Science Objectives

- Students will explore factors affecting the trajectory taken by a projectile (ignoring air resistance).
- Students will observe and compare horizontal and vertical velocity components as the projectile travels along its trajectory.
- Students will observe maximum range values and determine the optimum launch angle.
- Students will observe the effect of an elevated landing area on the range of a projectile.

Vocabulary

- apex
- parametric equations
- projectile
- projectile range
- trajectory
- vector components

About the Lesson

- This lesson simulates projectile trajectories, neglecting air resistance.
- This is a conceptual activity and as a result, students will:
  - Observe the connections between launch speed, launch angle, and vertical and horizontal velocity components.
  - Observe how a trajectory changes as the initial launch conditions change.
  - Determine a maximum range and optimum launch angle.
  - Explore the effect of an elevated landing area on range and optimum launch angle.

TI-Nspire™ Navigator™

- Send out the *Projectile_Trajectories.tns* file.
- Monitor student progress using Screen Capture.
- Use Live Presenter to spotlight student answers.

Activity Materials

- *Projectile_Trajectories.tns* document
- TI-Nspire™ Technology

TI-Nspire™ Technology Skills:
- Download a TI-Nspire document
- Open a document
- Move between pages
- Drag objects to observe results
- Trace a plot
- Capture data
- Plot data & analyze plots

Tech Tips:
Selecting an object to drag when it is near other objects can be a challenge. Watch for the tab symbol and the name of the object as the cursor hovers over it.

Lesson Materials:
Student Activity
- Projectile_Trajectories_Student.doc
- Projectile_Trajectories_Student.pdf

TI-Nspire document
- Projectile_Trajectories.tns

Discussion Points and Possible Answers

Teaching Tip: As an introduction, take students to a basketball court and have several students shoot baskets with different trajectories. If this is not feasible, bring in several foam balls and do the same with a garbage can as the basket. Discuss the different trajectories.

Move to page 1.2

Problem 1: Projectile Motion Components

Students should read about what determines trajectory of a projectile.

Move to page 1.3.

1. Students can drag the V initial vector anywhere in the window. As they do, they should note how the horizontal ***Vx*** and vertical ***Vy*** velocity components change. They can use the Pythagorean Theorem to see how the components relate to the V initial vector.

Tech Tip: The degree of precision in this activity is generally set for 1 decimal place, thus any student calculations need to be rounded accordingly.

Move to page 1.4.

2. The trajectory of the projectile is shown here. When students press **Menu > Trace > Graph Trace**, they will be shown the horizontal and vertical position of the projectile at the given time.

Move to pages 1.5–1.9.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q1. What two things tend to make the projectile travel higher?

Answer: greater initial speed and steeper angle

Q2. What two things can you do to make the projectile travel farther?

Answer: greater initial speed and launch at the best (optimum) angle

Q3. Set a suitable trajectory (between 20° and 70°) on page 1.4. What is the maximum height of that trajectory? Lock in a trace point.

Answer: Answers will vary depending on what trajectory students set. Read from Graph Trace display.

Q4. When did the projectile reach this maximum height, and what was its horizontal displacement at this time?

Answer: Answers wlll vary depending on what trajectory students set. Read from Graph Trace display.

> **Teaching Tip:** Remind students to set the initial velocity so that the trajectory remains on the screen without changing window settings.

Q5. At what time and location did the projectile land on the ground?

Answer: Answers vary depending on what trajectory students set. Read from **Graph Trace** display.

Move to page 2.1.

Problem 2: Horizontal and Vertical Motions

Students should read about the way horizontal and vertical motions are separate and independent.

Move to pages 2.2 and 2.3.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q6. What is the equation to determine distance for uniform horizontal motion (in terms of initial velocity components Vxi and Vyi)?

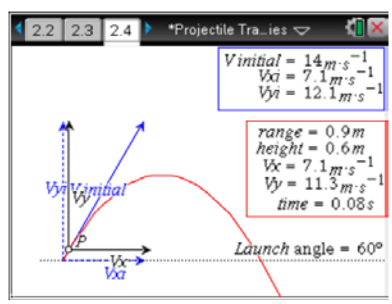
Answer: range or dx = Vxi · t

Q7. What is the equation for vertical height in free-fall motion (in terms of initial velocity components Vxi and Vyi)?

Answer: height or dy = Vyi · t + ½ g · t^2 or dy = Vyi · t -4.9 · t^2

Move to page 2.4.

3. Students can set the initial velocity as before. They can now also drag the projectile (point P) along the trajectory and see the data for the projectile as it moves. Note that Vx remains constant and Vy is positive going up and negative coming down.
   Dragging the point to ground level gives a maximum range, but that is the purpose of Problem 3.



Move to pages 2.5–2.8.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q8. What happens to the magnitude of the horizontal component of velocity (**Vx**) as the projectile moves along the trajectory?

Answer: Horizontal velocity (**Vx**) is constant.

Q9. What happens to the magnitude of the vertical velocity (**Vy**) as the projectile moves along the trajectory?

Answer: Vertical velocity (**Vy**) changes continuously and is + (up) and – (down).

Q10. When does the vertical velocity become zero? Why? What is this point of the trajectory called?

Answer: Vy becomes zero when projectile stops going up and starts going down (because of gravity). This happens when it "runs out" of vertical speed and starts to fall down. Because it stops going up at this point, this is the maximum height or apex of the trajectory.

Q11. What happens to the vertical velocity (**Vy**) after the projectile reaches maximum height? Explain.

Answer: At the apex (max height) the projectile starts to fall down. Vy becomes negative, and the object falls down faster and faster.

Move to page 2.9.

4. Students set a trajectory on page 2.4 so that they can see the full trajectory on the screen, including where the projectile would hit the ground. It does not matter what points they use to capture data, but they should use a wide range of points. Finding the apex is not necessary, although many will do this anyway.

Move to page 2.10.

Students should read directions for plotting data to find how velocity changes with time.

5. The data plot should be linear and show a negative slope. Different student graphs may look a little different because of the range of points they captured. Scaling occurs automatically.

Move to pages 2.11–2.21.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q12. What does the slope of any velocity vs. time graph represent?

Answer: The slope of any velocity versus time graph is acceleration (instantaneous acceleration if the graph is not linear).

Q13. What is the slope of this equation?

Answer: -9.8

Q14. What units should the slope have?

Answer: m/s^2

Q15. Explain what this slope means.

Answer: acceleration due to gravity

Q16. What is the vertical axis intercept (*y*-intercept)? What does this mean?

Answer: Intercept value varies. It is the initial vertical velocity (Vyi).

Q17. How does the *y*-intercept relate to the original trajectory (on page 2.4)?

Answer: It is the vertical velocity component (Vyi) of the initial velocity vector.

Q18. What does the point where the line crosses the *x*-axis represent? Where is this seen on the trajectory?

Answer: This is where the vertical speed is 0. This happens at the apex of the trajectory.

> **Teaching Tip:** Have students find the x-intercept value, that is the time when the vertical velocity is 0. Then have them go back to the trajectory, drag the projectile to the apex, and compare the times.

Q19. What happens to the vertical velocity at the point where the graph crosses the <u>*x*-axis</u>? Explain what is going on here.

<u>Answer</u>: The vertical velocity decreases in value and becomes 0 then negative as the graph crosses the *x*-axis. The projectile stops going up (+) and starts coming down (-).

Q20. Why is the vertical acceleration negative throughout the entire trajectory?

<u>Answer</u>: Vertical acceleration is caused by gravity, which acts downward (negative direction).

Q21. Does <u>horizontal</u> motion affect the <u>vertical</u> motion?

<u>Answer</u>: B. No

Q22. Does <u>vertical</u> motion affect the <u>horizontal</u> motion?

<u>Answer</u>: B. No

Problem 3: Maximize Range and Optimum Angle

Move to page 3.1.

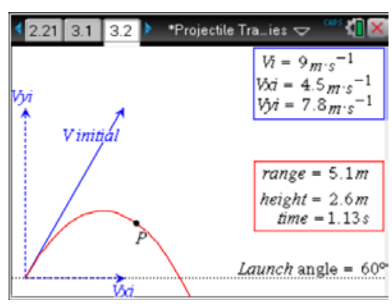
Have students answer the questions on either the handheld, on the activity sheet, or both.

Q23. How can you maximize the range of a projectile? Shooting or throwing it faster is obvious; what else can you do?

<u>Answer</u>: Find the best or optimal angle.

Move to page 3.2.

6. Students can change the launch angle only; the initial speed is fixed at 9 m/s. To estimate the maximum range, they should drag V initial vector and watch where the trajectory crosses "ground level." Set this to a maximum range then drag point P to the ground (height = 0) and find the range and time. Note the angle of launch.



Move to page 3.3.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q24. In this example, what launch angle gives the maximum range?

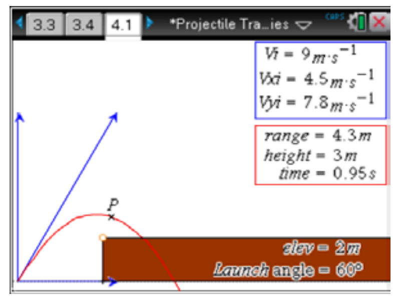
Answer: approx. 45°

Q25. What is the maximum range?

Answer: maximum range is approximately 8.2 m (N.B.: remember the low precision set in the system.)

Problem 4: Elevated Landing

Move to page 4.1.



7. This time students find maximum range where the trajectory meets the top edge of the elevated landing area (box). Again, students can change only the angle of launch for the projectile, but they can also change the elevation of the landing area by dragging the top left corner. Elevation can be set between 2 to 4 m. The bottom left corner can also be dragged to move the edge of the box to allow further investigations.

Move to pages 4.2–4.4.

Have students answer the questions on either the handheld, on the activity sheet, or both.

Q26. What happens to the range when the landing area is elevated above the launch location?

Answer: Range is reduced when the landing area is elevated.

Q27. Does elevation of the landing area affect the optimum angle for maximum range?

Answer: A. Yes

Q28. As the elevated landing area becomes higher, what happens to the angle needed for maximum range?

Answer: Greater elevation of landing area means the optimum launch angle will be greater than 45°. When the elevation is greater, a greater launch angle is needed for maximum range. There is a maximum: Elevation must be slightly less than apex height no matter what the launch angle.

Extension

As an extension to this activity, consider another trajectory where the launch point is elevated above the landing area. What does this do to the range and optimum angle for maximum range?

TI-Nspire Navigator Opportunities

Use TI-Nspire Navigator Screen Capture to monitor student progress and to retrieve the file from each student at the end of the class period. This is a conceptual lesson, Screen Capture can be used to illustrate student examples to explain concepts or remedy difficulties.

Wrap Up

When students are finished with the activity, revisit the application of shooting baskets, or other applications. Examples are slow pitch baseball, throwing horse-shoes or quoits, or artillery. Discuss how different shots can hit the same target, along with advantages and disadvantages of each.

Assessment

- Formative assessment will consist of questions embedded in the .tns file.
- Summative assessment may consist of questions/problems on a chapter test. It may also involve, conceptual understanding, as demonstrated through applications of the concept (e.g. shooting baskets).