One of the Many Ways

ID: 11885

Time required
50-60 minutes

Activity Overview

In this activity, students will graph polynomials to determine the value and number of zeros for a given polynomial. Students will also use the rational root theorem to determine the value of zeros for a polynomial and connect the number of zeros to the degree of the polynomial.

Topic: Polynomials

- *Zeros & Roots*
- *Rational Root Theorem*

Teacher Preparation and Notes

- *This activity has two parts. The teacher can use either part independently or both parts of the activity. Both parts of the activity ask similar questions about finding zeros and relating the number of zeros to the degree of the polynomial.*
- *Students may want or need to change the window to see the graphs. Students can change the window by pressing Menu > Window > Window Settings and enter appropriate settings for the window.*
- *For part II, if students know polynomial or synthetic division, they can find the roots in a more direct manner. Otherwise, students can use the guess-and-check feature of the activity.*
- *There is a CAS extension to the activity that draws connections to the type of zeros of a polynomial and the factored form of the polynomial.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "11885" in the quick search box.***

Associated Materials

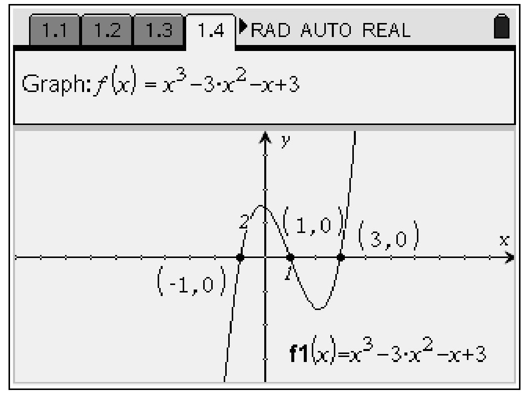
- *Alg2Week16_ZerosOfPoly_worksheet_TINspire.doc*
- *Alg2Week16_ZerosOfPoly.tns*
- *Alg2Week16_ZerosOfPoly_Soln.tns*

Suggested Related Activities

To download any TI-Nspire technology activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Discriminating Against the Zero — 11521*
- *Asymptotes and Zeros — 9286*
- *Watch Your P's and Q's — 8517*

Problem 1 – Finding Zeros Graphically



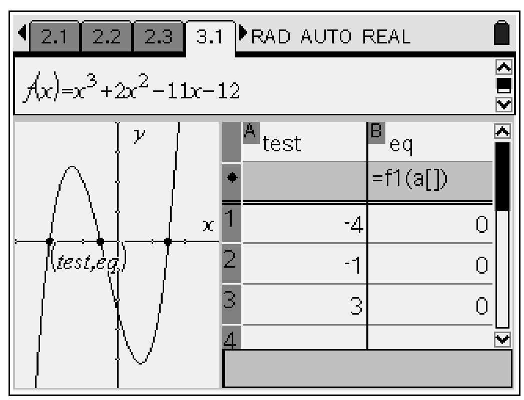
Students will graph polynomial functions and use the **Intersection Point(s)** tool (**MENU > Points & Lines > Intersection Point(s)**) and **Coordinates and Equations** tools (**MENU > Actions > Coordinates and Equations**) to find the values of the zeros. This process will display the coordinates of the points. Students will need to understand that the first number of the coordinate is the value of the zero. Explain to them that all real zeros are also *x*-intercepts.

Students are asked to observe the number of zeros compared to the degree of the polynomial. They should notice that the number of zeros is less than or equal to the degree of the polynomial.

Discussion Questions

- How does one know by plugging a number into the equation that it is a zero?
- For any polynomial, how many zeros are there and what is the degree of the polynomial?
- Why are there less zeros than the degree of the polynomial?
- What is common about where the number of zeros is less than the degree of the polynomial?

Problems 2-6 – Rational Root Theorem



In this part of the activity, students will use the Rational Root Theorem to find and test possible zeros of the graphed polynomial. As students enter a possible value, the point is plotted on the graph to determine if it is a zero. The window scale is hidden for these problems. If students know polynomial or synthetic division, they can use a more direct method for finding zeros. Students should notice the relationship between the number of zeros and degree of polynomial.

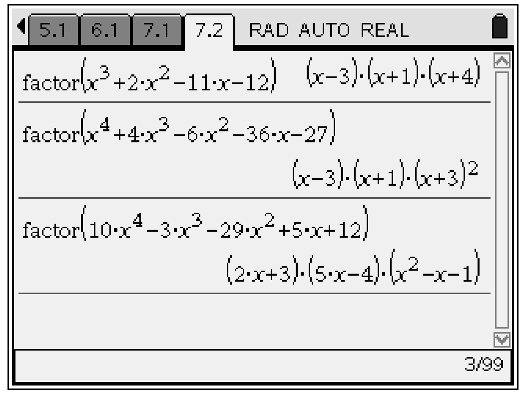
Discussion Questions

- For any polynomial, how many roots are there and what is the degree of the polynomial?
- Why are there less zeros than the degree of the polynomial?
- Is there a particular feature of the graph of a polynomial with less roots than the degree?
- Did anyone find a way to determine the zeros faster than just plugging in values?

CAS EXTENSION – Factoring Polynomials from Roots

This extension activity can only be completed with the TI-Nspire CAS handhelds.

For an extension activity, students can write the polynomial in factored form on the student worksheet using the CAS feature of the TI-Nspire. Use the discussion questions below and on the worksheet to help students explain why the handheld produces the factored form.



Discussion Questions

- What happens to the factored form if there are less zeros than the degree?
- What kinds of roots are possible other than the rational and real roots explored in this activity?
- How would the factored form change if a zero is a multiple root?
- How would the factored form change if a zero is an imaginary root?

Solutions – student worksheet

Function	Zeros
$f(x) = x^3 - 3x^2 - x + 3$	1, –1, 3
$f(x) = x^3 - 3x - 2$	–1, 2
$f(x) = x^4 + 5x^3 + 3x^2 - 5x - 4$	–4, –1, 1
$f(x) = x^4 - x^3 - 7x^2 + x + 6$	–2, –1, 1, 3
$f(x) = x^4 - 3x^3 - 6x^2 + 28x - 24$	2, 3
$f(x) = x^5 - 2.6x^4 - 1.11x^3 - 3.74x^2 - 0.73x + 0.3$	–2.5, –1, –0.5, 0.2, 1.2

1. *Sample Answer:* The number of roots is less than or equal to the degree, n, of the polynomial.
2. Sometimes
3. False
4. 5
5. Two
6. $x = -3$ and $x = 2$

Function	Possible Zeros	Rational Zeros
$f(x) = x^3 + 2x^2 - 11x - 12$	±1, 2, 3, 4, 6, 12	–3, –1, 3
$f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$	±1, 3, 9, 27	–3, –1, 3
$f(x) = 10x^4 - 3x^3 - 29x^2 + 5x + 12$	±1, 2, 3, 4, 6, 12, $\frac{1}{2}, \frac{3}{2}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{12}{5}, \frac{1}{10}, \frac{3}{10},$	$-\frac{3}{2}, \frac{4}{5}$
$f(x) = x^4 - 2x^3 - x^2 - 2x - 2$	±1, 2	No Rational Zeros

1. *Sample Response:* The polynomial cannot be completely factored or will have terms with powers.
2. The factor with the multiple root will have a power.
3. The polynomial will not be completely factorable, so it will have a quadratic or higher term as one of the factors.

Function	Factored Form
$f(x) = x^3 + 2x^2 - 11x - 12$	$(x-3)(x+1)(x+4)$
$f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$	$(x-3)(x+1)(x+3)^2$
$f(x) = 10x^4 - 3x^3 - 29x^2 + 5x + 12$	$(2x+3)(5x-4)(x^2-x-1)$
$f(x) = x^4 - 2x^3 - x^2 - 2x - 2$	$(x^2+1)(x^2-2x-2)$