# Project

THE MANDELBROT SET

You have seen geometric **fractals** such as the Sierpiński triangle, and you may have seen other fractals that look much more complicated. The Mandelbrot set is a famous fractal that relies on repeated calculations with complex numbers. To create the Mandelbrot set, you use the recursive formula $z_0 = 0$ and $z_n = z_{n-1}^2 + c$ where $n \geq 1$. Depending on the complex number you choose for the constant, c, one of two things will happen: either the magnitude of the values of z will get increasingly large, or they will not. (The magnitude of a complex number is defined as its distance from the origin of the complex plane, or $\sqrt{a^2 + b^2}$.) You already explored a few values of c in Exercise 15. Try a few more. Which values of c make the magnitude of z get increasingly large? Which values of c make z converge to a single value or alternate between values?
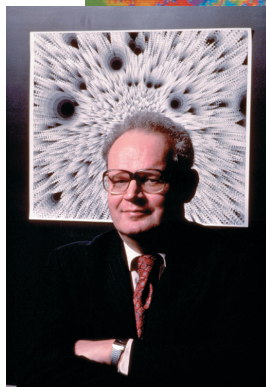
This Mandelbrot set shows how fractal geometry creates order out of what seem like irregular patterns. Points that are not in the Mandelbrot set are colored based on how quickly they diverge. Benoit Mandelbrot (b 1924), left, was the first person to study and name fractal geometry.
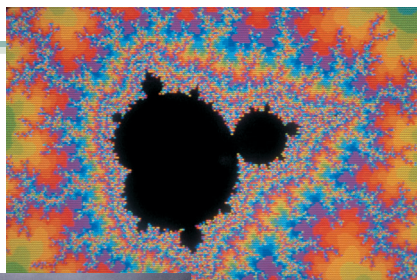
Use your calculator to determine what happens if $z_0 = 0$ and $c = 0.25$. What happens if c or z is a complex number, for example, if $z_0 = 0$ and $c = -0.4 + 0.5i$?

The Mandelbrot set is all of the values of c that do not make the magnitude of z get increasingly large. If you plot these points on a complex plane, then you'll get a pattern that looks like this one. Your project is to choose a small region on the boundary of the black area of this graph and create a graph of that smaller region. [▶️🖥️ **Calculator Note 7F** includes a program that analyzes every point in the window to determine whether it is in the Mandelbrot set. ◀] Look at this graph, select a window, and then run the program. It may take several hours.
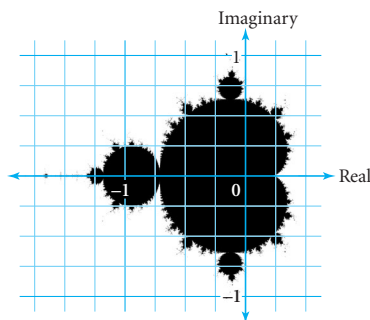
Your project should include

▶ A sketch of your graph.

▶ A report that describes any similarities between your portion of the Mandelbrot set and the complete graph shown above.

▶ Any additional research you do on the Mandelbrot set, or fractals in general.

You can learn more about the Mandelbrot set and other fractals by using the links at www.keymath.com/DAA .

Supporting the Project

The color of a point is determined by how fast the value at that point gets very large (that is, the number of iterations it takes for the value to *escape*). The black points, which don't escape but have a numerical long-term value, are in the Mandelbrot set. Note that running the calculator program may take several hours.

OUTCOMES

▶ The report includes a sketch of one graph created by the student and notes the window used. The window must be on the boundary between black and white of the original image.

▶ Similarities between the student's sketch and the original image are described in detail.

• The student "zooms in" more times and notes similarities in successive graphs.
• The report compares the Mandelbrot fractal to other fractals.
• The report includes further research on fractals.