# Exploring the Pythagorean Theorem

## Mathematics Objectives

- Students will analyze relationships to develop the Pythagorean Theorem.
- Students will find missing sides in right triangles using the Pythagorean Theorem.
- Students will connect the Pythagorean Theorem to finding the distance between two points in the coordinate plane.

## Applications and Skills

**Geometry**
Dragging points
Measuring area
Constructing intersection points
Constructing segments
Measuring lengths

**Lists and Spreadsheet**
Manual data capture
Entering formulas
Writing equations

**Calculator**
Calculating using Pythagorean theorem

## Materials

- TI-Nspire handheld
- TNS file: lesson11.tns
- *Exploring Right Triangles* (pages 137–138; page137.pdf)
- *Using the Pythagorean Theorem* (pages 139–140; page139.pdf)
- *Pythagorean Theorem and the Distance Formula* (pages 141–142; page141.pdf)

## Starting the Lesson

After loading the TNS file (lesson11.tns) on each handheld, begin the exercise by instructing students to do the following:

1. Turn on the TI-Nspire by pressing (on).
2. Press (on) and choose **My Documents**.
3. In the folder *Geometry TCM*, choose *lesson11*.
4. Remind the students how to navigate through the TNS file. To move forward through the pages, press (ctrl) ▶. To move backward through the pages, press (ctrl) ◀. To choose a particular page, press (ctrl) ▲, position the cursor on the desired page, and press (enter). To undo previous steps, press (ctrl) [Z] or (ctrl) (esc). Show students that any time they are using a menu
that they wish to exit, they should press (esc).

***Note:*** *Page numbers refer to the TI-Nspire file lesson11.*

## Explaining the Concept

### Problem 1—Exploring Right Triangles

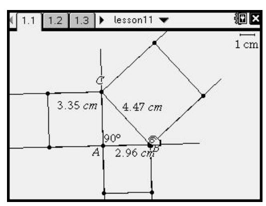
**Step 1** Distribute copies of *Exploring Right Triangles* (pages 137–138) to the students, and have them open to page 1.1.

**Step 2** Explain to students that the sketch on this page will enable them to explore the special relationship that exists with right triangles, such as in ΔABC. Also, point out to students that a square has been constructed on each side of the right triangle and the side lengths of the triangle have been measured.

**Teacher Note:** The sketch shows right triangle ABC. A square has been constructed on each side of the triangle, and the side lengths have been measured. The points of the square were built using parallel and perpendicular lines and measurement transfers, and then the shapes were constructed using the Shapes menu so that the areas can be measured.

**Step 3** Instruct students to grab and drag points B and C slowly and watch how the squares change. (See page 194 for more detailed instructions on grabbing and dragging points.)

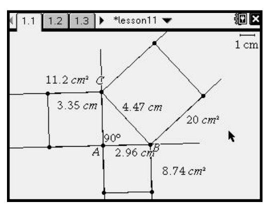
Step 3




**Step 4** Discuss students' observations as a class, and ask if one square always appears to be larger than the other two. Then, allow students to complete question 1 on their activity sheets.

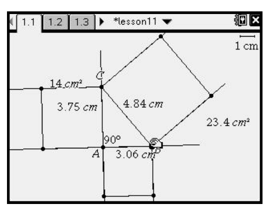
**Step 5** Now, have students measure the area of each square. To do this, instruct students to press (menu), select **Measurement**, and then select **Area**. Then, students must move their cursors (cursor) to hover over the perimeter of the square they wish to measure. When the cursor changes to a hand (hand), the perimeter darkens, and a "ghost" measurement appears, instruct students to press (enter) or (click). Then, have students use the ClickPad to move the measurement to a desired place on the page, and drop it by pressing (enter) or (click). Students must repeat these steps for each square and then press (esc) to release the Area Tool. (See page 200 for more detailed directions on measuring the area of a square.)

Step 5




**Step 6** Next, instruct students to grab and drag points B and C and watch how the areas change.

Step 6



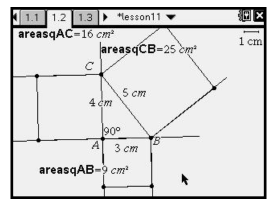
*__Note:__ Page numbers refer to the TI-Nspire file lesson11.*

## Explaining the Concept *(cont.)*

### Problem 1—Exploring Right Triangles *(cont.)*

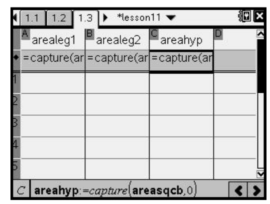
**Step 7** Discuss students' observations as a class, and ask them to explain how the area of each square is related to the triangle. Then, allow students to complete questions 2 and 3 on their activity sheets.

Step 8




**Step 8** Have students press ctrl ▶ to tab to page 1.2. Explain to students that the sketch on this page is like the sketch on page 1.1, but that the areas of the three squares have been defined as variables so that they can be captured in the spreadsheet on page 1.3.

Step 9




**Step 9** Have students press ctrl ▶ to tab to page 1.3 and examine the spreadsheet. Explain that this spreadsheet is set up to capture the three areas in columns A, B, and C when they command it to. Go over each column heading so that students understand what data will be captured in each column as the problem progresses.

**Step 10** Instruct students to press ctrl ◀ to tab back to page 1.2. Then, tell students to grab and drag points B or C. When they want to capture the areas for their spreadsheets, tell them to press ctrl . to capture the three measurements. Have students press ctrl ▶ to tab to page 1.3 to make sure that the data was captured correctly. Students should see a measurement in each of the three columns in the first row. (See pages 194–195 for more detailed directions on grabbing points, dragging points, and manual data capture.)

**Step 11** Have students repeat the process described in step 10 a total of 10 times. Then, allow them to complete question 4 on their activity sheets.

**Step 12** Lead a discussion about students' observations of the relationship between the data captured in the three columns of the spreadsheet. Allow students time to test their observations and ideas by typing equations into column D (and beyond, if necessary). (See page 205 for more detailed directions on typing formulas into spreadsheets.)

***Note:*** *Page numbers refer to the TI-Nspire file lesson11.*

## Explaining the Concept *(cont.)*

### Problem 1—Exploring Right Triangles *(cont.)*

**Step 13** Allow students time to complete questions 5 and 6 of their activity sheets. Circulate among the students at this time to aid those who struggle with this part of the activity.

**Step 14** Conduct a class discussion of the key points from this problem, making certain that the relationship between the three areas is found and explained. Particularly discuss how the equation looks and to what each part of the equation refers. If needed, review the algebra that will be needed to solve the equation for an unknown value.

**Step 15** Write the following statement on the board or overhead:

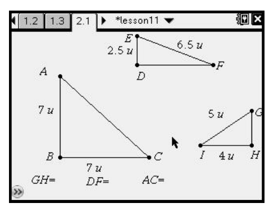
The Pythagorean Theorem states that for a right triangle:

$(\text{leg } 1)^2 + (\text{leg } 2)^2 = (\text{hypotenuse})^2$ OR $\_\_\_\_^2 + \_\_\_\_^2 = \_\_\_\_^2$

Discuss with students how to generalize the formula by using single-letter variables to fill in the blanks of the statement. Then, instruct students to complete the statement on their activity sheets in question 7. *(a; b; c)*

### Problem 2—Using the Pythagorean Theorem

Step 1




**Step 1** Distribute copies of the *Using the Pythagorean Theorem* (pages 139–140) to the students, and have them press ctrl ▶ to tab to page 2.1.

**Step 2** Explain to the students that this sketch shows three triangles which are each missing a leg length.

**Step 3** As a class, examine each triangle on the page, making sure to discuss the name of the triangle, the labels of the vertices, and all of the given lengths. Make sure students understand that the missing leg lengths can be determined using the Pythagorean Theorem.

**Step 4** Have students press ctrl ▶ to tab to page 2.2, and tell them that this page can be used to calculate the missing leg lengths for each of the triangles on page 2.1.

***Note:*** *Page numbers refer to the TI-Nspire file lesson11.*

## Explaining the Concept *(cont.)*

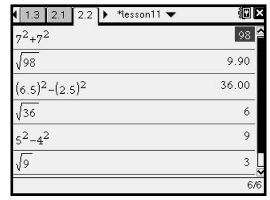
### Problem 2—Using the Pythagorean Theorem *(cont.)*

**Step 5** Complete the calculations for each triangle as a class using page 2.2. Then, have students use the Text Tool to insert the measurements in the proper places on page 2.1. Have students record the calculations in question 1 of their activity sheets. (See page 196 for more detailed directions on using text.)

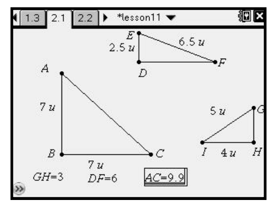
**Step 6** Allow students to check their calculations by measuring the lengths of the missing sides. To do this, have students press (menu), select **Measurement**, and then select **Length**. Then instruct students to choose the first side they want to measure and click both points on that side. Once the "ghost" measurement appears, students can use the ClickPad to drag it near their calculated measurements and press (enter) or [click] to release the measurement. If both measurements are the same, students will know that they performed their calculations correctly. (See page 197 for more detailed directions on measuring length.)

**Step 7** Instruct students to complete the rest of their activity sheets independently. Circulate around the room to aid struggling students. Remind students to use page 2.2 for any calculations they need to perform.

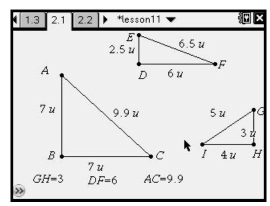
Step 5




Step 5



Step 6



## Applying the Concept

### Problem 3—Pythagorean Theorem and the Distance Formula

**Step 1** Distribute copies of *Pythagorean Theorem and the Distance Formula* (pages 141–142) to the students, and have them press (ctrl) ▶ to tab to page 3.1. Explain to students that they will be using the sketches on pages 3.1 and 3.2 to apply the Pythagorean Theorem to find the distance between two points in the coordinate plane.

**Step 2** Tell students that the sketch on page 3.1 shows points A and B plotted in the first quadrant. A line parallel to the $x$-axis has been constructed through point A. Another line parallel to the $y$-axis has been constructed through point B.

***Note:*** *Page numbers refer to the TI-Nspire file lesson11.*

## Applying the Concept *(cont.)*

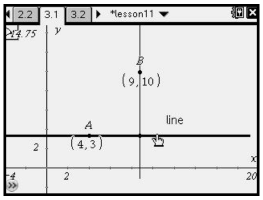
### Problem 3—Pythagorean Theorem and the Distance Formula *(cont.)*

**Teacher Note:** These lines will help students see the horizontal and vertical distances and how they connect to the legs of a right triangle.

**Step 3** Allow students time to observe the sketch and answer question 1 on their activity sheets.

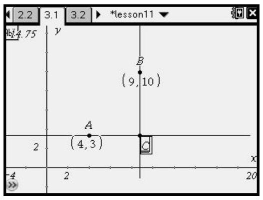
**Step 4** Now, instruct students to construct the point of intersection for lines A and B. (See page 200 for more detailed directions on constructing intersection points.)

Step 4



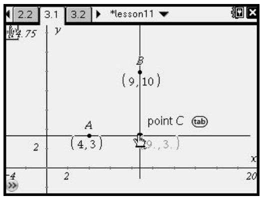

**Step 5** Next, have students label the intersection point C. To label point C, have students press [menu], select **Actions**, and then select **Text**. Tell students to move their cursors ([text cursor]) to hover over the point until it darkens. Have students press [enter] or [click] and a text box will appear. Students must type the text to name the point using the green letter keys. In this case, have students press [shift] [C] and then [enter] to release the Text Tool. (See page 197 for more detailed directions on labeling points.)

Step 5




**Step 6** Tell students that they must now find the coordinates of point C. To do this, have students press [menu], select **Actions**, and then select **Coordinates and Equations**. Now, tell students to move their cursors ([arrow]) to hover over point C. When the cursor changes to a hand ([hand]) and the point darkens, students must press [enter] or [click]. Next, a "ghost" coordinate appears, and students must use the ClickPad to move it to a desired place on the page and press [enter] or [click] to release it. (See page 206 for more detailed directions on finding coordinates of a point.)

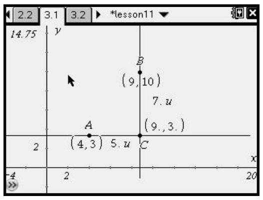
Step 6




**Teacher Note:** You may also wish for students to measure $\overline{AC}$ and $\overline{CB}$. This can be done in two different ways. Students can measure $\overline{AC}$ by subtracting the *x*-coordinates. They can measure $\overline{CB}$ by subtracting the *y*-coordinates. Or, students can use the Measurement Tool to find the lengths of both of the segments.

**Step 7** Give students time to answer question 2 on their activity sheets. If students have difficulty with this question, instruct them to connect points A and B using a segment. (See page 201 for detailed directions on constructing segments.)

Step 7



***Note:*** *Page numbers refer to the TI-Nspire file lesson11.*

## Applying the Concept *(cont.)*

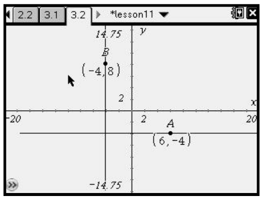
### Problem 3—Pythagorean Theorem and the Distance Formula *(cont.)*

**Step 8** As a class, discuss how to find the answer to question 3 on the activity sheet. After the discussion, allow students time to complete the calculations and record their answers.

**Teacher Note:** Once students use the Pythagorean Theorem to calculate the distance from A to B, measure the segment to verify.

**Step 9** Now, have students press ctrl ▶ to tab to page 3.2. Tell students that the sketch on this page shows points A and B plotted in the second and fourth quadrants. A line parallel to the *x*-axis has been constructed through point A. Another line parallel to the *y*-axis has been constructed through point B.

Step 9

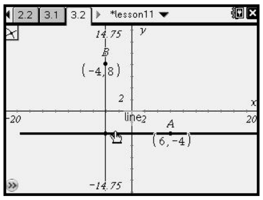



**Teacher Note:** These lines will help students see the horizontal and vertical distances and how they connect to the legs of a right triangle.

**Step 10** Allow students some time to observe the sketch and answer question 4 on their activity sheets.

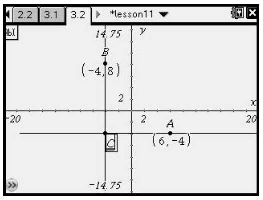
**Step 11** Now, instruct students to construct the point of intersection for lines A and B. (See page 200 for more detailed directions on constructing intersection points.)

Step 11



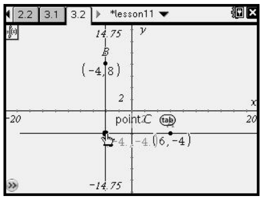

**Step 12** Next, have students label the intersection point C. Directions for this process can be found in step 5. (See page 197 for more detailed directions on labeling points.)

Step 12




**Step 13** Tell students that they must now find the coordinates of point C. Directions for this process can be found in step 6.

Step 13




**Teacher Note:** You may also wish for students to measure $\overline{AC}$ and $\overline{CB}$. This can be done in two different ways. Students can measure $\overline{AC}$ by subtracting the *x*-coordinates. They can measure $\overline{CB}$ by subtracting the *y*-coordinates. Or, students can use the Measurement Tool to find the lengths of both of the segments.

**Step 14** Give students time to answer question 5 on their activity sheets. If students have difficulty, instruct them to connect points A and B using a segment. (See page 201 for detailed directions on constructing segments.)

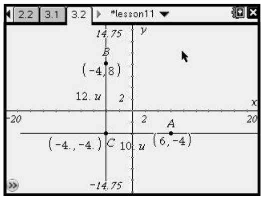
*Note: Page numbers refer to the TI-Nspire file lesson11.*

## Applying the Concept *(cont.)*

### Problem 3—Pythagorean Theorem and the Distance Formula *(cont.)*

**Step 15** As a class, discuss how to find the answer to question 6 on the activity sheet. After the discussion, allow students time to complete the calculations and record their answers.

Step 15



**Teacher Note:** Once students use the Pythagorean Theorem to calculate the distance from A to B, measure the segment to verify.

**Step 16** Write the distance formula on the board or overhead as shown below. Explain to students that this formula is used to find the distance between two points in the coordinate plane.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Step 17** Now, have students answer question 7 on their activity sheets.

**Step 18** Conduct a class discussion of the key points from this problem. The most important idea is that distances in the coordinate plane can be determined by finding the horizontal distance between the points (a leg of a right triangle), the vertical distance between the points (the other leg of a right triangle), and using the Pythagorean Theorem.

#### Differentiation

- **Below Grade Level**—Have students measure $\overline{AC}$ and $\overline{CB}$ on sketches 3.1 and 3.2, and provide direct instruction to help them connect the lengths to the differences between the *x*- and *y*-coordinates. You may wish to allow students to complete this activity sheet in pairs.
- **Above Grade Level**—Allow students to use a coordinate plane and construct their own right triangles. Encourage students to plot the points in quadrants II, III, or IV and to find the length of the hypotenuse using the distance formula or the Pythagorean Theorem.

## Extending the Concept

- Investigate the converse of the Pythagorean Theorem.
- Investigate what would happen if a shape other than a square was constructed from each leg of the right triangle (equilateral triangles, circles with the triangles' sides as their diameters, etc.).