Applied Fundamental Theorem of Calculus
TIMATH: CALCULUS

TEACHER NOTES

Math Objectives

- Students will be able to identify the graphical connections between a function and its accumulation function.
- For a given function, students will recognize the accumulation function as an antiderivative of the original function.
- Students will use the relationship between velocity and distance to solve a real-world travel problem.
- Students will be able to apply and explain the second Fundamental Theorem of Calculus.

Vocabulary

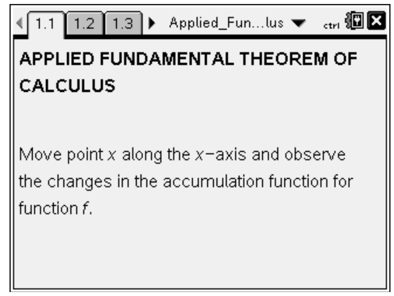
- accumulation function
- definite integral
- antiderivative
- velocity
- distance

About the Lesson

- The intent of this lesson is to help students make visual connections between a function and its definite integral.
- Students will use the accumulation function with a fixed starting point to find definite integrals of a function over different intervals.
- Students will observe that the accumulation function is an antiderivative of the original function.
- This lesson is a follow-up to *Second Fundamental Theorem of Calculus.*

TI-Nspire™ Navigator™ System

- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- Use Class Capture to demonstrate that students can grab and drag the point *x* properly.
- Use Quick Poll to assess student understanding.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- Press ctrl G to either hide the function line or access the function line in a *Graphs & Geometry* page.

Lesson Materials:
Student Activity

- Applied_Fundamental_Theorem_of_Calculus_Student.pdf
- Applied_Fundamental_Theorem_of_Calculus_Student.do

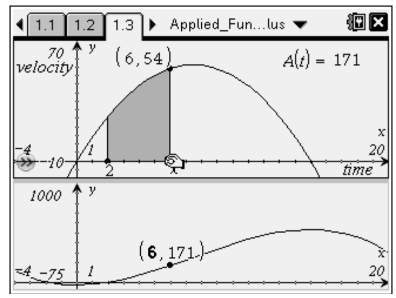
TI-Nspire document

- Applied_Fundamental_Theorem_of_Calculus.tns

Discussion Points and Possible Answers

> **Tech Tip:** If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand ( ) getting ready to grab the point. Also, be sure that the word *point* appears, not the word *text*. Then press ctrl to grab the point and close the hand ( ).

Move to page 1.3.



The top graph represents the velocity of a car on a 15-hour road trip. The *x*-axis represents the time in hours, and the *y*-axis represents the velocity of the car in miles per hour. The accumulation function, *A*(*t*), gives the value of the definite integral of *f*(*t*) between *a* and *x*. In this graph, the value of *a* is set to 2 hours.

1. Move point *x* along the axis to find each of the following values.

   a. What is the velocity of the car at 6 hours?

   <u>Answer</u>: 54 miles per hour

   b. When is the car going 50 miles per hour?

   <u>Answer</u>: after 5 hours and 10 hours

   c. $A(5) = \int_2^5 f(x)\,dx =$ ______________

   <u>Answer</u>: 171

   d. $A(14) = \int_2^{14} f(x)\,dx =$ ______________

   <u>Answer</u>: 528

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 1 at the end of this lesson.

2. The bottom graph shows the accumulation function as a function of x, with a = 2. What appears to be the relationship between the two graphs? Explain.

Answer: The top graph, the accumulation function, appears to be the derivative of the bottom graph. The bottom graph starts to decrease when the top graph goes in the negative (after 15 hours, which is outside the range included in the real-world scenario). The bottom graph changes concavity from concave to convex when the top graph changes from increasing to decreasing.

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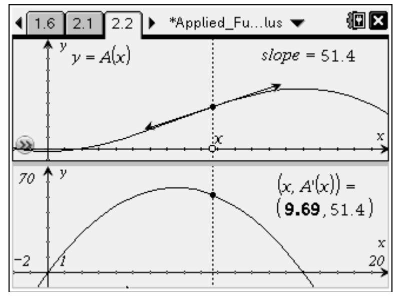

The top graph is the graph of the accumulation function, $y = A(x)$, for the function f from the previous pages, and the bottom graph shows the graph of its derivative, $y = A'(x)$.

Move point x back and forth along the graph.

3. What do you notice about the relationship between the two graphs?

Answer: The y-values of the bottom graph are the slopes of the tangent line to the matching point on the top graph. Students should recognize that this means the bottom graph, the original function, is the derivative of the top graph. This is consistent with the conclusion from Question 2. Therefore, the top graph is the antiderivative of the bottom graph.

TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 2 at the end of this lesson.

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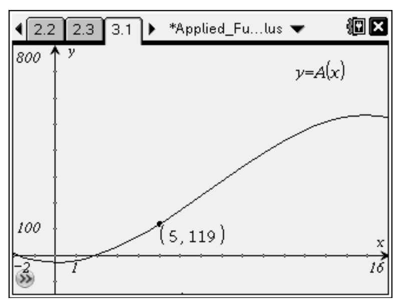

4. The velocity function graphed is $F'(x) = -x^2 + 15x$. How could you find the equation for $F(x)$ using the information from the graph?

Answer: The antiderivative of $F'(x) = -x^2 + 15x$ is $F(x) = \frac{-x^3}{3} + \frac{15x^2}{2} + C$. Given one point on the function, such as (5, 118.5), it is possible to solve for the constant, C.

$118.5 = \frac{-(5^3)}{3} + \frac{15(5^2)}{2} + C$ $\qquad 118.5 = \frac{875}{6} + C$ $\qquad C = -\frac{82}{3}$

$F(x) = \frac{-x^3}{3} + \frac{15x^2}{2} - \frac{82}{3}$

5. The accumulation function is the antiderivative of the velocity function. What does an accumulation of 477 represent in terms of the problem?

   Answer: The derivative of the distance is the velocity; therefore, the antiderivative of the velocity is the distance. An accumulation of 477 in this context means that the car has traveled a distance of 477 miles (which occurred in the interval from 2 hours to 12 hours).

6. How many miles did the driver travel between 2 and 6 hours from the start of the road trip?

   Answer: 171 miles

TI-Nspire™ Navigator™ Opportunity: ***Quick Poll***
See Note 3 at the end of this lesson.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

- The second Fundamental Theorem of Calculus.
- The relationship between velocity and distance.

TI-Nspire^{TM} Navigator^{TM} Opportunities

Note 1
Problem 1, *Quick Poll*
You may choose to use Quick Poll at this point to assess student understanding of the questions given on the student worksheet or additional questions, such as:

- *What is A*(7)?
- What is the integral formula for A(9)? $\left(\int_2^5 f(x)\,dx\right)$

Note 2
Problem 2, *Class Capture*
You may want to use Class Capture to verify students are able to move the point along the graph and use this as a point of discussion about the relationship between the two graphs.

Note 3
Problem 3, *Quick Poll*
You may choose to use Quick Poll to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask.