EXPLORATIONS

Activity 1

Integers! It All Adds Up!

Objective

- To explore addition of integers using a number line.
- To develop strategies for adding integers.

Materials

- TI-73 Calculator

Teacher Notes

Introduction

Integers (the collection of whole numbers, the opposites of the nonzero whole numbers, and 0) are quite familiar to people today, but at one time they were not accepted as true numbers in the mathematical community. Many mathematicians referred to integers as “absurd,” “ugly,” or “fictitious” numbers even though they would arise as solutions to problems. For example, an East Indian text of about 1150 A.D. indicates a solution to an equation as -5, but then makes fun of such a useless response!

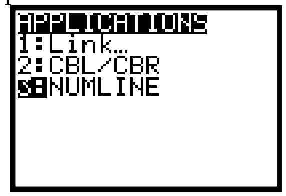
In 1545, the Italian mathematician Girolamo Cardano, published the rules governing operations with negative numbers in his text *Ars Magna* (*Great Art*).

If earlier mathematicians questioned the validity of these numbers, then it is understandable that many middle school students struggle with making sense of the operations with integers! The following activity provides students the opportunity to construct some rules for themselves based upon using the Number Line application.

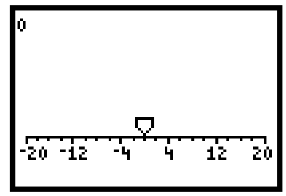
Procedure

1. Distribute the Student Activity pages. Have students work in pairs using one calculator between them. Roles that can be assigned are that of reader/recorder and Number Line "Operator."

2. Have students access the Number Line application by pressing [APPS] and selecting the NUMLINE application. (It will typically be the third option, but this depends on other applications that you have installed.)



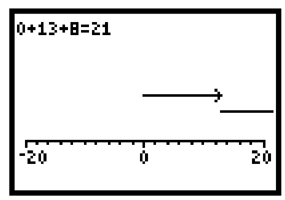
   Press [ENTER] twice to get to the NUM/FRAC LINE menu and select option **1: Number Line**.
   If the standard or default window doesn't appear with the line indicator at 0 and a view of the number line from -20 to 20, have students either press [WINDOW] and change the **Min**, **Max**, **Scale**, and **Start** values (default values are -20, 20, 2 and 0, respectively) OR press [ZOOM] and select option **2**.



   If no values are displayed on the Number Line other than **Min** and **Max,** check to be sure that **LabelOn** is selected in the WINDOW settings. If previous work is displayed, press [CLEAR] **1** to clear the window.

   Note: The first option under [ZOOM] is **ZoomFit**. This activity does not make use of this option since the intent is to have students do mental arithmetic as much as possible. Students will certainly see **ZoomFit** and test its capabilities, letting you know that option is much faster than setting the viewing window! You may wish to postpone the use of **ZoomFit** for this activity to encourage the development of mental arithmetic skills and number sense.

3. Students can initially investigate something familiar to them on the Number Line by adding positive integers or whole numbers, and then explaining how addition of two or more numbers can be represented on the number line.



   When students enter 13 + 8, the screen will show 0 + 13 + 8. The starting value will always appear and you do not need to enter + 13 + 8. As you begin to type in numbers, the application automatically assumes addition.
   If students make errors while typing in the entry, they can use the arrow keys to backspace and type over the error, or use the [DEL] key. If they have already pressed [ENTER] and find an error, they should press [ENTER] again, press [◄] or [►] to move to the error and correct it, then press [ENTER] or [GRAPH].

4. Ask students to explain how the "number rays" or vectors represent the operation of addition. Introduce students to the term ***vector*** and how it is used throughout the activity.
   The responses should have some reference to the length of the vectors being the same as the absolute value of the integer. The vectors should be pointed in the positive direction on the number line, with one vector beginning at the end of the other, and the sum being at the endpoint (arrow head) of the final vector.

5. Ask students to add 15 to the previous sum of 21 and determine an appropriate **Max** for the viewing window before they press [GRAPH]. Encourage students to share their mental arithmetic strategies. They only need an estimate to determine a new **Max**, but many will try to find the exact sum. A sample strategy might be 20 + 10 is 30 and 5 + 1 is 6 so the new sum is 36. If you view many "36's" for the maximum value as you roam the room, you know students were searching for the exact sum. You might want to talk about why the max could be 38, 39, 40, etc.

6. Students clear the window and draw a sketch of their prediction for 0 + -12. Sketches should show one vector, starting from zero and ending at -12.

Make sure that students know the difference between the [−] and [(−)] keys. The TI-73 displays the negative sign as a raised character.

7. Students add ⁻8 to ⁻12. Ask how the vectors show addition of negative integers. An explanation similar to the one for positive integers could be used, but the vectors are pointed in the negative or opposite direction.

8. Students add ⁻5 to ⁻20 and select a new **Min**. They should notice that the continued sum of negative integers will produce vectors that appear further to the left, away from zero. As long as you are adding negative integers, the vectors will not change direction. Addition of negative integers is similar to adding positive integers in that you continue moving in the same direction on the number line as you add. A difference is the direction in which you move on the line; one in a positive direction, the other in the negative direction.

You might want to talk about the idea that although the absolute value of the sum is increasing, the sum is getting smaller. A movement towards the left on the number line will produce a number smaller than the starting point (and movement towards the right on the number line will produce a number greater than the starting point). Students usually believe this idea on the positive side of the number line, but struggle with it on the negative side.

You could make "real world" comparisons

- to temperature: Which is colder, 10 degrees below zero or 20 degrees below zero?
- to money: Would you rather be $10 in debt or $50 in debt?
- to sports: Would you rather have a 5-yard loss or 15-yard loss in a football play?

Each example may help illustrate the idea that negative integers closer to zero have a greater (or more preferred) value than those that are further away, or, as you move to the left, the integer value gets smaller.

If students have had some previous experience with integer arithmetic, some might say that the vector could point in the positive direction since "a negative and a negative equals a positive." They are recalling multiplication rules without meaning. Repeatedly adding ⁻5, for example, would model *n x ⁻5*, which would be a negative value. They can use that thinking when they discuss multiplication.

9. Here you can observe from the student sketches whether students understand that addition of a negative number moves one in the negative direction. Instead of zero being the starting point, we have moved first to 15. From there, we will add ⁻9, which should move us 9 spaces to the left.

10. Encourage students not to change their sketches in step 9 if they don't exactly match what is produced by the application. Have them discuss the differences and why they are acceptable or not. The sketches provide important information for you to use in determining their understanding of the addition.

11. Students are asked if ⁻19 added to the previous sum would produce a sum that is still in the viewing window. This question focuses on number sense. Students might think "If I'm on the right side of zero, it would take adding a number less than ⁻20 to get me out of the viewing window. Negative 19 is bigger than ⁻20 so I shouldn't be out of the window range." Here is another opportunity to encourage appropriate thinking. Students might say "I need to add a number bigger than ⁻20 to move out of the viewing window" when they need to think *smaller* than ⁻20.

12. The feeling students might get when adding positive and negative integers is one of "changing directions," "going back and forth," or being "pulled in opposite directions" on the Number Line.

Remind students to use only the [◄] and [►] keys when "taking a ride." Using the [▼] and [▲] keys produces a different kind of "ride."

13. Through their own examples, students should see the commutative property at work and determine that order doesn't matter. Some are surprised when this happens. "How can you start off in a positive direction one time and a negative direction the other and still end up at the same place?" The calculator allows them to explore many examples to help see why this is the case.

14. Students may write that to find opposites, you simply "turn around and go back as far as you came" or "if you're pulled 13 away from zero, the opposite pulls you 13 spaces back."

Having them think about always returning to zero should help them understand that the sum of two opposites is zero. They can "build" several sets of opposites without having to press [CLEAR] since the starting point is always zero when adding the opposites.

15. Students may write that for the sum of positive integers, you always move in a positive direction or to the right on the number line. When adding negative integers, you always move in the negative direction or to the left. Adding positives and negatives moves you in both directions.
Some students may comment that adding positives and negatives is like subtraction. It is, but students must pay attention to the sign of the integer which is greater in absolute value in order to determine the sign of the sum. Having the visual image of the number line may help many students understand why the sum of one set of positive and negative integers can be positive while another sum can be negative.

16. Encourage students to share their strategies either orally or perhaps as a journal entry. Use writing activities with caution when sharing mental strategies. Having students write their thoughts often discourages mental arithmetic since "I have to write it down anyway!"

Student Activity

Name

Date ______________________________

Activity 1

Integers! It All Adds Up!

Objective: *In this activity, you will use the number line to investigate how to add integers and to justify and support the rules that you develop. You will use the Number Line application on the TI-73 calculator.*

Integers (the collection of whole numbers, the opposites of the nonzero whole numbers, and 0) are quite familiar to people today, but at one time they were not accepted as true numbers in the mathematical community. Many mathematicians referred to them as "absurd," "ugly," or "fictitious" numbers even though integers would arise as solutions to problems. For example, an East Indian text from about 1150 A.D. indicates a solution to an equation as -5 but then makes fun of such a useless response!

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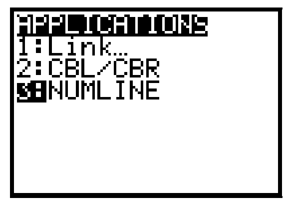
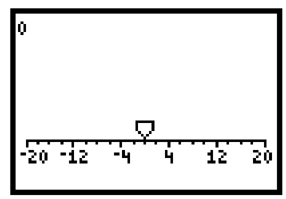


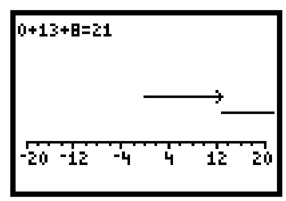
1. Access the Number Line application by pressing [APPS] and pressing [▼] to move down to the NUMLINE application. Press [ENTER] twice to get to the NUM/FRAC LINE menu and select the Number Line option.

Your window should look like the one at the right. If not, press [WINDOW] and set **Min= -20**, **Max= 20**, **Scale= 2**, and **Start= 0**. Press [GRAPH] to view the Number Line again. If the window needs to be cleared, press [CLEAR] and select **1:Yes**.

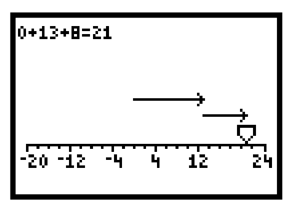


2. Find a solution to the sum 13 + 8 using the Number Line application.

3. Type in **13** [+] **8** and press [ENTER].



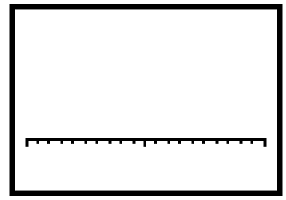
4. To capture the entire picture, press [WINDOW]. You need to select a new maximum value. There are many reasonable values that you could choose. Try 24 by pressing [▼] to move down to **Max =** and entering **24**. Press [GRAPH] to return to the Number Line. Explain how the "number rays" or vectors represent the operation of addition.



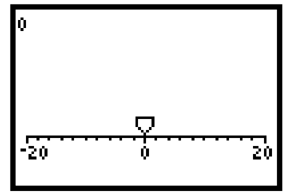
5. Suppose you add 15 to the sum from question 4. You know that the sum would appear off of the current window. Before finding the sum, press [WINDOW] and choose a new value for **Max** that will allow you to see all of the vectors. Again, there are many acceptable values, but try to select a maximum that is no more than 5 units greater than the new sum.

The value of **Max** we chose *before* viewing the sum is: ______________ .

Now, add 15 to the sum by pressing [+] **15** [ENTER].
Draw a sketch of your Number Line window on the blank screen at the right.

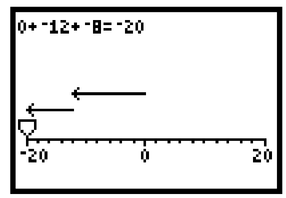


6. Press [CLEAR] to clear the window and press [WINDOW] to change the **Max** value to **20**. Press [GRAPH] to return to the Number Line window.
This time, add negative integers and see what happens. What is your prediction of how the application will show adding **0 + -12**? Draw a sketch of your prediction on the screen at the right.

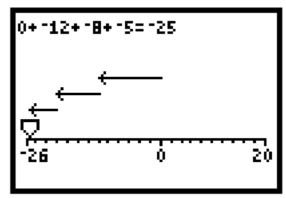


Press [+] [(-)] **12**,then [ENTER]. (The [(-)] key allows you to enter negative values or to "take the opposite of" the current entry. The [-] key is used for the operation of subtraction.) Does your previous sketch match the current Number Line view?

7. Now, add -8 to the current sum of -12. Press [+] [(-)] **8**, then [ENTER].
How do the vectors show addition of negative integers when using a number line?



8. Add -5 to the sum. By doing so, you will be out of the current viewing window. Press [WINDOW] to change the **Min** value so that you will be able to see all vectors. Since you're adding -5, a minimum of -25 would just capture all of the vectors, so -26 would allow them to be viewed more easily. Type in **-26** and press [GRAPH]. Add -5 if you haven't already done so.



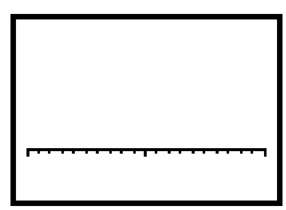
a. If you continue adding negative integers, how would the vectors would look in the viewing window?

b. Would a vector ever be pointed in a positive direction if you continued adding negative integers? Explain.

c. How is adding positive integers similar to adding negative integers? How is it different?

9. Now try adding positive and negative integers.
What do you predict the sum of 15 + -9 will be?

Draw a sketch of the Number Line representation of the sum on the screen at the right. Provide some labels on the Number Line to help with reading the vector length.

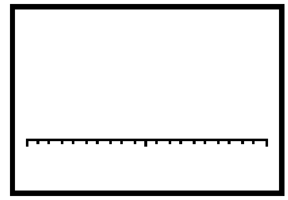


10. Press [CLEAR] to clear the window, and press [WINDOW] to reset the **Min** value to **⁻20**. Press [GRAPH] to return to the number line window. Press [+] **15** [ENTER]. The line indicator moves to 15 on the number line. Now, press [+] [(-)] **9** [ENTER]. Adding the ⁻9 produced a new vector beginning at 15, 9 units long and pointed in the negative direction. The sum is at the end of this vector.

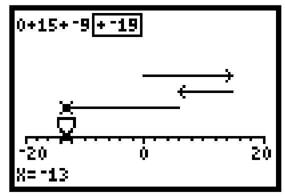
11. If you added ⁻19 to the current sum, would the new sum still be in the current viewing window or would you have to change the window settings? Explain.

In which direction will the vector for adding ⁻19 be pointed? Explain.

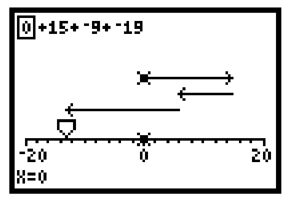
Test your prediction. Draw a sketch of the result of adding ⁻19 to the previous expression 15 + ⁻9. Notice that this is the same as adding ⁻19 to 6.



12. To help understand the movement along the Number Line, press the [TRACE] key to move along the vectors. Notice a box appears around the last addend of ⁻19 and flashing cursors appear on the end of the vector and on the number line.

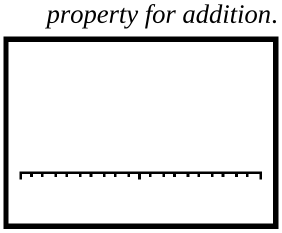
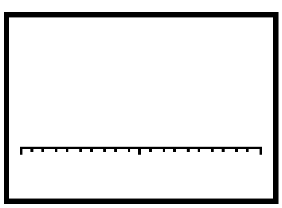
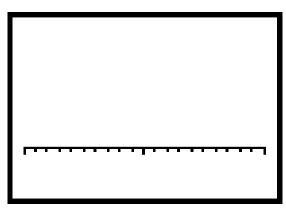
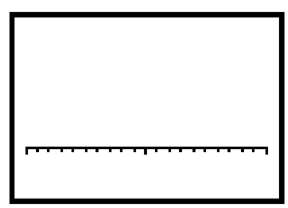
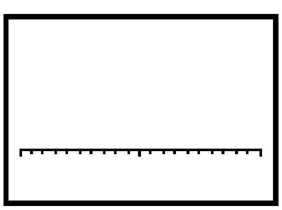
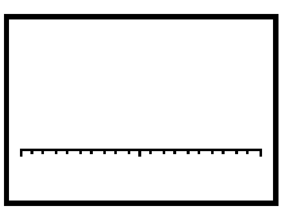
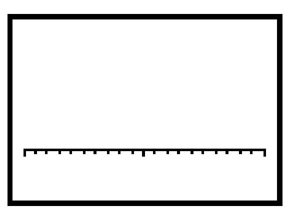
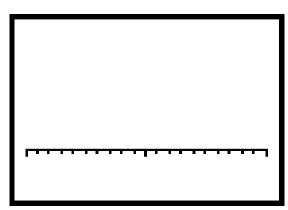


Press the [▶] and [◀] keys to "untrace the path" of the vectors until you are at the original starting point of 0. When you reach the end of a vector, continue pressing the [▶] and [◀] keys to move up (or down if you're moving forward through the sum) to the next vector in the sum. Press the [▶] and [◀] keys to "trace the path" of the sum.



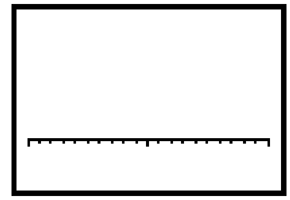
Imagine sitting on top of the cursor and taking "a ride" along the vectors (as you "move" through the expression **0 + 15 + ⁻9 + ⁻19**). Describe the "ride" of adding positive and negative integers.

13. Try a few more additions of positive and negative integers. Record your sketches of final sums in the boxes below. For one of your examples, enter in the addends in the opposite order and see what happens. For example, if you have done 15 + ⁻9; try ⁻9 +15. These examples help in understanding the *commutative property for addition*.

14. Predict what will happen when you add 13 to $^{-}13$.

a. Test your prediction and record a sketch of the window. Press the TRACE key to move along the vectors.



b. 13 and $^{-}13$ are called *opposites*. How does the Number Line application help make sense of the term *opposite*?

c. Test a few more opposites on the calculator. Do you always return to zero? (If you are trying very large numbers, remember to change the viewing window so you can see all of the vectors.)

15. Think about adding positive integers. How would you describe the general movement of the TRACE key in those sums? If you need to, enter a sum of positive integers to do the tracing.

a. Describe the movement of the TRACE key when adding only negative integers.

b. What is the biggest difference between
- adding just positive or just negative integers, and
- adding positive AND negative integers?

16. Find the following sums by using mental arithmetic and visualizing the movement on the Number Line.

a. 19 + 17
b. 13 + $^{-}27$
c. $^{-}27$ + $^{-}13$
d. 26 + $^{-}5$
e. 24 + 6
f. 15 + 15