Vectors Introduction

Student

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TI-Nspire | Investigation | Student | 30 min

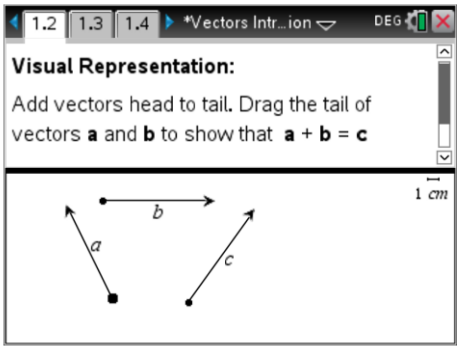
Introduction

Vectors are a powerful tool for solving a range of geometric, mechanics and kinematics based problems. Once the basic operational procedures and concepts have been understood, vectors can make difficult problems much easier to solve. Imagine trying to determine an equation for the distance between two objects moving independently in three dimensional space; it is very easy with vectors. This activity introduces vector representation physically and numerically and provides corresponding examples and problems for vector addition and subtraction.

Geometric Representation of a Vector

Open the TI-*n*spire document: "Vectors 1" and navigate to page 1.2. To move a vector, use the touchpad to place the mouse over the tail of the vector where it will automatically change to an open hand. To grab the vector click and hold or press Ctrl and then click.

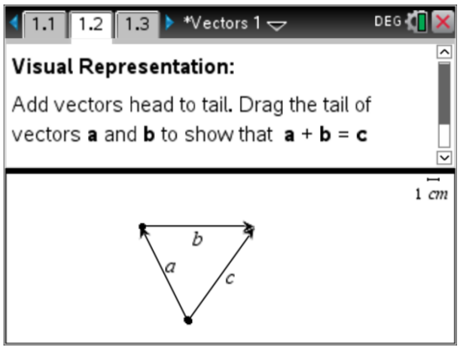
Move the vectors to show that $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$



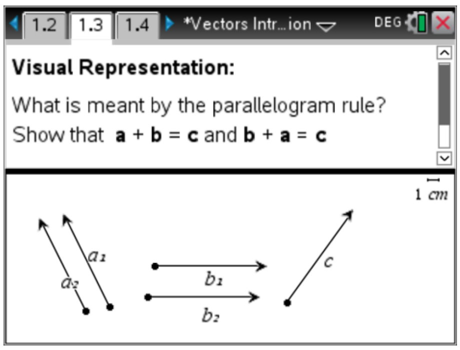
Question: 1.

Draw a diagram to show that: $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$, does it matter where on the page the vector diagram is placed?

Students should show vectors *a* and *b* added 'head to tail' and vector c as the resultant.



Navigate to page 1.3 where a copy of the original diagram is located with two sets of vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$. Arrange the vectors to show that $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$ and $\underset{\sim}{b}+\underset{\sim}{a}=\underset{\sim}{c}$.



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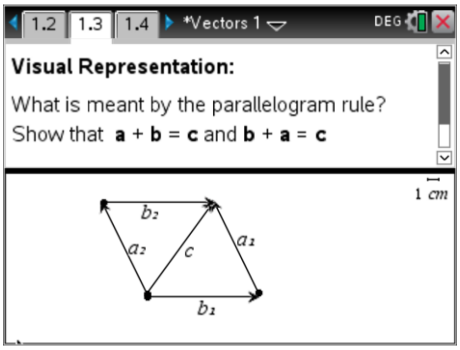
Question: 2.

In reference to $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$ and $\underset{\sim}{b}+\underset{\sim}{a}=\underset{\sim}{c}$ explain what is meant by the ‘parallelogram rule’.

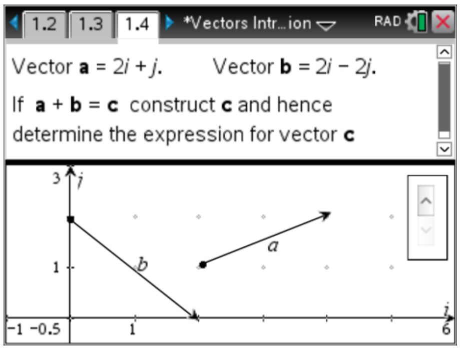
The parallelogram rule shows that $\underset{\sim}{b}+\underset{\sim}{a}=\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{c}$ by producing a parallelogram. As shown opposite b_1 has the same magnitude and direction (parallel) as b_2, similarly with a_1 and a_2.



Rectangular Representation of a Vector (2 Dimensions)

Navigate to page 1.4 where vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ have been represented using two perpendicular axes: $\underset{\sim}{i}, \underset{\sim}{j}$.

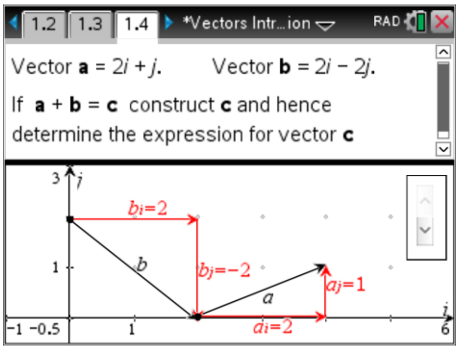
The $\underset{\sim}{i}$ and $\underset{\sim}{j}$ components of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ can be displayed using the slider on the far right of the screen.



Question: 3.

Add vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$, hence determine the values of x and y where: $\underset{\sim}{a}+\underset{\sim}{b}=x\underset{\sim}{i}+y\underset{\sim}{j}$.

The slider can be used to highlight the vector components such that $b_i = 2$ and $a_i = 2$ therefore $x = 4$. Similarly the j components: -2 + 1 = -1, therefore $y = -1$



Vectors as Matrices

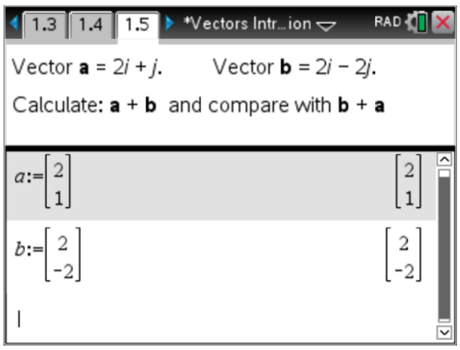
Navigate to page 1.5 where vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ have been defined using column matrices.

The vectors can easily be converted to row matrices starting with $\underset{\sim}{a}$ press:

[menu] > Matrix & Vector > Transpose

Vector $\underset{\sim}{a}$ is converted into a row vector.

The transpose operation also works in ‘reverse’ converting a row vector into a column vector.



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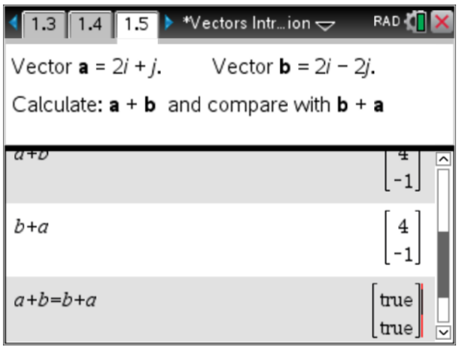
It is useful to be able to work with vectors in row or column form. Column form is generally found in most mathematics texts. Computer programs often use the row format because they have traditionally been easier to represent, similarly for word processing and websites. For example: [2, 3, 4] can be written without special mathematical tools or software.

Question: 4.

Verify that $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{b}+\underset{\sim}{a}$ and use the result to check the answer to the previous question.

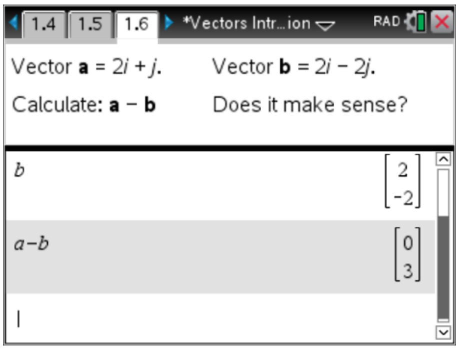
The vectors (matrices) can be recalled directly from the VAR menu on the calculator or using the keypad. Students can confirm that $\underset{\sim}{a}+\underset{\sim}{b}=\underset{\sim}{b}+\underset{\sim}{a}$ is true by checking each individually or by typing it as an equation.



Question: 5.

Navigate to page 1.6 and determine the result for: $\underset{\sim}{a}-\underset{\sim}{b}$. Draw an accurate vector diagram that supports this result.

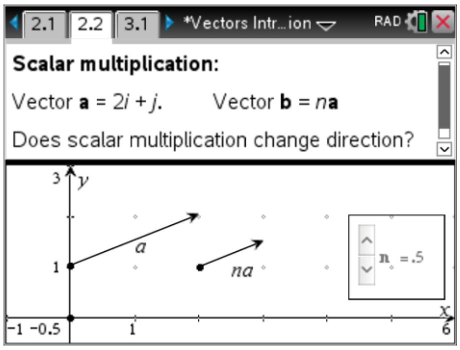
The numerical result should be self-explanatory; however students should be encouraged to draw a diagram of the same.



Scalar Multiplication

Navigate to page 2.2 where two vectors $\underset{\sim}{a}$ and $\underset{\sim}{b}$ have been defined. Vector $\underset{\sim}{b}$ however is defined as: $\underset{\sim}{b}=n\underset{\sim}{a}$ where n is a scalar quantity, a numerical value.

Click on the slider to change the value of n and see how it changes vector $\underset{\sim}{b}$.



Question: 6.

Change the value of n using the slider. What changes does this make to vector $\underset{\sim}{b}$ in terms of direction and size?

Direction does not change, only the 'size' (magnitude) of the vector changes.

Question: 7.

Based on your response to Question 6, what would be required to prove that two vectors are parallel?

The vector b is a scalar multiple of vector a.

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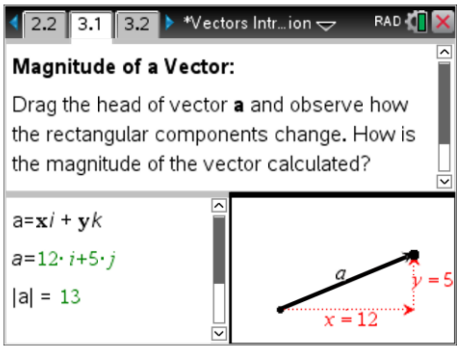
Magnitude of a Vector

Navigate to page 3.1 where vector $\underset{\sim}{a}$ is dynamic.

Manipulate the head of the vector and observe how the symbolic representation of the vector changes. Study the geometric representation carefully and consider the following: "How is the magnitude of a vector calculated?"



| a | = Magnitude of vector a.

Question: 8.

The calculator application on page 3.2 contains four vectors that have been defined (matrix format):

$\underset{\sim}{b} = 5\underset{\sim}{i} + 12\underset{\sim}{j}$ $\quad \underset{\sim}{c} = 8\underset{\sim}{i} + 15\underset{\sim}{j}$ $\quad \underset{\sim}{d} = \underset{\sim}{i} + \underset{\sim}{j}$ $\quad \underset{\sim}{e} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

Use: **norm(vector)** command to determine the magnitude of each of the vectors: $\underset{\sim}{b}$, $\underset{\sim}{c}$, $\underset{\sim}{d}$ and $\underset{\sim}{e}$. The norm command can be accessed via: **[Menu]** > **Matrix & Vector** > **Norms** > **Norm**

$\underset{\sim}{b} = 5\underset{\sim}{i} + 12\underset{\sim}{j}$ $\quad \underset{\sim}{c} = 8\underset{\sim}{i} + 15\underset{\sim}{j}$ $\quad \underset{\sim}{d} = \underset{\sim}{i} + \underset{\sim}{j}$ $\quad \underset{\sim}{e} = 3\underset{\sim}{i} + 4\underset{\sim}{j}$

$|\underset{\sim}{b}| = \sqrt{5^2 + 12^2}$ $\quad |\underset{\sim}{c}| = \sqrt{8^2 + 15^2}$ $\quad |\underset{\sim}{d}| = \sqrt{1^2 + 1^2}$ $\quad |\underset{\sim}{e}| = \sqrt{3^2 + 4^2}$

$|\underset{\sim}{b}| = 13$ $\quad |\underset{\sim}{c}| = 17$ $\quad |\underset{\sim}{d}| = \sqrt{2}$ $\quad |\underset{\sim}{e}| = 5$

Question: 9.

Suppose $\underset{\sim}{v} = n\underset{\sim}{b}$ and $|\underset{\sim}{v}| = 1$ determine the value of *n* where *n* is a scalar quantity.

The purpose of this question is to introduce the notion of a unit vector without explicitly stating 'unit vector'.

$$|n\underset{\sim}{b}| = 1$$

$$n\underset{\sim}{b} = 5n\underset{\sim}{i} + 12n\underset{\sim}{j}$$

$$|n\underset{\sim}{b}| = \sqrt{5^2 n^2 + 12^2 n^2}$$

$$|n\underset{\sim}{b}| = |n|\sqrt{5^2 + 12^2}$$

$$\frac{1}{13} = n$$

In the case of a unit vector the positive value of *n* is used so the direction of the vector is not changed, in this question however students could use +/- *n* .

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