Exponentially Fast Derivative

ID: 11608

Time Required
40–45 minutes

Activity Overview

In this activity, students will investigate the derivative of exponential functions. For e^x, students will explore the slope of the tangent graphically and use CAS functionality to discover the chain rule for exponential functions. Similarly, a^x is explored graphically and CAS functionality is used to discover the rule.

Topic: Exponential Differentiation

- *Derivative of e^x and the chain rule.*
- *Derivative of $b \cdot a^{u(x)}$*

Teacher Preparation and Notes

- *Students will need to know that the derivative of position is velocity, and the derivative of velocity is acceleration. For questions at the end, students are expected to know the derivative of trig functions and the product rule.*
- *The activity is designed to be a student-centered discovery and instruction of differential calculus for exponential functions. Teaching the basics before beginning the activity is optional. After completing the activity, students should be more successful with AP* questions like 2003formBAB4, 2002formB AB2&BC2, 1991BC2, and multiple choice questions 1998AB16, and 2003AB9&17.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "11608" in the quick search box.***

Associated Materials

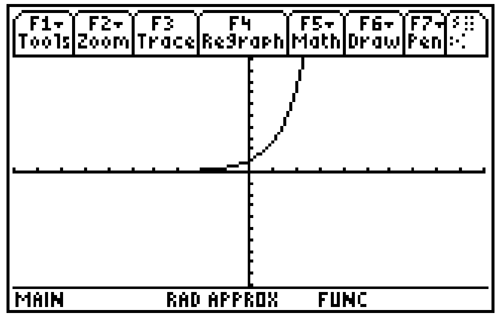
- *CalcWeek06_ExpFast_Worksheet_TI89.doc*

Suggested Related Activities

- *Exponential Differentiation (TI-Nspire technology) — 8980*
- *The Exponential Derivative (TI-89 Titanium) — 8979*
- *Exponent Rule (TI-84 Plus) — 9730*
- *Derivative Trace (TI-Nspire technology) — 8110*
- *Derivatives of Logarithmic Functions (TI-Nspire CAS technology) — 9093*

Part 1 – Warm up

Students are to graph exponential functions and consider the range. This knowledge is applicable to many exam questions involving e^x. Next, students are asked question about basic knowledge of the domain and value of the natural logarithm. Answers are graphically reinforced. These facts are important in the discovery and derivation of the derivative of a^x.



<u>*Further Discussion*</u>

- *If needed, review some logarithm properties. For example, ask students for an expression equivalent to $\ln(e^2)$. Recognizing e^x and $\ln(x)$ are inverses can also be drawn out by asking what $e^{\ln(2)}$ is.*

Student Solutions

1. $f(x) = e^x$ is always positive and so is $f(x) = e^{-x}$

2. The domain of $y = \ln(x)$ is $x > 0$

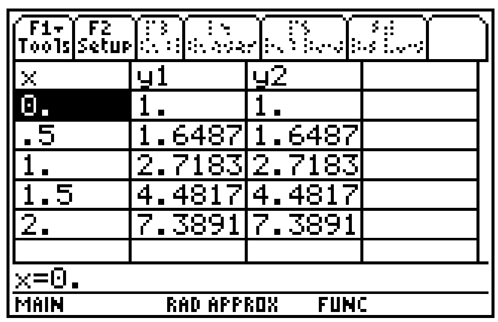
3. a. $x > 1$

b. $1 < x < e$

c. $x > e$

Part 2 – Discover the derivative of e^x

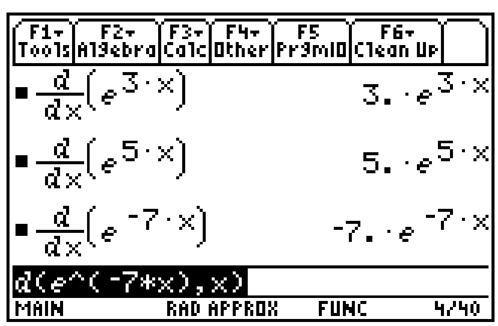
Students begin the part of the activity by comparing the values of the function e^x to the values of its derivative using a table. They should realize that the derivative of e^x is unlike the derivative of a polynomial or a power function like x^n.



<u>*Discussion Question*</u>

- *What is the derivative of a polynomial of degree n? How does that compare with the derivative of e^x?*

Students use CAS functionality to do a series of problems to observe the pattern of the derivative of $e^{u(x)}$ with respect to x. Students will then apply this rule to other problems. The valuable exercise of reconciling their solution with the answer that their graphing calculator gives is particularly helpful for multiple-choice exam questions.



Student Solutions

4. $\frac{d}{dx}\left(e^{x}\right)=e^{x}$

5. $\frac{d}{dx}\left(e^{a\cdot x}\right)=a\cdot e^{a\cdot x}$

6. $\frac{d}{dx}\left(e^{bx^{n}}\right)=bnx^{n-1}e^{bx^{n}}$

7. a. Since $x^{0.5}=\sqrt{x}$, $\frac{d}{dx}\left(e^{4x^{0.5}}\right)=4(0.5)x^{-0.5}e^{4x^{0.5}}=\frac{2e^{4\sqrt{x}}}{\sqrt{x}}$

b. Since $e^{a}e^{b}=e^{a+b}$, $\frac{d}{dx}\left(e^{e^{3x}}\right)=e^{e^{3x}}3e^{3x}=3e^{e^{3x}+3x}$

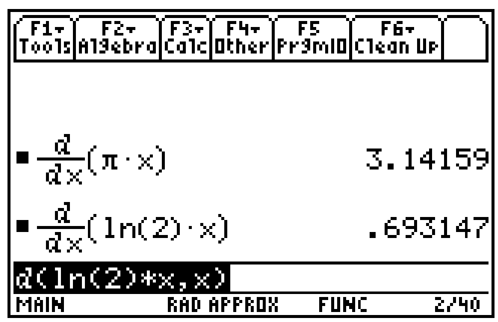
c. $\frac{d}{dx}\left(\sin\left(e^{-2x}\right)\right)=-2e^{-2x}\cos\left(e^{-2x}\right)$

d. $\frac{d}{dx}\left(\cos^{3}\left(e^{x}\right)\right)=3\cos^{2}\left(e^{x}\right)\left(-\sin\left(e^{x}\right)\right)e^{x}$

e. Because e^{x} and ln(x) are inverse functions, $e^{\ln(5x)}$ = 5x. So the derivative is 5.

Part 3 – Investigate a^{x}

Students review the derivative of a constant times x to set the groundwork for finding the derivative of 2^{x}. When the constant is not an integer or decimal, students tend to have difficulties.



Students examine the steps to finding the derivative of $y=2^{x}$ using implicit differentiation.

Students differentiate several exponential functions to find a rule for differentiate exponential functions.

Student Solutions

8. a. ln(2) ≈ 0.69314

b. ln(e) = 1

c. ln(4) ≈ 1.38629

9. a. π

b. ln(2)

10. a. $y = \left(\frac{3}{2}\right)^x \Rightarrow y' = 0.40547\left(1.5\right)^x$

b. $y = 3^x \Rightarrow y' = 1.0986\left(3\right)^x$

c. $y = 4^x \Rightarrow y' = 2\ln 2\left(4^x\right) = 1.3863\left(4^x\right)$

d. $y = 0.5^x \Rightarrow y' = \ln\left(\frac{1}{2}\right)0.5^x = -\ln 2\left(0.5^x\right) = -0.69315\left(0.5^x\right)$

e. $y = 2\cdot 5^x \Rightarrow y' = 2\ln 5\left(5^x\right) = 3.2189\left(5^x\right)$

11. $\frac{d}{dx}\left(ba^{u(x)}\right) = ba^{u(x)}\ln(a)\cdot u'(x)$

12. a. $\frac{d}{dx}\left(3^{x^7}\right) = 7x^6 3^{x^7}\ln 3$ There is only a slight difference due to commutative property of multiplication.

b. $\frac{d}{dx}\left(5^{2x+3}\right) = 5^{2x+3}\ln 5\cdot(2) = 2\cdot 5^{2x}5^3\ln 5 = 2\ln(5)\cdot\left(5^2\right)^x 125 = 250\ln\left(5\right)25^x$

c. $\frac{d}{dx}\left(\left(\frac{3}{2}\right)^{2x+2}\right) = \left(\frac{3}{2}\right)^{2x+2}\ln\left(\frac{3}{2}\right)2 = 2\ln\left(\frac{3}{2}\right)\cdot\left(\frac{3}{2}\right)^{2x}\left(\frac{3}{2}\right)^2 = \frac{9}{2}\ln\left(\frac{3}{2}\right)\cdot\left(\left(\frac{3}{2}\right)^2\right)^x = \frac{9\ln\left(\frac{3}{2}\right)\cdot\left(\frac{9}{4}\right)^x}{2}$

d. $\frac{d}{dx}\left(\left(1/2\right)^{3x}\right) = \left(\frac{1}{2}\right)^{3x}\ln\left(\frac{1}{2}\right)\cdot 3 = 3\ln\left(2^{-1}\right)\left(\left(\frac{1}{2}\right)^3\right)^x = -3\ln 2\left(\frac{1}{8}\right)^x$

Part 4 – Extension/Homework: Exam practice questions

These questions are great practice questions for the AP exam. The questions are non-calculator questions. With the use of the CAS functionality the questions become fairly easy.