Volumes of Rectangular Prisms - Connections Between Algebra and Geometry

Geometry Major Topics: Volumes of Prisms

NCTM Principles and Standards:

Content Standards

Algebra:

Use mathematical models to represent and understand quantitative relationships.

Draw reasonable conclusions about a situation being modeled.

Geometry:

Analyze characteristics and properties of two- and three-dimensional shapes and develop mathematical arguments about geometric relationships.

Analyze properties and determine attributes of two- and three-dimensional objects.

Use visualization, spatial reasoning, and geometric modeling to solve problems.

Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.

Measurement:

Apply appropriate techniques, tools, and formulas to determine measurements.

Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.

Objectives: This lesson will introduce students to many of the algebraic graphing features of the TI-89 calculator, especially the **Maximum** command. This will be accomplished by solving geometric application problems involving volumes of right rectangular prisms. It is suggested the first activity be teacher directed, and the second activity be given to the students as an in-class exercise. This activity should take at least one 50-minute class period to complete.

Materials: TI-89 Graphing Calculator

Prerequisites: The students should have previously used the TI-89 calculator and should have a familiarity with the definitions and vocabulary associated with the solid figures.

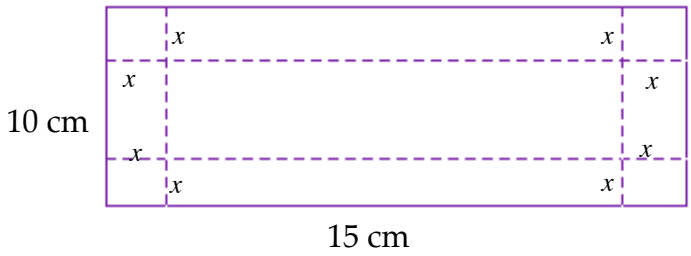
Procedure:

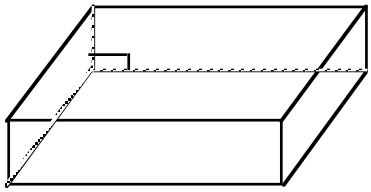
Activity 1: The following problem will be set up and solved:

Matt Matician cuts squares from the corners of a 10 cm. by 15 cm. piece of cardboard and then folds up the edges to make a box without a top.

A. Draw a picture and express the volume of the box as a function of the side of the square, x. Leave in factored form.
B. State the appropriate domain and range for this problem situation. Sketch your graph.
C. Approximate the maximum volume. Round your answer to the nearest hundredth.
D. Find the dimensions of the box that give the maximum volume. Round your answer to the nearest hundredth.

1. Two pictures, the original piece of cardboard and the right rectangular prism that results from folding the cardboard, are shown below.

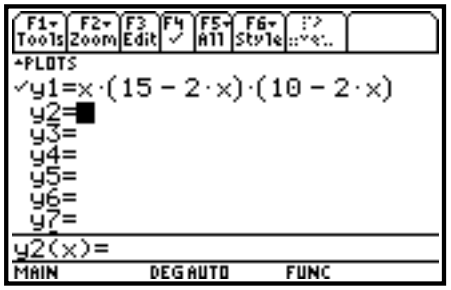


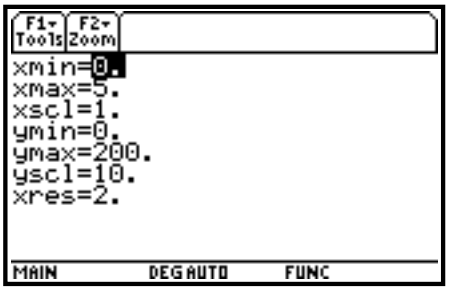


Since the volume of a right rectangular prism is the product of the length, width, and height, the volume of the box in the problem is : $\mathbf{V_{box} = x\,(15-2x)\,(10-2x)}$.

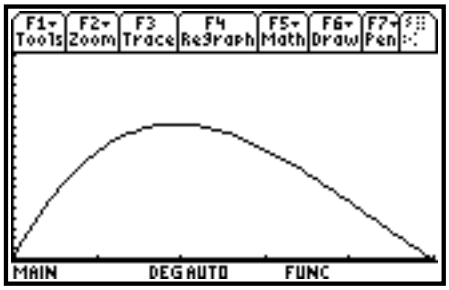
2. The dimensions of the box must all be greater than 0; thus, to find the domain and an appropriate viewing window, the inequalities $x > 0$, $15-2x > 0$, and $10-2x > 0$ must be solved. The resulting domain is $0 < x < 5$. Since the volume of a box will be nonnegative, a minimum y value will be 0. As the equation is graphed, the students will need to experiment to get an appropriate maximum y value.

3. The equation and the values for the viewing window will be entered into the calculator. Press • **y=** and enter the equation. It must be emphasized that on the TI-89 calculator, the multiplication sign must be typed between the first two dimensions in order for the calculator to realize that a function is not being defined. Press • **WINDOW** to enter the minimum and maximum values for the x and y values. Your screens should be similar to the ones below.

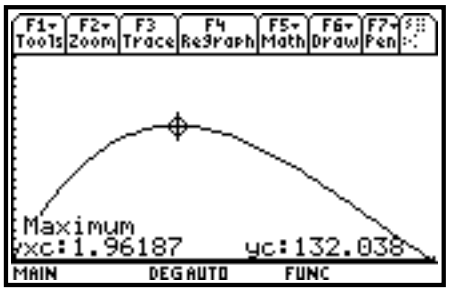




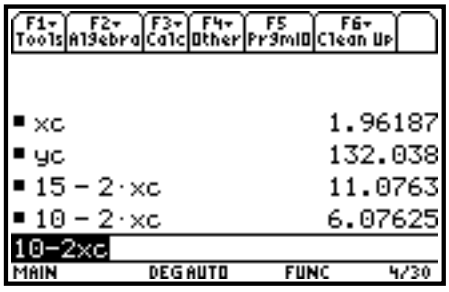
4. Press • **GRAPH** to graph the function.



5. Press F5 Math and select **4:Maximum**. The graph is displayed with the prompt "Lower Bound?" . Move the cursor to the left of the maximum y-value and press ENTER. The prompt "Upper Bound?" is displayed on the screen. Move the cursor to the right of the maximum y-value and press ENTER. The values for the calculated maximum y-value are displayed.



6. The values for xc and yc are stored in the calculator. Press the HOME button on the calculator to return to the HOME screen. Typing xc and pressing ENTER, then typing yc and pressing ENTER will confirm this fact. The connection between the algebraic solutions and the geometric context of the problem can now be established. The value of xc represents the side of the square that must be cut from the corner to achieve a maximum volume for the box; it also represents the height of the box. The value of yc represents the maximum volume that can be obtained. To determine the other dimensions, type 15-2xc and then 10-2xc on the entry line; these represent the length and width of the box. To complete the solution to the given problem, the final displayed values should be rounded to the nearest hundredth.



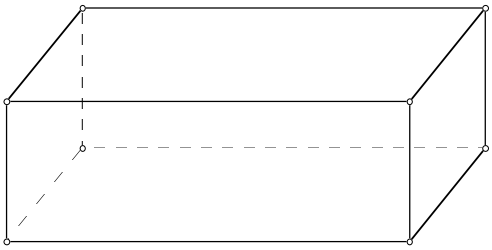
Activity 2: The following problem should be set up and solved:

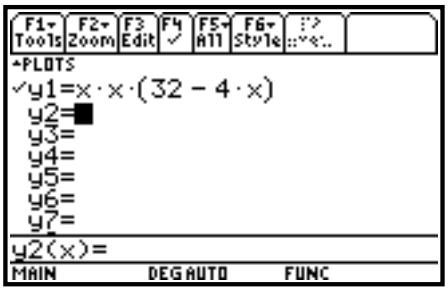
Patty Prism wants to create a right rectangular prism with a square base. The sum of the perimeter of the square base and height of the prism must be 32 cm.

A. Draw a picture and express the volume of the prism as a function of the side of the square, x. Leave in factored form.
B. State the appropriate domain and range for this problem situation. Sketch your graph.
C. Approximate the maximum volume. Round your answer to the nearest hundredth.
D. Find the dimensions of the prism that give the maximum volume. Round your answer to the nearest hundredth.

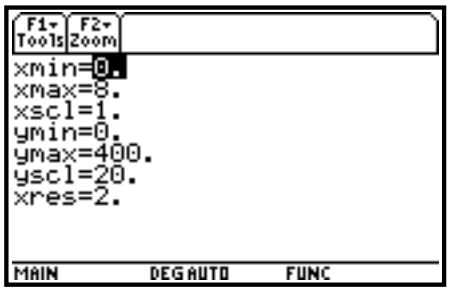
Teacher Note: The procedure from Activity 1 should be modeled. The resulting solutions should be similar to the ones shown below.

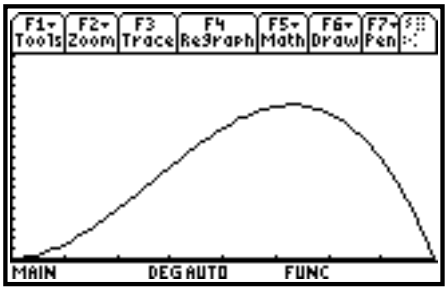
A.



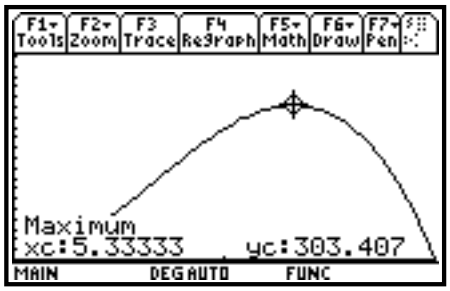


B.

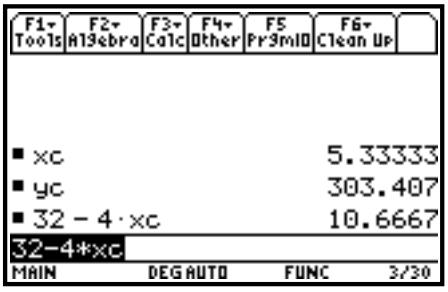




C.



D. The maximum volume of 303.41 cu. cm. is obtained if the square base of the prism has a side of length 5.33 cm. and the height of the prism is 10.67 cm.



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