



# MATH AND SCIENCE @ WORK

AP\* CALCULUS Educator Edition



## SPACE SHUTTLE ASCENT

### Instructional Objectives

Students will

- create and analyze scatter plots given a table;
- find regression functions using a graphing technology;
- apply the first derivative test to find local extrema;
- find inflection points to analyze the concavity of a function; and
- make connections to real world problems.

### Degree of Difficulty

This problem requires students to have a good understanding of the TI-Nspire handheld. Students will also be connecting data and making multiple conjectures relating graphs and tables to a real life situation.

- For the average AP Calculus AB student the problem may be moderate to very difficult depending on their experience with the TI-Nspire.
- For the average AP Calculus BC student the problem may be moderately difficult.

### Media Resources

This problem has two media files for use during the introduction and post discussion of the problem. These files will engage students in a space shuttle launch and introduce them to the different events that take place during the space shuttle’s ascent into space.

- Space Shuttle Launch Video (10 minutes). This video shows the launch of Space Shuttle Discovery STS 121 mission on July 4, 2006. To access the video, follow the link provided and choose the video titled: “The Rocket’s Red Glare.” This should be shown to students during the introduction of the problem and is necessary for them to answer part of the problem. It is highly recommended that the instructor become familiar with the video before showing it to students.

[http://www.nasa.gov/mission\\_pages/shuttle/shuttlemissions/sts121/launch/sts121-allvideos.html](http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts121/launch/sts121-allvideos.html)

*Appendix A provides a discussion guide that can be used with the video.*

**Grade Level**  
11-12

**Key Topic**  
Applications of Differentiation

**Degree of Difficulty**  
Calculus AB:  
Moderate to Difficult  
Calculus BC: Moderate

**Teacher Prep Time**  
10 minutes

**Problem Duration**  
60-90 minutes

**Technology**  
TI-Nspire Handheld, computer with projector and movie player

**Materials**  
Student Edition including:  
- Background handout  
- Problem worksheet  
- ShuttleAscent.tns

**NCTM Principles and Standards**  
- Algebra  
- Data Analysis and Probability  
- Problem Solving  
- Communication

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- Google Earth Space Shuttle Ascent Trajectory (5-10 minutes). The file, AscentTrajectory\_Tour.kmz, shows the ascent phase of the Space Shuttle Discovery during STS 119 in Google Earth. The file is provided for download with this problem.

*Appendix B provides instructions for using this file.*

## Background

*This problem is part of a series of problems that apply math and science principles to human space exploration at NASA.*

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries, and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the International Space Station, or ISS. However, by 2010 NASA plans to retire the space shuttle in favor of a new Crew Exploration Vehicle, or CEV. Until then, space exploration depends on the continued success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.



*Figure 1: Space Shuttle Discovery at lift-off during STS 121.*



*Figure 2: Onlookers view the launch of STS 121.*

The ascent phase begins at liftoff and ends at insertion into a circular or elliptical orbit around the Earth. To reach the minimum altitude required to orbit the Earth, the space shuttle must accelerate from zero to 8,000 meters per second (almost 18,000 miles per hour) in eight and a half minutes. It takes a very unique vehicle to accomplish this.

There are three main components of the space shuttle that enable the launch into orbit. The orbiter is the main component. It not only serves as the crew's home in space and is equipped to dock with the International Space Station, but contains maneuvering engines for finalizing orbit. The external tank, the largest component of the space shuttle, supplies the propellant to the orbiter's three main maneuvering engines. The two solid rocket boosters, the third component, provide the main thrust at launch and are attached to the sides of the external tank (Figure 1). The components of the space shuttle experience changes in position, velocity and acceleration during the ascent into space. These changes can be seen when one takes a closer look at the entire ascent process (Figure 3).



The ascent process begins with the liftoff from the launch pad. Propellant is being burned from the Solid Rocket Boosters, or SRB, and the external tank, or ET, causing the space shuttle to accelerate very quickly. This high-rate of acceleration as the space shuttle launches through the Earth's atmosphere causes a rapid increase in dynamic pressure, known as  $Q$  in aeronautics. The structure of the space shuttle can only withstand a certain level of dynamic pressure (critical  $Q$ ) before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 70%. At about one minute after launch the dynamic pressure reaches its maximum level (max  $Q$ ). The air density then drops rapidly due to the thinning atmosphere and the space shuttle can be throttled to full power without fear of structural damage.

At about 2 minutes after launch, the atmosphere is so thin that the dynamic pressure drops down to zero. The SRB, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the external tank. The jettison of the booster rockets marks the end of the first ascent stage and the beginning of the second.

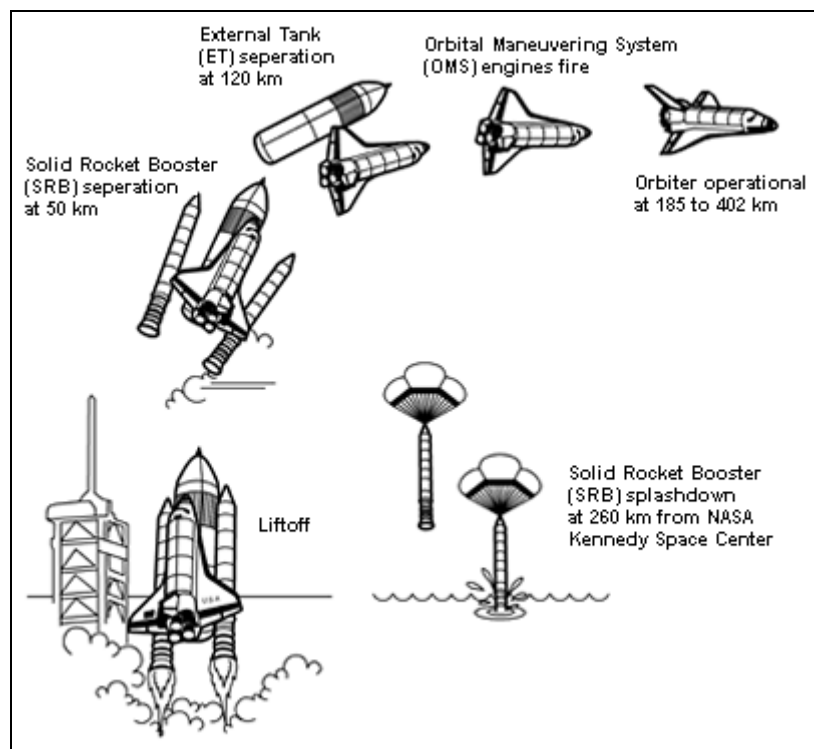


Figure 3: Space Shuttle Ascent Process

The second stage of ascent lasts about six and a half minutes. The space shuttle gains more altitude above Earth and the speed increases to the nearly 7,850 m/s (17,500 mph) required to achieve orbit. The main engines are commanded by the onboard computer to reduce power, ensuring that acceleration of the space shuttle does not exceed  $29.4 \text{ m/s}^2$  (3 g). Within thirty seconds the space shuttle reaches Main Engine Cut-Off, or MECO. For the next eleven seconds, the space shuttle coasts through space. At nine minutes, the command to jettison the nearly empty external tank is given by the onboard computer leaving the space shuttle's maneuvering engines to control any movement needed to achieve final orbit.

As we look to the future of space exploration, the ascent stage will remain a critical part of any successful mission.



## AP Course Topics

### Functions, Graphs, and Limits

- Analysis of Graphs
  - With the aid of technology, graphs of functions are often easy to produce. The emphasis is on the interplay between the geometric and analytic information and on the use of calculus both to predict and to explain the observed local and global behavior of a function

### Derivatives

- Concept of the derivative
  - Derivative presented graphically, numerically, and analytically
- Derivative as a function
  - Corresponding characteristics of graphs of  $f$  and  $f'$
  - Relationship between the increasing and decreasing behavior of  $f$  and the sign of  $f'$
- Second derivatives
  - Corresponding characteristics of the graphs of  $f$ ,  $f'$ , and  $f''$
  - Relationship between the concavity of  $f$  and the sign of  $f''$
  - Points of inflection as places where concavity changes
- Application of derivatives
  - Analysis of curves, including the notions of monotonicity and concavity
  - Interpretation of the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration

## NCTM Principles and Standards

### Algebra

- Understand patterns, relations, and functions.
- Represent and analyze mathematical situations and structures using algebraic symbols.
- Use mathematical models to represent and understand quantitative relationships.

### Data Analysis and Probability

- Select and use appropriate statistical methods to analyze data.

### Problem Solving

- Build new mathematical knowledge through problem solving.
- Monitor and reflect on the process of mathematical problem solving.

### Communication

- Organize and consolidate their mathematical thinking through communication.
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others.
- Use the language of mathematics to express mathematical ideas precisely.

## Problem

*Note: Students are given the following problem information within the TI-Nspire document, ShuttleAscent.tns. They are also provided Table 1 on the student handout that they can reference while doing the problem. Students can submit their answers within the ShuttleAscent.tns file, or they can write their answers on the student handout.*

Before each mission, projected data is compiled to assist in the launch of the space shuttle to ensure safety and success during the ascent. To complete this data, flight design specialists take into consideration a multitude of factors such as the weight of the space shuttle, propellant used, mass of



payload being carried to space, and mass of payload returning. They must also factor in atmospheric density, which is changing throughout the year. After running multiple scenarios through computer modeling, information is compiled in a table showing exactly what should happen each second of the ascent. This information helps the NASA engineers and flight controllers to quickly evaluate whether or not the space shuttle is operating as expected during the launch phase.

*Table 1: STS-121 Ascent Data (TI-Nspire page 1.7)*

<b>Time</b> (s)	<b>Altitude</b> (m)	<b>Velocity</b> (m/s)	<b>Acceleration</b> (m/s <sup>2</sup> )
0	-8	0	2.45
20	1244	139	18.62
40	5377	298	16.37
60	11,617	433	19.40
80	19,872	685	24.50
100	31,412	1026	24.01
120	44,726	1279	8.72
140	57,396	1373	9.70
160	67,893	1490	10.19
180	77,485	1634	10.68
200	85,662	1800	11.17
220	92,481	1986	11.86
240	98,004	2191	12.45
260	102,301	2417	13.23
280	105,321	2651	13.92
300	107,449	2915	14.90
320	108,619	3203	15.97
340	108,942	3516	17.15
360	108,543	3860	18.62
380	107,690	4216	20.29
400	106,539	4630	22.34
420	105,142	5092	24.89
440	103,775	5612	28.03
460	102,807	6184	29.01
480	102,552	6760	29.30
500	103,297	7327	29.01
520	105,069	7581	0.10

*Note: Notice from the table that the altitude is negative at liftoff. Zero altitude can be described as a specific distance from the center of the Earth. Since the Earth is not perfectly spherical the location of the launch just happens to be below this specified point. Also, because this is a calculated number, some degree of error may be present.*

On July 4, 2006 Space Shuttle Discovery launched from Kennedy Space Center on mission STS-121, to begin a rendezvous with the ISS. Data for mission STS-121 showing the altitude, velocity, and acceleration of the space shuttle every 20 seconds from liftoff to MECO, is displayed in Table 1.



In the following questions you will use the data from Table 1 to determine a position function, a velocity function, and an acceleration function. *Position* is defined as a location in a coordinate system. In this problem, the altitude of the space shuttle in relation to time after launch is the space shuttle's vertical position. *Velocity* is defined as the rate of change of the vertical position. It describes how fast the space shuttle is moving. It is measured in meters per second (m/s). *Acceleration* is defined as the rate of change of velocity. It describes how the velocity of the space shuttle is changing and is measured in meters/second<sup>2</sup> (m/s<sup>2</sup>).

Because of the power and speed of lift off, precision is of utmost importance. Keep this in mind as you work with the data in this problem.

Open the TI-Nspire document (ShuttleAscent.tns), read through the problem set-up, and complete the questions in the document.

- A. Answer the following based on Table 1(TI-Nspire page 1.7).
  - I. Create a scatterplot of the space shuttle's altitude in relation to time. Remember to adjust your viewing window.
  - II. Use the regression functions on your TI-Nspire handheld to determine which type of function best fits the data (i.e. linear, quadratic, cubic, quartic, exponential, power, etc.). Store the equation in  $f1(x)$ .
- B. Write the function describing the space shuttle's altitude. Let  $h$  stand for altitude and  $t$  stand for time. In writing the equation, you will need to decide what decimal place to round the coefficients to. An alternative to this is to write the coefficients using significant figures. In that case, you will need to decide how many significant figures to use. Explain your reasoning for your decision. (In your handheld, keep the original equation in  $f1(x)$  to be used for following questions.)
- C. Set  $f2(x)$ , the velocity function, to be the first derivative and apply the first derivative test to  $f2(x)$  to determine the relative extrema on the interval  $[0, 520]$ . Remember to adjust your viewing window. Explain what the extrema represent about the altitude of the space shuttle. How does this compare with the data in Table 1(TI-Nspire page 1.7)?
- D. Determine the points of inflection of the altitude function. Keep your function needed to find the points of inflection in  $f3(x)$ . Describe the concavity of the altitude function.
- E. Analyze the altitude function along the interval  $[0, 160]$ . What is happening to the space shuttle when the concavity changes?
- F. Answer the following based on Table 1(TI-Nspire page 1.7).
  - I. Make a scatterplot of the velocity versus time.
  - II. Explain what is happening to the velocity of the space shuttle over the interval  $[0, 520]$ .
  - III. Determine the regression equation for velocity. Write the velocity function letting  $v$  stand for velocity and  $t$  stand for time. Store this equation in  $f4(x)$ .
- G. Answer the following based on Table 1(TI-Nspire page 1.7).
  - I. Create a scatterplot of acceleration vs. time.

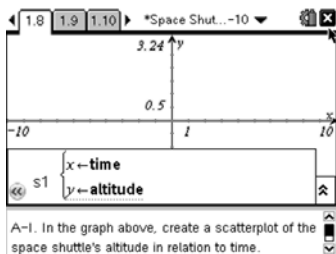


- II. Identify where the following points occur on the graph.
    - Max Q
    - Solid Rocket Booster Separation
  - III. Why does the space shuttle's acceleration increase in a quadratic fashion along the interval  $[120, 460]$ ?
  - IV. What is happening to the acceleration on the interval  $[480, 500]$  and why?
  - V. Why does the acceleration drop drastically along the interval  $[500, 520]$ ?
- H. Using the scatterplot from question G, part I, answer the following:
- I. Is there one function that best fits the acceleration data? Why or why not?
  - II. What is the acceleration function that describes the interval  $[120, 460]$ ? Let  $a$  stand for acceleration and  $t$  stand for time. Store this equation in  $f5(x)$  on your handheld. The data for time and acceleration during this interval are stored in **maintime** and **mainacceleration** in columns E and F of the Lists and Spreadsheets page.
- I. Compare the graphs in  $f2(x)$  (the first derivative of the altitude function) and in  $f4(x)$ , (the velocity function found by doing the regression of the data). Do this by hiding all other equations and plots. Do not clear the other equations. Remember to adjust your viewing window. How do these graphs compare? Explain the differences in what they are representing.
- J. Now compare the graphs in  $f3(x)$  (the second derivative of the altitude function) and in  $f5(x)$  (the acceleration function found by doing the regression of the data). How do these graphs compare? Explain the differences in what they are representing.

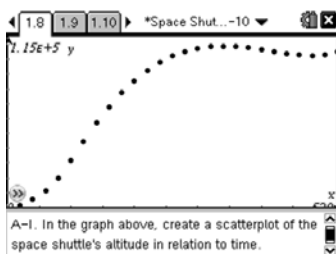
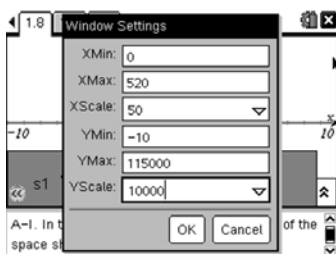
### Solution Key (One Approach)

*Note: A TI-Nspire document with solutions shown (ShuttleAscent\_Solutions.tns) is provided for download with this problem.*

- A. Answer the following based on Table 1(TI-Nspire page 1.7).
- I. Create a scatterplot of the space shuttle's altitude in relation to time. Remember to adjust your viewing window.  
**Step 1:** Change the graph type (TI-Nspire page 1.8) to a Scatter Plot.  
**Step 2:** Enter **time** for the x-list and **altitude** for the y-list.



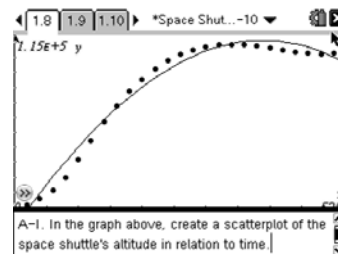
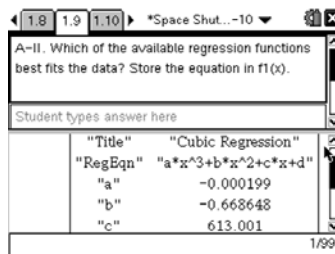
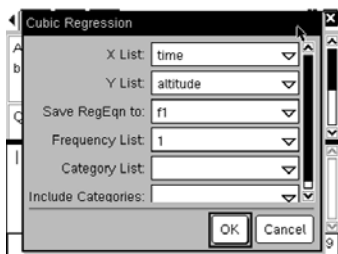
**Step 3:** Change the window settings to fit the table data and graph. An example is shown.



- II. Use the regression functions on your TI-Nspire handheld to determine which type of function best fits the data (i.e. linear, quadratic, cubic, quartic, exponential, power, etc.). Store the equation in  $f1(x)$ .

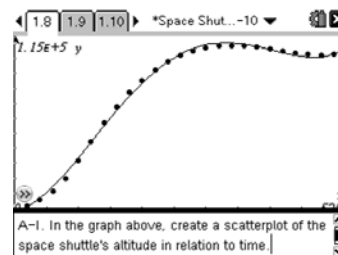
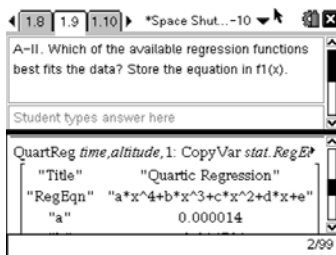
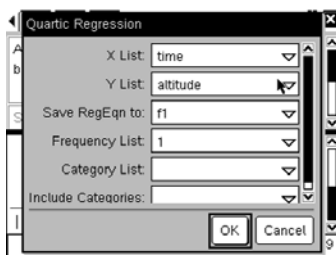
Students should see that the data is not very linear and that it is not shaped like a parabola, ruling out linear and quadratic functions. They can do the regressions for cubic and quartic then graph the equations to see which matches the graph better. When they try to do the regressions for exponential and logarithmic, they will get a domain error on their handheld ruling them out.

Cubic Regressions gives the following:



On visual inspection, students will see that the first inflection point is missing and can thus rule out cubic.

Quartic regression gives the following:







The students should see the quartic regression best matches the data.

*Note: You may want to discuss the  $R^2$  value shown in the regression and what that means about the data.*

- B. Write the function describing the space shuttle’s altitude? Let  $h$  stand for altitude and  $t$  stand for time. In writing the equation, you will need to decide what decimal place to round the coefficients to. An alternative to this is to write the coefficients using significant figures. In that case, you will need to decide how many significant figures to use. Explain your reasoning for your decision. (In your handheld, keep the original equation in  $f1(x)$  to be used for following questions.)

Rounding to three decimal places would not be appropriate in this case because that would round the coefficient of  $x^4$  to 0 making the function cubic rather than quartic. If you round the coefficients you would need to round to at least 6 decimal places in order for the function to fit the data well. Using three significant figures would also be appropriate and would still fit the data.

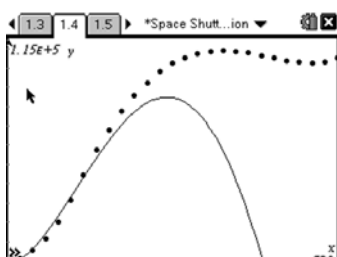
Rounding coefficients to six decimal places gives:

$$h(t) = 0.000014t^4 - 0.014791t^3 + 4.150837t^2 + 79.754386t - 2475.799324$$

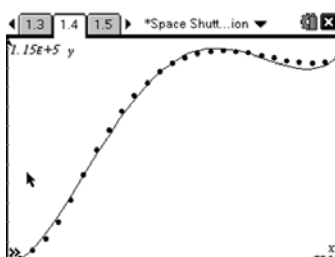
Expressing coefficients in three significant figures gives:

$$h(t) = (1.40 \times 10^{-5})t^4 - (1.48 \times 10^{-2})t^3 + 4.15t^2 + 79.8t - 2480$$

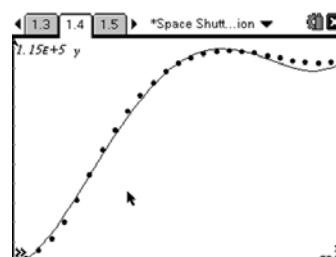
*Note: This problem would be good to discuss as a class and come to a consensus on how to represent the equations as a class. Some students will be more familiar with using significant figures than others depending on which Science classes they have taken. This would be a great time to discuss why significant figures are the rule in science rather than rounding. The following screen shots show what students will find as they explore which method to use.*



Coefficients rounded to 5 decimal places



Coefficients rounded to 6 decimal places

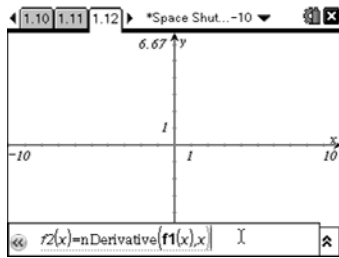


Coefficients expressed as 3 significant figures

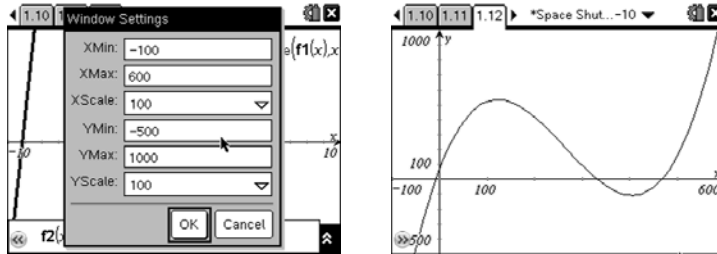
- C. Set  $f2(x)$  the velocity function, to be the first derivative and apply the first derivative test to  $f2(x)$  to determine the relative extrema on the interval  $[0, 520]$ . Remember to adjust your viewing window. Explain what the extrema represent about the altitude of the space shuttle. How does this compare with the data in Table 1 (TI-Nspire page 1.7)?

The local extrema can be found by finding the zeros of the first derivative of the function.

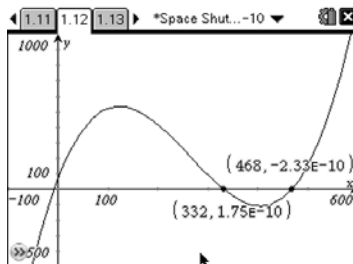
**Step 1:** Enter the derivative of  $f1(x)$  into  $f2(x)$  as shown.



**Step 2:** Change the window settings to fit the data. An example is shown.



**Step 3:** Find the zeros by using the Zero feature in the Analyze menu. You may need to drag the coordinates of the zeros in order to see them clearly.



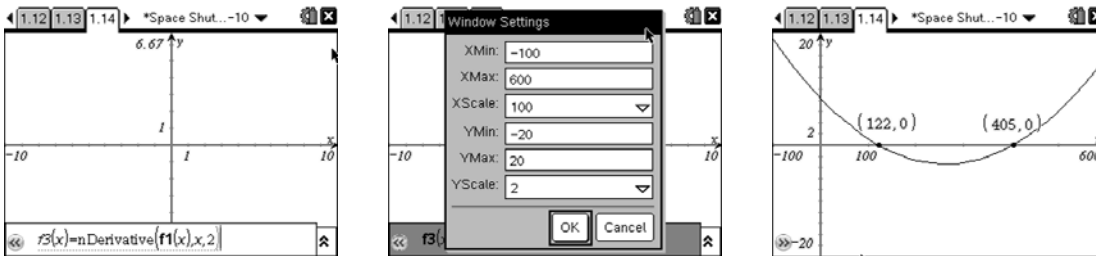
The local extrema are at  $t = 332$  seconds and  $t = 468$  seconds.

By looking at the altitude function one can see when time reaches 332 seconds we have a local maximum and when time reaches 468 seconds we have a local minimum.

On our function when time is 332 seconds the altitude is 110,718 meters. When time is 468 seconds, the altitude is 100,928 meters. Looking at the table, the highest altitude the space shuttle reaches is 108,942 meters and occurs at 340 seconds. The space shuttle then dips down a bit to an altitude of 102,552 meters at 480 seconds before again increasing altitude. So our function is pretty closely aligned with the data in the table.

- D. Determine the points of inflection of the altitude function. Keep your function needed to find the points of inflection in  $f3(x)$ . Describe the concavity of the altitude function.

Find where the second derivative of the function equals zero. To do this on the handheld enter the derivative function into  $f3(x)$ , change the window, and use the Zero feature in the Analyze menu to find the zeros.



The points of inflection can be found at  $t = 122$  seconds and  $t = 405$  seconds.

The altitude function is concave up on the interval  $[0, 122]$  concave down on the interval  $[122, 405]$  and concave up on the interval  $[405, 520]$ .

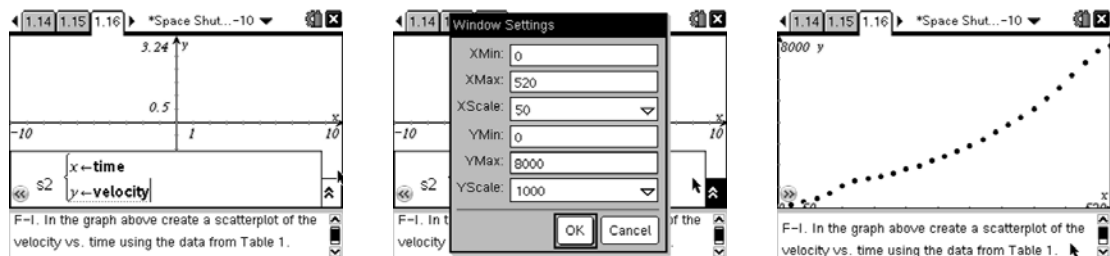
- E. Analyze the altitude function along the interval  $[0, 160]$ . What is happening to the space shuttle when the concavity changes?

At approximately 2 minutes (120 seconds) the Solid Rocket Boosters, or SRB, separate from the rocket.

- F. Answer the following based on Table 1(TI-Nspire page 1.7).

- I. Make a scatterplot of the velocity versus time.

On the handheld first change the graph type to a Scatter Plot and hide S1. Then plot S2 as shown. Adjust the window and graph.



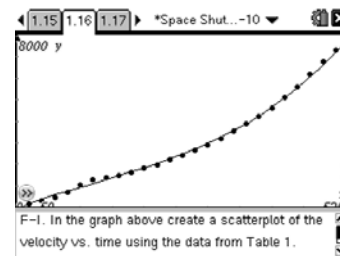
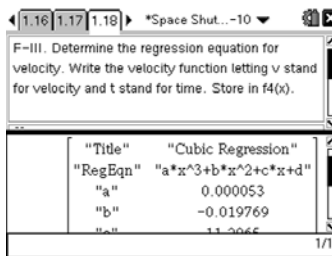
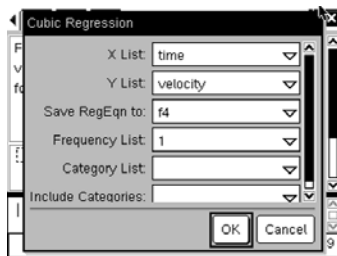
- II. Explain what is happening to the velocity of the space shuttle over the interval  $[0, 520]$ .

On the interval  $[0, 120]$  the velocity is increasing at a fluctuating rate. It continues to increase in an almost quadratic manner until it gets to 500 seconds. Then the velocity appears to stay constant.

- III. Determine the regression equation for velocity. Write the velocity function letting  $v$  stand for velocity and  $t$  stand for time. Store this equation in  $f4(x)$ .

Since we used a quartic equation for the altitude function, the velocity function should be a cubic function because it should be the derivative of the altitude function.

Calculate the cubic regression equation and place it in  $f4(x)$ . Check the graph to be sure it matches the data.



Rounding coefficients to six decimal places gives:

$$v(t) = 0.000053t^3 - 0.019769t^2 + 11.2065t - 51.5758$$

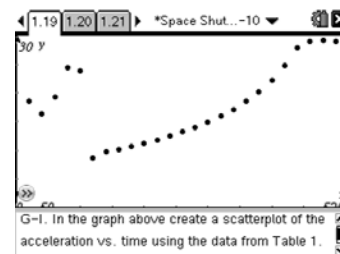
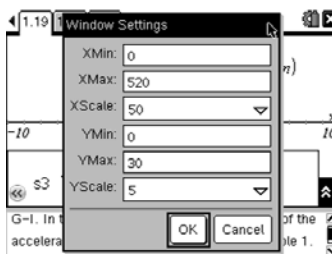
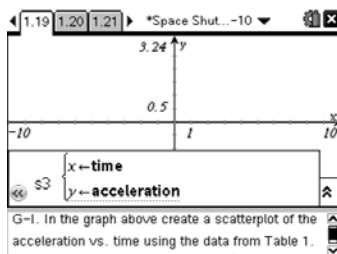
Expressing coefficients in three significant figures gives:

$$v(t) = (5.28 \times 10^{-5})t^3 - (1.98 \times 10^{-2})t^2 + 11.2t - 51.6$$

G. Answer the following based on Table 1 (TI-Nspire page 1.7).

- I. Create a scatterplot of acceleration vs. time.

On the handheld hide plot S2 and all current equations. Then set plot S3 as shown. Adjust the window and graph.



- II. Identify where the following points occur on the graph.

- Max Q is at about 40 seconds where the acceleration begins to increase again.
- Solid Rocket Booster Separation occurs at about 120 seconds. The acceleration drops and then begins increase smoothly.

- III. Why does the space shuttle's acceleration increase in a quadratic fashion along the interval [120, 460]?

Propellant in the ET is burning (reducing mass). As the mass continues to decrease the acceleration increases at a faster rate until the space shuttle reaches its maximum acceleration of 3 g at 450 seconds.

- IV. What is happening to the acceleration on the interval [480, 500] and why?

The space shuttle is at maximum acceleration (3 g) where it stays constant until it is ready for orbit.

- V. Why does the acceleration drop drastically along the interval [500, 520]?



The main engines are cut off (MECO).

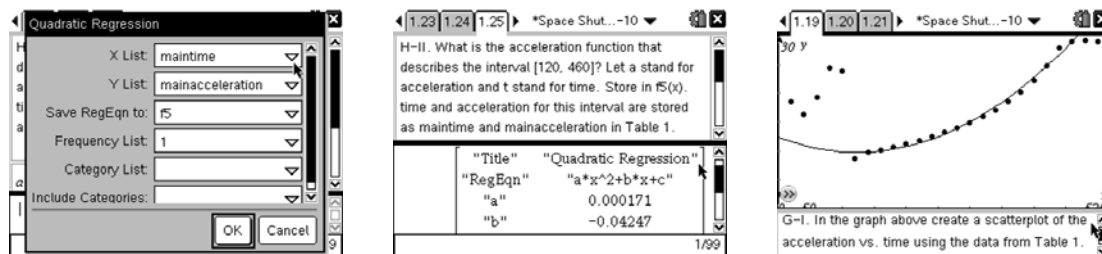
H. Using the scatterplot from question G, part I, answer the following:

I. Is there one function that best fits the acceleration data? Why or why not?

There is not one function that would fit this data. It appears to be a piecewise function. There would be a different function for each of the intervals discussed in question F.

II. What is the acceleration function that describes the interval [120, 460]? Let  $a$  stand for acceleration and  $t$  stand for time. Store this equation in  $f5(x)$  on your handheld. The data for time and acceleration during this interval are stored in **maintime** and **mainacceleration** in columns E and F of the Lists and Spreadsheets page.

To find the equation on the interval [120, 460], students should calculate the quadratic regression as shown below and then graph it on the plot made in question G to verify.



Rounding coefficients to six decimal places gives:

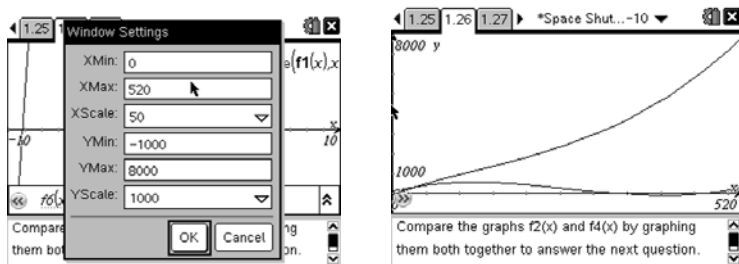
$$a(t) = 0.000171t^2 - 0.04247t + 12.3591$$

Expressing coefficients in three significant figures gives:

$$a(t) = (1.71 \times 10^{-4})t^2 - (4.25 \times 10^{-2})t + 12.4$$

I. Compare the graphs in  $f2(x)$  (the first derivative of the altitude function) and in  $f4(x)$ , (the velocity function found by doing the regression of the data). Do this by hiding all other equations and plots. Do not clear the other equations. Remember to adjust your viewing window. How do these graphs compare? Explain the differences in what they are representing.

Hide all plots and all equations but  $f2(x)$  and  $f4(x)$ , adjust window, and graph.



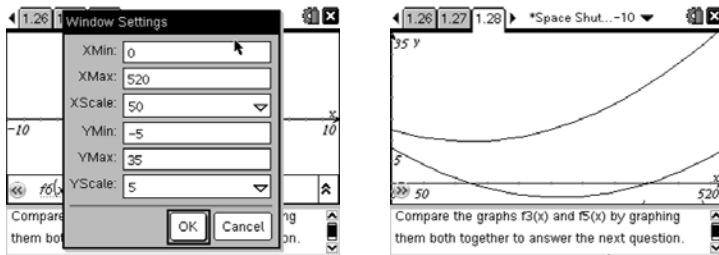
J. These graphs should both describe the velocity of the space shuttle, however the derivative of the altitude function explains only the vertical velocity of the space shuttle and the data given in



the table is the total velocity of the space shuttle. The shuttle is not moving just vertically but at an angle in order to go into orbit. Thus there is a vertical velocity and a horizontal velocity.

- K. Now compare the graphs in  $f_3(x)$  (the second derivative of the altitude function) and in  $f_5(x)$  (the acceleration function found by doing the regression of the data). How do these graphs compare? Explain the differences in what they are representing.

Hide  $f_2(x)$  and  $f_4(x)$  and turn on  $f_3(x)$  and  $f_5(x)$ , adjust window, and graph.



These graphs both describe the acceleration of the space shuttle, but again the second derivative of the altitude function describes the vertical acceleration and the data given in the table describes the total acceleration.



**Scoring Guide**

Suggested 21 points total to be given.

<b>Question</b>	<b>Distribution of points</b>
<b>A</b> <i>2 points</i>	1 point for the scatter plot 1 point for the quartic regression equation
<b>B</b> <i>2 points</i>	1 point for the equation 1 point for the explanation
<b>C</b> <i>2 points</i>	1 point for the local extrema 1 point for the explanation of what the extrema represent
<b>D</b> <i>2 points</i>	1 point for the inflection points 1 point for the description of concavity
<b>E</b> <i>1 point</i>	1 point for the conclusion of SRB separation
<b>F</b> <i>3 points</i>	1 point for the scatter plot 1 point for the explanation 1 point for the velocity function
<b>G</b> <i>6 points</i>	1 point for the scatter plot 1 point for locating maxQ 1 point for locating SRB separation 1 point for explanation of acceleration increase on [120,460] 1 point for explanation of acceleration on [480,500] 1 point for identifying main engine cut off on [500,520]
<b>H</b> <i>1 points</i>	1 point for the acceleration function
<b>I</b> <i>1 points</i>	1 point for explaining the differences in the velocity functions
<b>J</b> <i>1 points</i>	1 point for explaining the differences in the acceleration functions



## Appendix A

### Discussion Guide for STS 121 Launch Video “The Rocket’s Red Glare”

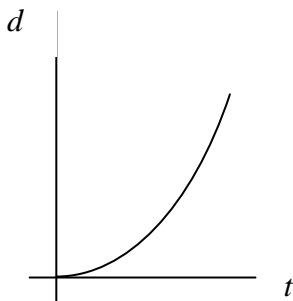
Before playing the launch video, encourage students to pay close attention to the communication that goes on between the space shuttle crew and mission control and to strive to gain a sense of what the space shuttle is physically experiencing during the climb to orbit.

After the students have watched the video discuss what they observed. The following questions and answers are just a few examples of questions that could be used in a class discussion of the video.

1. Name some of the forces that are involved in the ascent process.

*Throttle down occurs as the space shuttle approaches and passes through the sound barrier where the pressure of the atmosphere produces enormous force on the space shuttle system and the crew inside the vehicle. “Accelerating more slowly” reduces these forces. Throttle up occurs after this point of maximum dynamic pressure (Max Q) has passed and allows the space shuttle to accelerate continuing its ascent.*

2. After the space shuttle has cleared the tower the video image is from the Solid Rocket Booster (SRB). You can literally see Florida “fall down” below the rising space shuttle. Sketch a  $d$ - $t$  graph of this portion of the ascent or explain what it would look like.



*The graph is concave up, does not have a constant slope, and its origin is at the launch pad.*

3. As the Solid Rocket Boosters (SRBs) burn out and the space shuttle approaches SRB separation, what is happening to its acceleration?

*As the SRBs burn out they are no longer able to contribute significantly to accelerating the space shuttle to orbit. Therefore, the space shuttle, although still accelerating due to its own engines, slows its rate of acceleration rapidly. After SRB separation there is a good size jolt as the space shuttle is able to accelerate more quickly now that it has lost a huge amount of mass.*

4. Consider the statement “from zero to 17,500 mph in 8.5 min.” Ponder the enormity of these values in comparison to the fastest car you’ve ever heard of. What are the factors that contribute to the space shuttle achieving such a magnificently huge orbital velocity?

*Some factors include decreasing mass (propellant use and jettison of SRBs), constant thrust, and decreasing atmospheric drag.*

5. Pay special attention to the camera angle provided of the Earth as it slips past in the background. How would you describe the space shuttle’s progress (constant speed, accelerating, at rest...)?



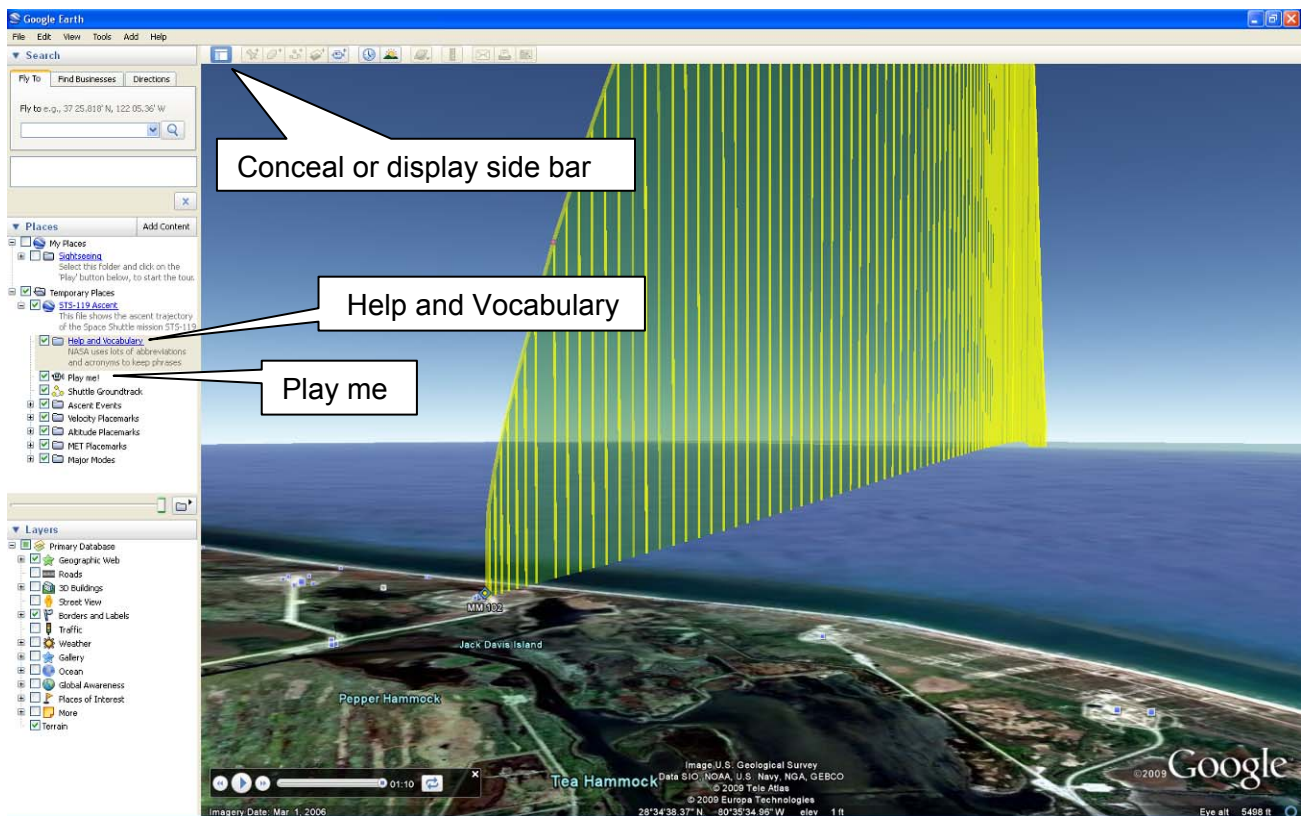


*Because the Space Shuttle's altitude is so great now it is nearly impossible to tell from the video that it is still accelerating. Recall, however, that the main engines are still running, propellant mass is decreasing quickly and there is no atmospheric drag. One can conclude that the space shuttle is accelerating.*

## Appendix B

### Instructions for Google Earth Space Shuttle Ascent Trajectory with Tour

1. Prior to presenting this file to your students, you should become familiar with its features and the features of Google Earth.
2. To access the file, you must first have Google Earth installed on the computer. To download a free version of Goggle Earth, follow the link below and choose "Download Google Earth 5". <http://earth.google.com>. The Google Earth user guide can be found under the help menu and gives a good overview of the software.
3. Download the provided file AscentTrajectory\_Tour.kmz.
4. Open the file (AscentTrajectory\_Tour.kmz) by selecting File > Open and select the file.



5. To learn more about this file, click on **Help and Vocabulary**, located on the left frame under **Temporary Places**.
6. An animated tour of the ascent trajectory is provided with this file and is 1:10 minutes long. To start the animation, double click on "**Play Me!**"



7. While watching the tour you can conceal the side bar by clicking the Conceal button located at the top left of the tool bar.
8. The tour begins with an overview of the entire space shuttle ascent from several different angles. This overview is 20 seconds long.
9. After the overview, the sequence will begin again. There are three 5 second pauses for inserting explanations of the ascent events: SRB Separation, MECO, and ET Separation. Each event is marked with a black and red circle.
10. There are additional features that you may want to explore with students at the conclusion of the tour. You may also want to use the following subtopics to unclutter the animation, (for example, uncheck the altitude and velocity folders).

To expand a folder in the left frame, click on the “+” next to the folder. To collapse the folder, click on the “-” next to it. For each subtopic there is a pop-up information box that can be opened by clicking on the subtopic. To close the pop-up information box, click on the “x” in the upper right corner of the box (or uncheck the small checked box to the left of the subtopic by clicking on it).

- Ascent Events Folder (expanded)
  - Click on an event and a pop-up information box will appear on the graph.
  - Notice the bullets for the events are the same as the markers (red and black circles) that appear on the graph for these events.
- Velocity Placemarks Folder (expanded)
  - Click on a velocity and a pop-up information box will appear on the graph.
  - Notice the bullets for the velocities are the same as the markers (green circles) that appear on the graph for these velocities.
- Altitude Placemarks Folder (expanded)
  - Click on an altitude and a pop-up will information box will appear on the graph.
  - Notice the bullets for the altitudes are the same as the markers (pink circles) that appear on the graph for these altitudes.
- MET (Mission Elapsed Time) Placemarks Folder (expanded)
  - Click on an MET and a pop-up information box will appear on the graph.
  - Notice the bullets for the METs are the same as the markers (orange circles) that appear on the graph for these METs.
- Major Modes (MM) Folder (expanded)
  - Click on a major mode and a pop-up information box will appear on the graph.
  - Notice the bullets for the major modes are the same as the markers (blue diamonds) that appear on the graph for these major modes.



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