Exponentially Fast Derivative

ExponentialFastDerivative.tns

Name ______________________

Class ______________________

Part 1 – Warm up

1. **a.** Where is $f(x) = e^x$ positive?

   b. Where is $f(x) = e^{-x}$ positive?

2. What is the domain of ln(x)?

3. Where is $y = \ln(x)$:

   a. Positive?

   b. Positive number less than 1?

   c. Greater than 1?

Part 2 – Investigate the derivative of e^x

On page 3.2, display the slope of the tangent line to the point on the graph of e^x. Select the **Tangent Line** tool (**MENU > Points & Lines > Tangent Line**) and click on the marked point on the graph. Then select **MENU > Measurement > Slope** and click the tangent line twice. Press play.

4. What is the derivative of e^x?

5. $\frac{d}{dx}\left(e^{2x}\right) =$

   $\frac{d}{dx}\left(e^{3x}\right) =$

   $\frac{d}{dx}\left(e^{5x}\right) =$

   $\frac{d}{dx}\left(e^{-7x}\right) =$

   What is the pattern?

6. $\frac{d}{dx}\left(e^{x^2}\right) =$

   $\frac{d}{dx}\left(e^{x^3}\right) =$

   $\frac{d}{dx}\left(e^{5x^4}\right) =$

   $\frac{d}{dx}\left(e^{-2x^{10}}\right) =$

   What is the pattern?

7. Try the follow problems using the Chain Rule. Then use the *Calculator* application on page 4.4. Record both answers and explain why the two solutions are equivalent.

By Hand	Using Technology	Explanation of Equivalence
a. $\frac{d}{dx}\left(e^{4x^{0.5}}\right)=$		
b. $\frac{d}{dx}\left(e^{e^{3x}}\right)=$		
c. $\frac{d}{dx}\left(\sin\left(e^{-2x}\right)\right)=$		
d. $\frac{d}{dx}\left(\cos^3\left(e^{x}\right)\right)=$		
e. $\frac{d}{dx}\left(e^{\ln(5x)}\right)=$		

Part 3 – Investigate a^x

8. On page 5.1, use ctrl + enter to find the decimal approximation for the following numbers.

a. ln(2) **b.** ln(*e*) **c.** ln(4)

On page 5.2 press play or grab and slide the white point on the slider. Observe the relationship between the bold graph of the function a^x and its derivative.

To understand how the derivative of 2^x is derived on page 5.4, you need to know the derivative of a constant times *x*, even when it might not look like a constant at first glance.

9. **a.** $\frac{d}{dx}(\pi x)=$ **b.** $\frac{d}{dx}\left(x\ln(2)\right)=$

10. Differentiate the following functions and compare the decimal approximation solution with the graph explored on page 5.2.

a. $y = \left(\frac{3}{2}\right)^x$ **b.** $y = 3^x$ **c.** $y = 4^x$

d. $y = 0.5^x$ **e.** $y = 2 \cdot 5^x$

11. What is the rule applied to exponential functions?

12. Try the following questions on your own first. Then use the *Calculator* application on page 5.9 and explain why the two solutions are equivalent.

By Hand	Using Technology	Explanation of Equivalence
a. $\frac{d}{dx}\left(3^{x^7}\right) =$		
b. $\frac{d}{dx}\left(5^{2x+3}\right) =$		
c. $\frac{d}{dx}\left(\left(\frac{3}{2}\right)^{2x+2}\right) =$		
d. $\frac{d}{dx}\left(\left(\frac{1}{2}\right)^{3x}\right)$		

Part 4 – Extension/Homework: Exam practice questions

13. a. When playing a video game, a button is pressed causing a point to move along the x-axis described by $x(t) = -2t + 3 + e^{1-2t}$, where t is time. Find the acceleration when $t = 2$.

b. If the positive x direction is forward, describe the speed of the particle at $t = 2$. Explain your reasoning.

14. Let $f(x) = \ln(e^{-5x} + 2x - 3)$. Find $f'(0)$.

15. Find $f'(x)$ if $f(x) = e^{-\cos(x)}$

16. Let $f(x) = x \cdot e^{-2x}$.

a. Solve for x when $f'(x) = 0$.

b. Solve for x when $f''(x) = 0$.