t Distributions

ID: 9618

Time required
40 minutes

Activity Overview

Students first compare the t-distribution to the standard normal distribution to see that while the basic shape is the same, the t distribution has greater variability and more area in the tails than near the mean. Students then use the ***invt*** *command to find critical values for a t-distribution and again note that as n increases, the critical values become closer to those used for a normal distribution. They construct confidence intervals and finally, they use the* ***tinterval*** *command to find the confidence intervals.*

Topic: Continuous Distributions and their Properties

- *Graph the probability density function*
- *Find probability using t Distribution.*
- *Calculate a confidence interval*

Teacher Preparation and Notes

- *This activity is designed to be used for students studying Statistics and Probability.*
- *Students should have been introduced to the t distribution. They should already be familiar with the normal distribution and its characteristics, as well as finding and interpreting confidence intervals for normal distributions (with known population standard deviation or sample size larger than 30).*
- *This activity is intended to be mainly* ***teacher-led****, with breaks for individual student work. Use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9618" in the keyword search box.***

Associated Materials

- *t_Distribution_Student.doc*
- *t_Distribution.tns*
- *t_Distribution_Soln.tns*

Suggested Related Activities

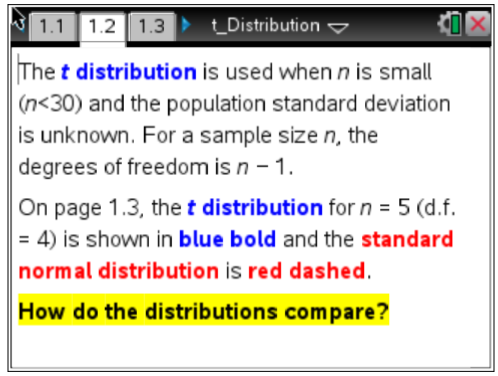
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Special Cases of Normally Distributed Data (TI-89 Titanium) — 4363*
- *One and Two Sample t-Distributions (TI-84 Plus and TI-Navigator) — 1967*
- *F Distribution (TI-Nspire technology) — 9781*
- *Chi-Square Distributions (TI-Nspire technology) — 9738*

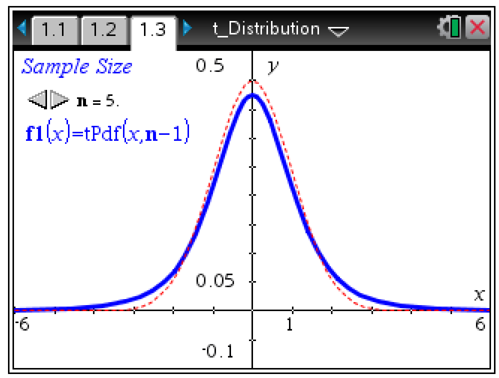
Problem 1 – Characteristics of the *t* Distribution

Begin with a discussion of when the *t* distribution is used (when *n* is small (less than 30) and the population standard deviation is unknown).

Explain to students that, in this activity, **d.f.** stands for degrees of freedom, and is one less than the sample size.

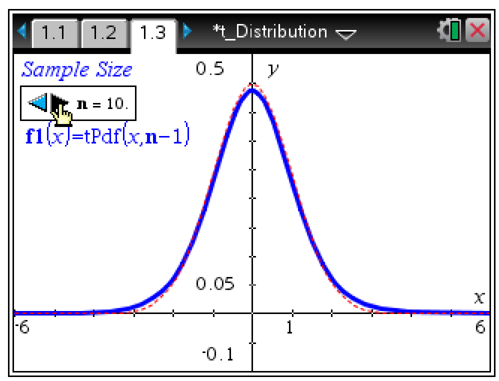


On page 1.3 students will see the standard normal distribution (red-dashed) along with the *t* distribution for a sample size of 5, which has 4 degrees of freedom (blue-bold). They should discuss how the distributions compare.



Both are bell-shaped with a mean of 0, but the *t* distribution has more area in the tails. The standard deviation in the *t* distribution must be greater than one.

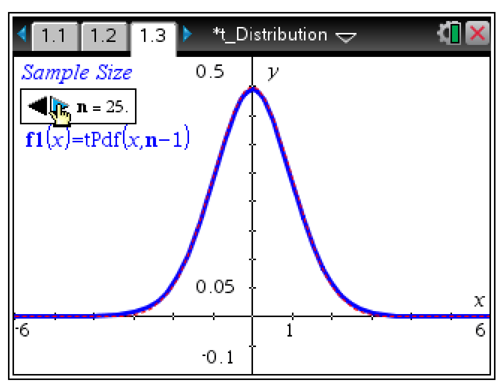
Students can change the sample size to 10 by clicking the arrows for *n*. When they click the arrow, the *n*-value will change and the graph will be updated.



Students are to repeat for sample sizes of 15, 20 and 25 (d.f. = 14, 19 and 24). Ask what happens to the graph and why they think this happens.

As *n* increases, the distribution more closely follows the normal distribution. Smaller samples have more variability than larger samples; when $n \geq 30$, the normal distribution can be used instead of the *t* distribution.

t distributions have more area, and therefore more probability in the tails, which will affect confidence intervals, as will be seen.



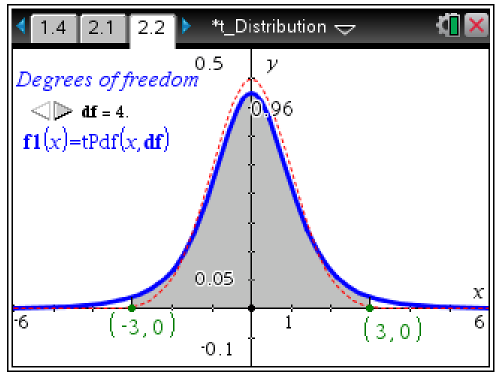
Problem 2 – Comparing Areas

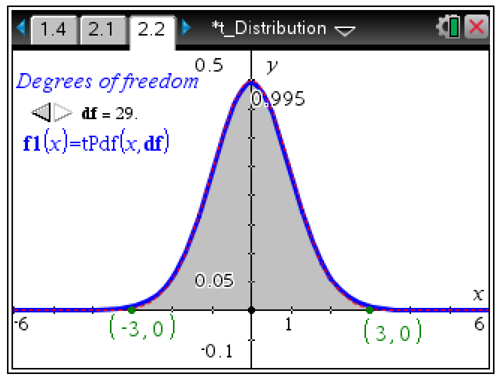
On page 2.2, students will see again the standard normal distribution and the *t* distribution for *n* = 4. Two points are marked on the *x*-axis. They are to use the **Integral** tool from the Measurement menu to first find the area between these points under the normal distribution and then to find the area between these points under the *t* distribution.

To use the **Integral** tool, students should click on the desired curve, then the left boundary point on the *x*-axis and then the right boundary point.

Students will notice that the area between those points is greater for the normal distribution than for the *t* distribution. For example, the area of the normal curve for degrees of freedom 14 is 0.985 while the area for the t distribution is 0.96.

Then they should change the degrees of freedom to 9, 14, 19, 24, and 29 to see the areas approach that for the normal distribution.





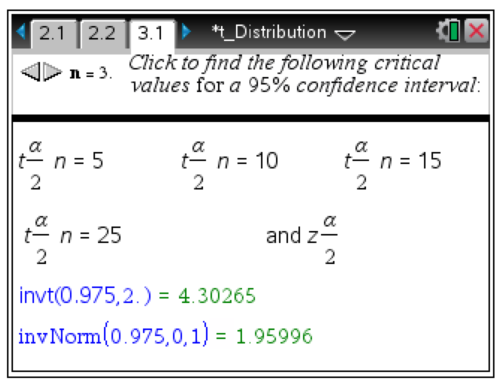
Problem 3 – Critical Values for a *t* Distribution

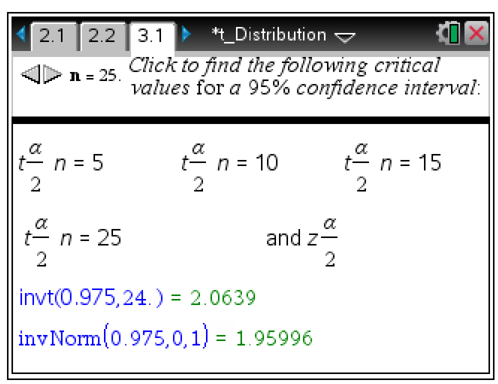
Students will now find the five critical values listed on page 3.1, each at the 95% level.

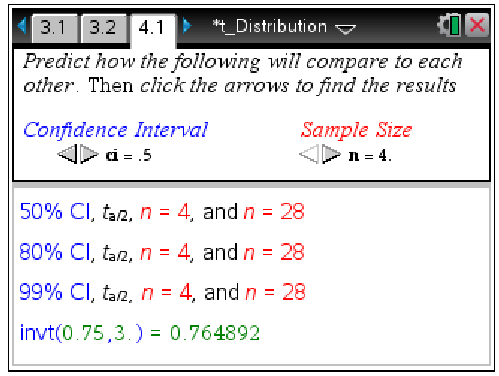
Students must remember that for a 95% confidence interval, 5% is left in *both* tails, so 2.5% is left in *each* tail. For the positive critical value (2.5% to the right), they are looking for the value where 97.5% of the area is to the left.

Students can click on the arrows by the *n*-variable to change its value. The calculated values at the bottom of the page will update with each click. The **invt** command will require the percent and the degrees of freedom (*n* – 1). The **invNorm** command will require the percent, mean, and standard deviation.

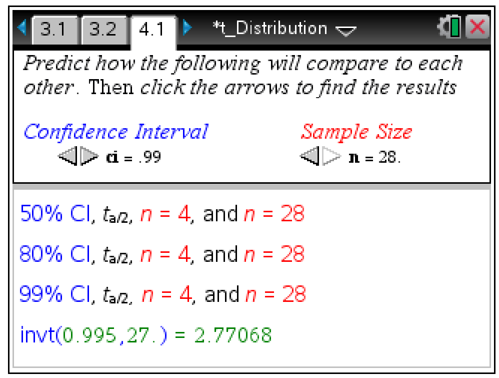
Students can also check their answers against a *t* distribution chart. Students should notice that as *n* increases, the critical values approach the *z* critical value of about 1.96.







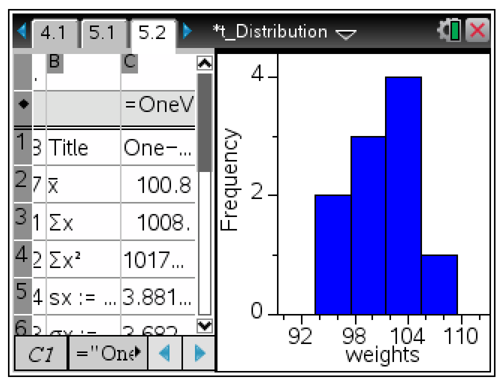
On page 4.1, students should predict how the critical values listed will compare. They should give their reasoning for their predictions.



Students should be able to make statements such as, *For n = 4, the critical values for an 80% confidence interval will be greater than for a 50% confidence interval because the distance (number of standard deviations) away from the mean increases as the confidence increases*. *For each percent, the critical value will be greater for n = 4 than for n = 28, because smaller samples have more possible error.*

After discussing what should happen generally, students will find the actual values on the next page.

Problem 4 – Constructing a Confidence Interval

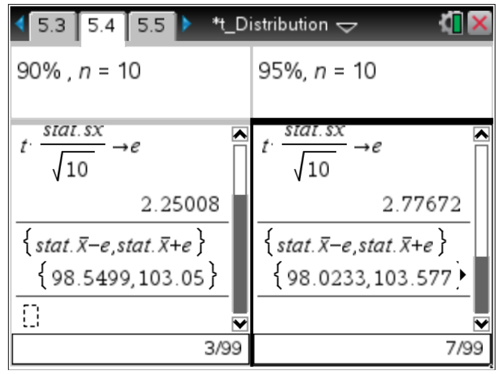


Students will find the mean and standard deviation of the weights listed on page 5.2 by pressing [menu] **> Statistics > Stat Calculations > One-Variable Statistics**. Press **OK** for one list, select ***weights*** for the x1 List, and **b[ ]** for the results. The mean is 100.8 and the standard deviation is about 3.881158.

To verify that the distribution is roughly normal, students should create a histogram of the weights. Have them choose **weights** for the *x*-axis variable and change the Plot Type to a histogram. Students now need to change the bar (bin) width from 1 to 4 by **clicking** on all the bars of the histogram, pressing [ctrl] [menu] and selecting **Bin Settings > Equal Bin Width** and entering 4.

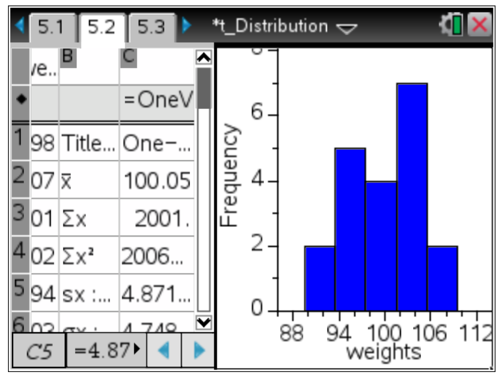
Now to readjust the vertical scale, have the students **click** on the *x*-label , ***weights*** and press [enter].

On page 5.4, students will construct 90% and 95% confidence intervals for the population mean by first finding the critical value, then the margin of error, and then subtracting and adding the error from and to the mean.



Students should use the confidence intervals in sentences, such as, *Based on these 10 data values, we are 95% confident that the true mean weight is between 98.0233 grams and 103.577 grams.*

Page 5.5 lists 10 more weights students need to add to those already in the spreadsheet on page 5.2.



The mean, standard deviation, and histogram will all automatically update (to rescale the vertical axis, just have the students click on the weights label again and press enter.

Students are to find the new confidence intervals on page 5.6. They should notice the smaller margins of error.

Lastly, students can find the confidence interval by pressing menu **> Statistics > Confidence Intervals > t interval**. (It is also in the catalog.) They should select **Data** for the Input Method, **weights** for List, and type in the desired C Level (0.90 or 0.95). The interval is (CLower, CUpper).