The Hinge Theorems

ID: 8853

Time required
45 minutes

Activity Overview

In this activity, students will explore the inequality relationships that arise when some of the triangle congruence conditions are in place but others are not. The SAS Inequality Theorem and the SSS Inequality Theorem are often referred to as the Hinge Theorem and its converse. These two theorems concern inequalities involving the sides and angles of two triangles.

Topic: Triangles and Congruence

- *Use necessary and sufficient conditions for congruence to conjecture theorems about congruent triangles.*

Teacher Preparation

This activity is designed to be used in a high school or middle school geometry classroom.

- *The SAS Inequality Theorem (Hinge Theorem) states:*
  If two sides of one triangle are congruent to two sides of another triangle, and the included angle of the first triangle is larger than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.
- *The SSS Inequality Theorem (Converse of Hinge Theorem) states:*
  If two sides of one triangle are congruent to two sides of another triangle, and the third side of the first triangle is longer than the third side of the second triangle, then the included angle of the first triangle is larger than the included angle of the second triangle.
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "8853" in the keyword search box.***

Associated Materials

- *TheHingeTheorems_Student.doc*
- *TheHingeTheorems.tns*

Suggested Related Activities

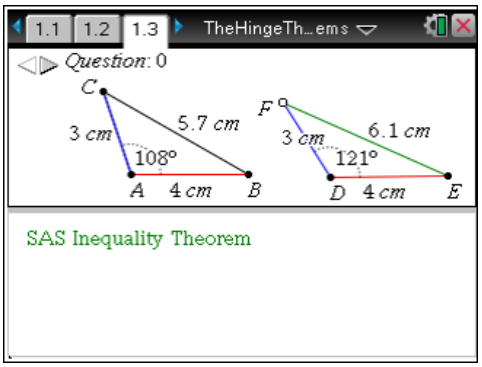
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Interior & Exterior Angles of a Triangle (TI-Nspire technology) — 8771*

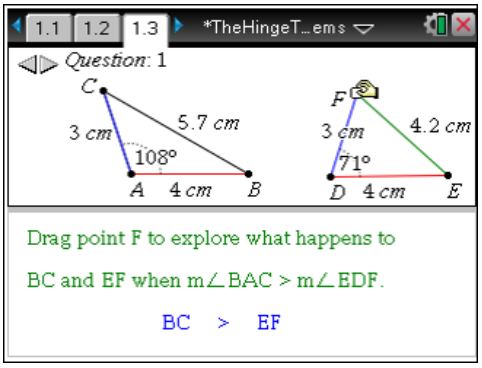
Problem 1 – SAS Inequality Theorem

Have students open the file and read the directions on page 1.2.

On page 1.3, students will work through the SAS inequality theorem by answering several questions about *BC* and *EF* by changing the relationship between ***m∠EDF*** and ***m∠BAC***. Students will click through the question slider to see new questions and answer them on their worksheet.



Students will verify that the property stated in the SAS Inequality Theorem holds true for this example. They should drag point *F* so that *m∠EDF* increases but remains less than *m∠BAC*. Then they can drag point *F* so that *m∠EDF* is greater than m∠*BAC*. This enables them to explore changes in the lengths of $\overline{BC}$ and $\overline{EF}$ as ∠*EDF* changes in size.



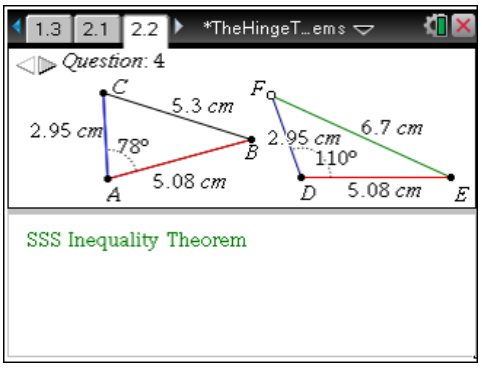
TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

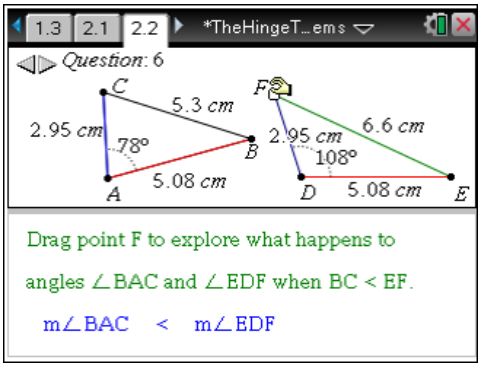
Problem 2 – SSS Inequality Theorem

Students should advance to page 2.1 and read the directions.

On page 2.2, students will work through the SSS inequality theorem by answering several questions about ***∠BAC*** and ***∠EDF*** by changing the relationship between ***EF*** and ***BC***. Students will click through the question slider to see new questions and answer them on their worksheet.



Students should drag point *F* so that the length of $\overline{EF}$ is less than the length of $\overline{BC}$. Then they can make $\overline{EF}$ longer than $\overline{BC}$. Lastly, students explore when the lengths are the same. This enables them to explore changes in the size of ∠*BAC* and ∠*EDF* as $\overline{EF}$ changes in length.



Sample Student Solutions:

1. What is the relationship between the two lengths of $\overline{BC}$ and $\overline{EF}$ when $m\angle BAC > m\angle EDF$? ***BC > EF***
2. What is the relationship between the two lengths of $\overline{BC}$ and $\overline{EF}$ when $m\angle BAC = m\angle EDF$? ***BC = EF***
3. What is the relationship between the two lengths of $\overline{BC}$ and $\overline{EF}$ when $m\angle BAC < m\angle EDF$? ***BC < EF***
4. Does the SAS Inequality Theorem hold true for $\triangle ABC$ and $\triangle DEF$? **Yes**
5. What is the relationship between the measurements of $\angle EDF$ and $\angle BAC$ when $BC > EF$? ***m∠BAC > m∠EDF***
6. What is the relationship between the measurements of $\angle EDF$ and $\angle BAC$ when $BC < EF$? ***m∠BAC < m∠EDF***
7. What is the relationship between the measurements of $\angle EDF$ and $\angle BAC$ when $BC = EF$? ***m∠BAC = m∠EDF***
8. Does the SSS Inequality Theorem hold true for $\triangle ABC$ and $\triangle DEF$? **Yes**

TI-Nspire Navigator Opportunities

Note 1
Problem 1, *Screen Capture*
This would be a good place to do a screen capture to verify students are working through the questions and are able to grab and move point F to investigate all cases.