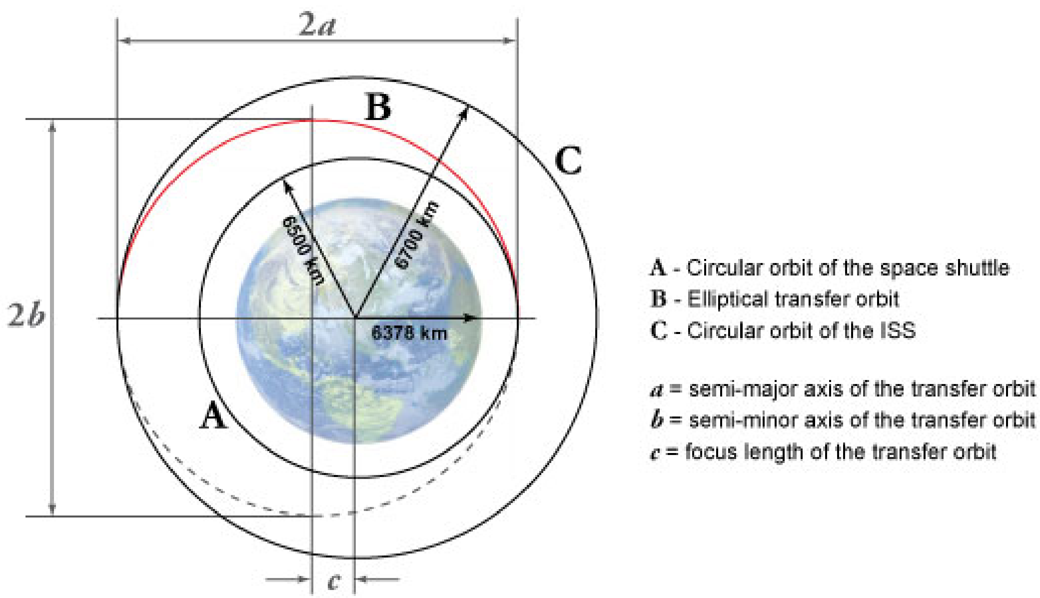


Figure 3: Simplified illustration of the space shuttle and ISS circular orbits and the elliptical transfer orbit. Note: Not to scale.

Directions: Show all work and justify your answers to all questions. Round all answers to the nearest whole number.

1.4 The circular orbits of the space shuttle and ISS are described by their radii. Calculate the long radius (semi-major axis) *a,* of the transfer orbit between the two circular orbits. *Note: The semi-major axis of the transfer orbit is the average of the radii of the two orbits*.

$$a = \frac{r_{ISS} + r_{Shuttle}}{2}$$

$$a = \frac{6700 + 6500}{2}$$

$$a = 6600\ km$$

1.5 The FDO flight controller also uses the *a*-value to describe the transfer orbit. The *a* value, calculated in question 1.4, is the length of the semi-major axis of the ellipse created by the transfer. The center of the Earth is one focus length, *c*, from the center of the transfer ellipse. Refer to Figure 3 and calculate *c*, the length of the focus.

$$c = a - r_{ss}$$
$$c = 6600 - 6500$$
$$c = 100\ km$$

1.6 Calculate the length of the semi-minor axis of the transfer ellipse.

$$c^2 = a^2 - b^2$$
$$b^2 = a^2 - c^2$$
$$b = \sqrt{(6600)^2 - (100)^2}$$
$$b = \sqrt{43550000}$$
$$b = 6599.24\ km$$
$$b=6599\ km$$

1.7 Assuming the coordinate axes are at the center of the ellipse, write the equation that represents the transfer ellipse in standard form.

$$\frac{x^2}{6600^2} + \frac{y^2}{6599^2} = 1$$

1.8 The radius of Earth is 6378 km. How far from the Earth is the circular orbit of the space shuttle?

$$d = r_{shuttle} - r_{earth}$$
$$d = 6500 - 6378$$
$$d = 122\ km$$

1.9 How far away from the Earth is the circular orbit of the ISS?

$$d = r_{iss} - r_{earth}$$
$$d = 6700 - 6378$$
$$d = 322\ km$$

1.10 Assuming your coordinate axes are at the center of the ellipse, use the given radius of the Earth to write the equation that would represent the shape of the Earth as a circle.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-100)^2+(y-0)^2=6378^2$$
$$(x-100)^2+y^2=6378^2$$

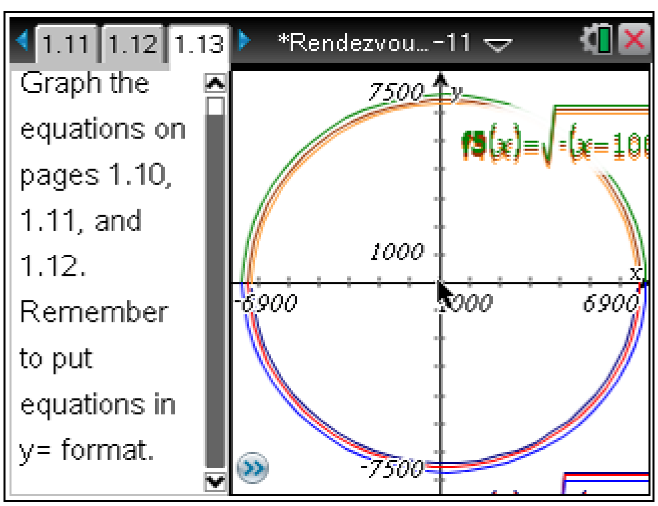
1.11 Assuming your coordinate axes are at the center of the ellipse, write the equation that represents the circular orbit of the space shuttle.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-100)^2+(y-0)^2=6500^2$$
$$(x-100)^2+y^2=6500^2$$

1.12 Assuming your coordinate axes are at the center of the ellipse, write the equation that represents the circular orbit of the ISS.

$$(x-h)^2+(y-k)^2=r^2$$
$$(x-100)^2+(y-0)^2=6700^2$$
$$(x-100)^2+y^2=6700^2$$

1.13 Graph the equations in questions 1.10, 1.11, and 1.12 using the TI Nspire.
Note to teacher: Remind students to rewrite equations in the y= format to graph.



1.14 The term *eccentricity* is commonly used by NASA to describe the shape of an elliptical orbit. Calculate the eccentricity of the ellipse created by the transfer. Leave your answer in fraction form.

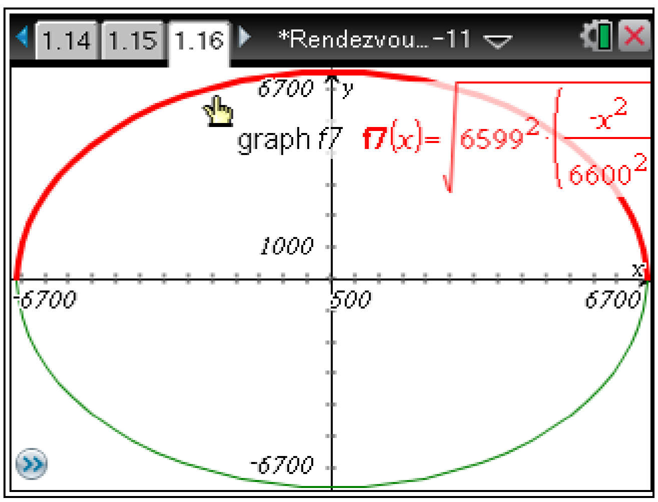
$$e = \frac{c}{a}$$

$$e = \frac{100}{6600}$$

1.15 What does the eccentricity tell you about the shape of the ellipse? Graph the ellipse on Nspire page 1.16 to verify your answer.

The eccentricity is very close to zero; therefore, the ellipse looks much like a circle.

1.16



Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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