Going Back To Your Roots

**Problem 1 – The Fundamental Theorem of Algebra**

Johann Carl Friedrich Gauss (1777–1855), German mathematician and physicist, first proved the Fundamental Theorem of Algebra.

This theorem states that every polynomial equation of degree greater than or equal to 1, with complex coefficients has at least one complex root.

Consider the polynomial $f(x) = x^3 + x^2$.

- Factor this polynomial so that the degree of each factor is 1.

Set your graphing calculator to best display this polynomial by pressing WINDOW and matching the screen to the right.

```
WINDOW
 Xmin=-2
 Xmax=2
 Xscl=1
 Ymin=-1
 Ymax=1
 Yscl=1
 Xres=1
```
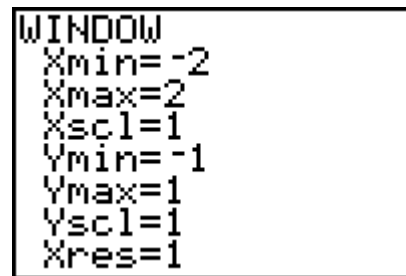
Graph this polynomial by pressing Y=, entering the equation, and pressing GRAPH.

Calculate the zero where the graph crosses the x-axis by pressing 2nd TRACE and selecting **zero**.
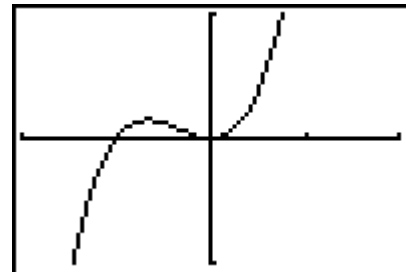
Now, use the arrow keys to move the cursor to

- the left of the zero and press ENTER
- the right of the zero and press ENTER
- the guess of the zero's location and press ENTER

Repeat this procedure for the other zero appearing on the graph.

- Which zero matches which factor of the polynomial?

Since the factor x shows up twice, in the polynomial $f(x) = x^3 + x^2 = x \cdot x \cdot (x + 1)$, the polynomial has two distinct roots or zeros. The root $x = 0$ has multiplicity 2 and $x = -1$ has multiplicity 1.

- What do you notice about the multiplicities of the roots and the degree of the polynomial?

- Rewrite the roots as complex numbers.

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Check For Understanding

1. How many complex roots does $f(x) = x^5 - 7x^2 + 3x + 2$ have?

2. What is the multiplicity of the root $x = 3$ in $f(x) = (x - 1)^4(x - 3)^2(x + 3)$?

3. What is the total number of complex roots of $f(x) = (x - 1)^4(x - 3)^2(x + 3)$?

Problem 2 – Beyond Real

What about the polynomial $f(x) = x^2 + 9$?

Can you factor it? How do you find the zeros? What type of roots does it have?

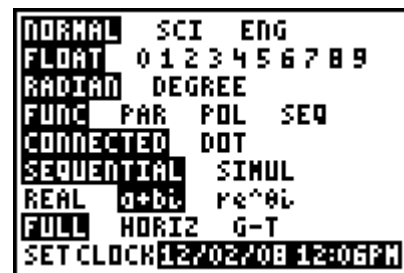Set your graphing calculator to best display this polynomial by pressing WINDOW and matching the screen to the right.

```
WINDOW
 Xmin=-4
 Xmax=4
 Xscl=1
 Ymin=-1
 Ymax=16
 Yscl=2
 Xres=1
```
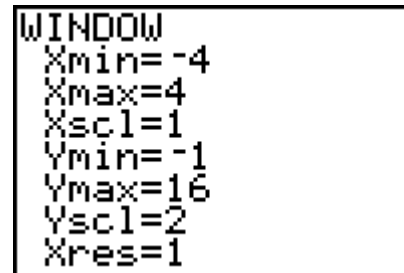
Graph this polynomial by pressing Y=, entering the equation, and pressing GRAPH.

- How many complex roots does the polynomial have?

- What type(s) of roots does the polynomial have?

- How did the graph enable you to determine the root type?

Press MODE and press ENTER on **a + bi**.

Return to the **Home** screen. Use the quadratic formula to determine the values of the roots.

```
NORMAL   SCI   ENG
FLOAT  0 1 2 3 4 5 6 7 8 9
RADIAN   DEGREE
FUNC   PAR   POL   SEQ
CONNECTED   DOT
SEQUENTIAL   SIMUL
REAL   a+bi   re^θi
FULL   HORIZ   G-T
SET CLOCK 12/02/08 12:06PM
```

Problem 3 – The Mixed Case

Set your graphing calculator to best display the polynomial $f(x) = x^3 + x^2 + 4x + 4$ by pressing WINDOW and matching the screen to the right.

```
WINDOW
 Xmin=-2
 Xmax=1
 Xscl=.5
 Ymin=-4
 Ymax=5
 Yscl=1
 Xres=1
```

Graph this polynomial by pressing Y=, entering the equation, and pressing GRAPH.

- How many complex roots or zeros does the polynomial have?

- How many of the roots or zeros are real?

- What are the real zeros?

- Rewrite it as a complex number as well as a factor.

- Divide the polynomial by this factor to aid you in the complete factorization of the polynomial. Make sure all the factors are linear when the factorization is complete.

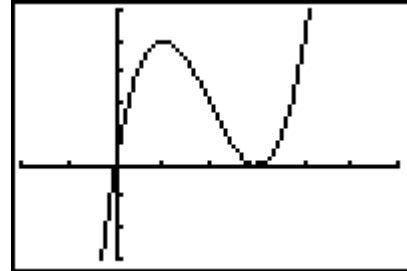- Identify all complex roots of the polynomial.

![Texas Instruments logo] **Going Back To Your Roots**

Extension – Even and Odd Multiplicity

What is meant by the term *multiplicity*?

The polynomial $x(x-3)^2$ has been graphed to the right.

- What is the multiplicity of the factor x?

- What is the multiplicity of the factor $(x-3)$?

- What do you notice about the graph at each zero?

The polynomial $(x-3)^3(x+4)^4$ has been graphed to the right.

- What is the multiplicity of the factor x?

- What is the multiplicity of the factor $(x-3)$?

- What do you notice about the graph at each zero?
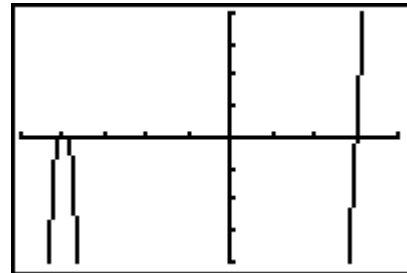
Investigate this further by using the Transformational graphing application by pressing APPS and selecting **Transfrm**.

Now press Y= enter **x^A**. Press ZOOM and select **ZStandard**.

Notice the variable A is located on the left side of the screen. Enter different values for A and notice the effects they have on the graph.

- Describe how the graph at the *x*-value of the roots differs for roots with even multiplicities and for roots with odd multiplicities.
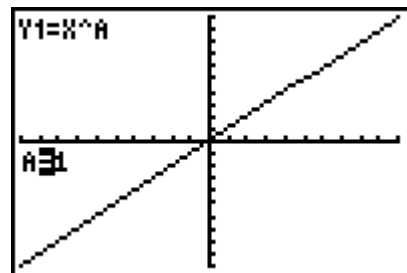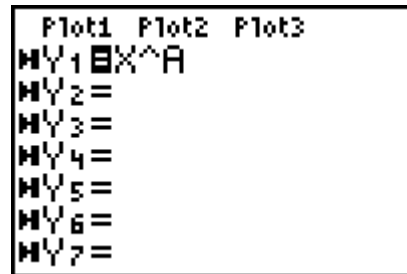
Application & Practice

The work for the first polynomial has been started. Find the two remaining factors, roots, and multiplicities for this polynomial. Complete the chart for the remaining polynomials.

Polynomial	Factor(s)	Roots	Multiplicity
$f(x) = x^4 - 9x^3 + 27x^2 - 31x + 12$	$x - 4$	4	1
$f(x) = x^3 - 7x^2 + 11x - 5$			
$f(x) = x^5 + 9x^4 + 31x^3 + 63x^2 + 108x + 108$			