

Activity 8

Two Dimensional Particle Motion

When a particle is shot into the air with a known velocity, v_0 , and angle to the horizontal, θ , its path can be plotted as parametric equations. If the starting position is denoted as (x_0, y_0) then the x position is given by $x(t) = x_0 + v_0 \cos(\theta)t$ and its y position by



$$y(t) = y_0 + v_0 \sin(\theta)t + \frac{1}{2}gt^2$$

where $g = -9.8 \text{ m/s}^2$ or -32ft/s^2


For this activity the starting position in the x direction, x_0 , will always be 0.

Exploration


As an example, a particle is fired into the air from the ground at 40 m/s and at an angle of 60° . Sketch a graph of the path taken by the particle. Then find the time at which it will reach its highest point and its position at that point. Finally, determine when and where the particle will strike the ground.


1. Open a new TI InterActive! document. Title this document **Two Dimensional Particle Motion**. Add your name and the date to this document.
2. Select Mode settings  and change the Angle Format to Degree and the Graph Type to Parametric. Click OK.
3. Select Math box  and define the y starting position as $y_0 = 0$.
4. Since the initial velocity is 40 m/s, in another math box define $v_0 = 40$.
5. Since the angle at which it is fired is 60° , in another math box define $\theta = 60$.

Note: θ can be found under .

6. Select Graph  and define $x_1(t) = v_0 \cos(\theta) \cdot t$ and $y_1(t) = y_0 + v_0 \sin(\theta) \cdot t - 0.5 \cdot 9.8 \cdot t^2$.

Note: Use the Symbol Palette  to enter θ .

7. Click on the graph style box  and choose to animate **with Path and Symbol**. Click OK.

8. Click on . Set up the window with these values:

Tstep = 0.1

t:[0, 8]

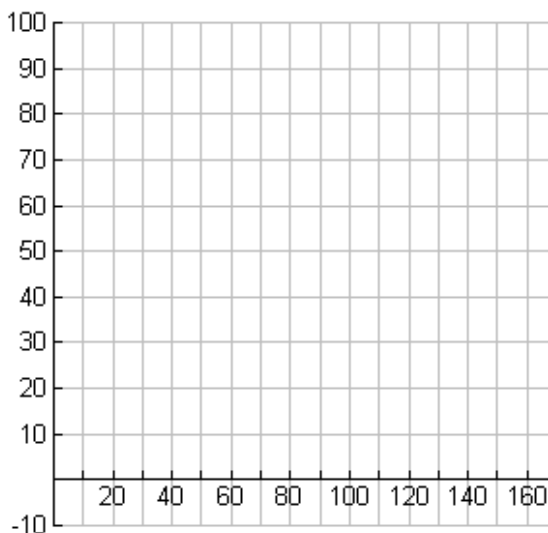
x: [0, 170]

Xscale = 10


y: [-10,100]

Yscale = 10


9. Click on the Animate tab and change the Draw Speed to Slow and the Draw Type to Sequential. Click on Apply, then click OK.



10. The path of the ball can now be viewed by clicking on Animate

Graph . Sketch the path taken by the ball on the grid provided.

Analysis

1. Click  and change the Trace Step to 0.01. Click on the graph near the maximum height. Trace on the graph to approximate the highest point and the time at which the highest point was reached.

$t =$ _____ $x =$ _____ $y =$ _____

2. Click on the graph near where the particle strikes the ground. Trace on the graph to approximate the time and position at which the particle strikes the ground.

$t =$ _____ $x =$ _____ $y =$ _____

3. Changing the value of v_0 will create a new path. Click on Save to Document



to paste the graph into TI InterActive! Double-click on $v_0 = 40$ and change the 40 to the values indicated. Click out of the math box and describe the effects on the graph. Adjust the window as needed.

$v_0 = 30$ _____

$v_0 = 50$ _____

4. Changing the value of θ will also create a new path. Double click on $v_0 = 50$ and change the 50 back to 40. Double click on $\theta = 60$ and change the 60 to values indicated. Click out of the math box and describe the effects on the graph. Adjust the window as needed.

$\theta = 30$ _____

$\theta = 70$ _____

5. Change the value of y_0 to create a new path. Double click on $\theta = 70$ and change the 70 back to 60. Double click on $y_0 = 0$ and change the 0 to the values indicated. Click out of the math box and describe the effects on the graph. Adjust the window as needed.

$y_0 = 10$ _____

$y_0 = 35$ _____

6. Generalize your findings about the effect of v_0 , θ , and y_0 on the path of the particle.

7. Save this document as **motion.tii**. Print a copy of this document.

Additional Exercises

For each of the situations described below, sketch a graph of the path taken by the particle. Then find the time at which it will reach its highest point and its position at that point. Finally, determine when and where the particle will strike the ground. For each problem assume that $x_0 = 0$.

1. A soccer ball is kicked from ground level at 20 m/s at an angle of 30° .

Highest Point:

$$t = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

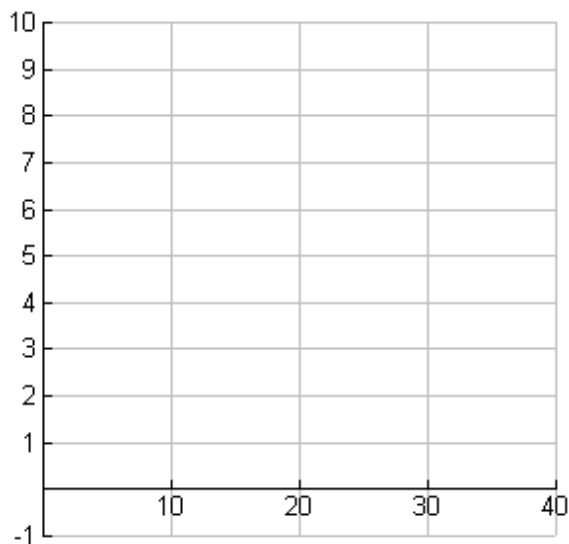
$$y = \underline{\hspace{2cm}}$$

Strikes the ground:

$$t = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$



2. A baseball is thrown from a height of 1.5 meters above the ground at 80 m/s at an angle of 20° to the horizontal.

Highest Point:

$$t = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

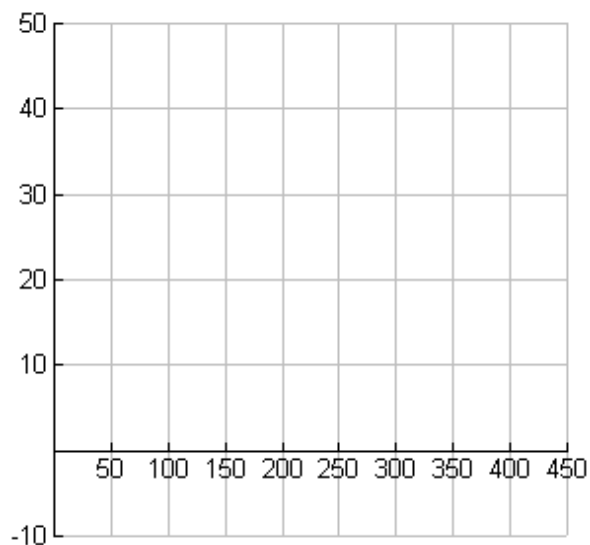
$$y = \underline{\hspace{2cm}}$$

Strikes the ground:

$$t = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$



3. A rocket is shot from 1 meter above the ground at 200 m/s at an angle of 45° .

Highest Point:

$$t = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

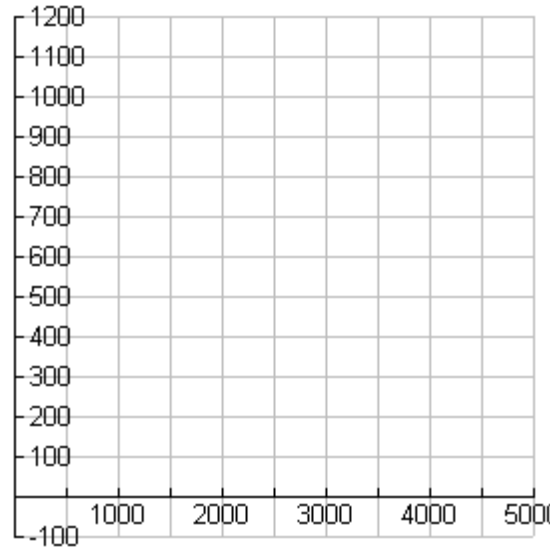
$$y = \underline{\hspace{2cm}}$$

Strikes the ground:

$$t = \underline{\hspace{2cm}}$$

$$x = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$



4. If an object is fired from the ground at any given velocity, at what angle will maximize the distance that the rocket will travel? Verify your results.

5. To verify your results, define $\theta := \{10, 20, 30, 40, 45, 50, 60, 70, 80, 90\}$.

Double-click on the graph and click on Animate Graph . Does this graph verify your answer to question 4? Explain.

6. Save the graph in the document. Save this document as **particle_motion.tii**. Print a copy of this document.