Grandfathers Equation

Introduction:

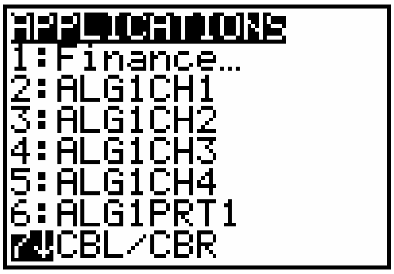
The beauty of a Grandfather clock is not just in the casing; the motion of the pendulum is often admired. Without the pendulum swing the mechanics of the clock might just as well be electronic in nature. Many aspects relating to the motion of a pendulum swing can be modelled mathematically. The first stage in modelling the motion is to collect some data. The Calculator Base Laboratory or CBR™ is a motion detector that can be connected to a TI-83 (plus) calculator. The position, velocity and acceleration at any time are recorded in the calculator's Stat. – List editor. The data can be used to draw graphs of the ball's motion. The motion of the pendulum in this series of investigations in only considered in a single horizontal plane.

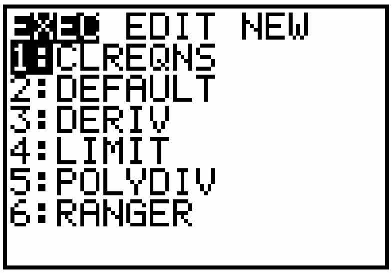
Equipment:

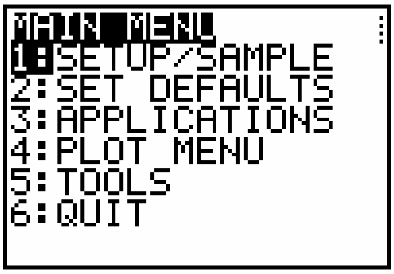
CBR
TI-83(Plus) Graphic Calculator
Calculator to calculator link cable
A pendulum bob attached to string approximately 1 metre in length. A 20 – 30cm diameter fishing buoy makes an excellent pendulum, they generally have a pre-drilled hole through the centre for attaching the string.

Getting Started – Collecting the data:

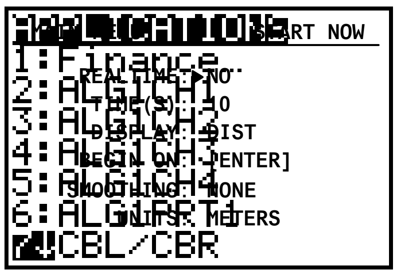
- Load the CBR *Ranger* program onto the graphics calculator.
- Make sure your pendulum is free to swing.
- Start the *Ranger* program or the CBL/CBR application.
  If you are using the CBR/CBL App. choose Ranger.





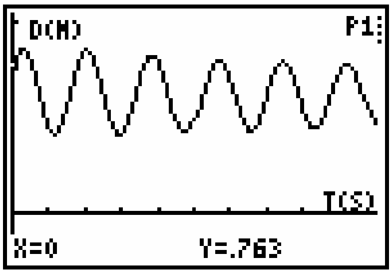


- From the main menu select: `SETUP/SAMPLE`.
- Use the [▲] [▼] arrow keys to move between options, use the [ENTER] key to toggle through options.
  - o Real time – No — *Real time data collection can only occur for sample times of 15secs.*
  - o Time (s) – 10 — *This may need adjustment depending on the length of the pendulum.*
  - o Display – Dist. — *Velocity and acceleration data will still be collected.*
  - o Begin – Enter — *Measurements begin when* [ENTER] *is pressed, [trigger] may also be selected.*
  - o Smoothing – None — *A good pendulum needs data smoothing*
  - o Units – Meters — *No explanation required!*

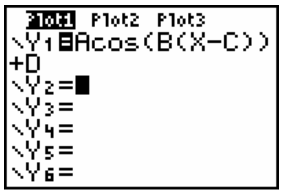
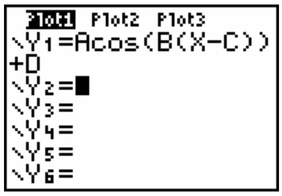


- Once all the selections have been made move the cursor to "`START NOW`"
- Press [ENTER] on the calculator.
- Point the CBR at the target. It is best placed on a bench or table.

- Hold the pendulum string so that the pendulum bob swings in front of the ranger. The pendulum bob should not get any closer than 50cms from the Ranger. If the pendulum comes closer than 50cms 'flat' spots occur in the data.
- Start the pendulum swinging. If you are holding the pendulum string in your hand, try and avoid swinging the pendulum with hand. For optimum results it is generally better to attach the top of the string to a fixed point.
- Follow the instructions on the calculator screen. If [trigger] was chosen in the set up menu the CBR may be detached during the experiment and returned later for data transfer.
- Press [ENTER] / [trigger] to start the CBR. The CBR will start ticking.
- When the CBR has stopped ticking the data collection is complete.
- Return the CBR to the calculator, plug it in and press [ENTER]; the data will be transferred and a graph of the data will be displayed. Press [ENTER] when you have finished viewing the graph.
- If the data/graph is not as clear as you would like, select option 5: `REPEAT SAMPLE`

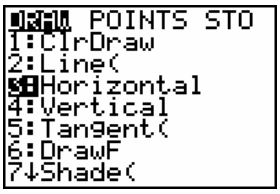


- If you are happy with the data select option [7] to quit the ranger program. Time, distance, velocity and acceleration data will automatically be saved to L_1, L_2, L_3 and L_4 respectively.

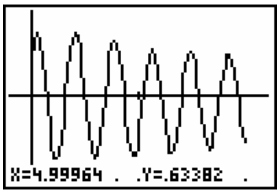
Instruction	Screen Shot
Enter the Y= editor [Y=] and type in the following equation: `Y1 = A cos(B(X – C)) + D` [ALPHA] [MATH] [COS] [ALPHA] [APPS] [(] [X,T,Θ,n] [–] [ALPHA] [PRGM] [)] [)] [+] [ALPHA] [x^{-1}] [ENTER] **Important:** Make sure all the brackets are included and in the correct locations. Plot1 is switched on from the Ranger program and should be set up as a scatter – plot using L_1 and L_2 as data.	
Switch the equation off by using the [▲] [▼] [◄] [►] arrow keys to move over the top of the '=' and press [ENTER] to switch the equation off. Notice in the two screen shots shown, the top screen shot has the equation selected. The one shown to the right shows the equation has been switched off.	

Determining vertical translation:

<table>
<tr><th>Instruction</th><th>Screen Shot</th></tr>
<tr><td>Press the [GRAPH] button to return to the graph. The Draw menu allows you to draw directly over graphs.

Press [2nd] [PRGM] and select option [3] - Horizontal (Line).</td><td>
</td></tr>
<tr><td>When the horizontal line appears across the graph, DO NOT press [ENTER] or the line will be drawn across the screen.

Use the [▲] and [▼] arrows to adjust the line until the graph is symmetrical about the line.

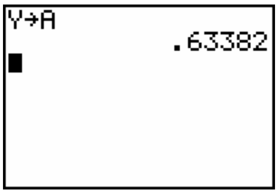
The vertical translation of the graph is now indicated at the base of the screen: Y = 0.63382 (This value depends on the data collected.)</td><td>
</td></tr>
</table>

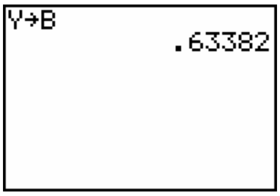
Press [2nd] [MODE] to return to the home screen. You can recall the value for Y by pressing [ALPHA] [1].

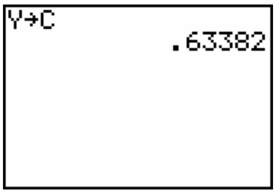
Press [STO▸] and then the appropriate parameter:

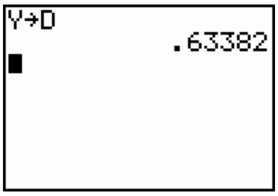
Y_1 = A cos(B(X – C)) + D

A – [ALPHA] [MATH]   B – [ALPHA] [APPS]   C – [ALPHA] [PRGM]   D – [ALPHA] [x^{-1}]









- The three screen shots above illustrate examples of how to store Y into each one of the parameters. Only one of these screens represents the correct solution. You need to decide which of the parameters: A, B, C or D is responsible for a vertical translation of the function:

$$Y_1 = A \cos(B(X - C)) + D$$

- The value stored in Y will depend on the data you collected.

Question 1:

a) Which parameter A, B, C or D represents vertical translation in the equation: Y_1=Acos(B(X–C))+D?

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b) What was your vertical translation for your data?

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c) In the screen shots above the vertical translation of the graph is 0.633. What does this mean in relation to the physical characteristics of the pendulum and corresponding data collection?

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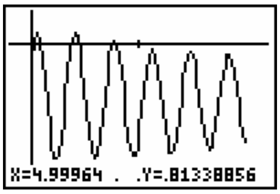
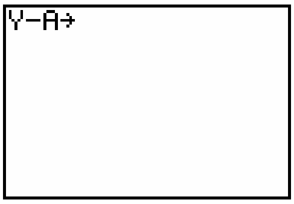
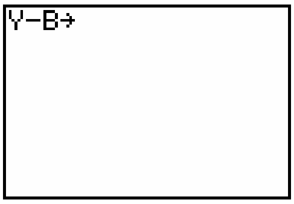
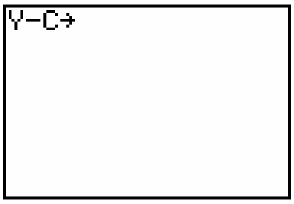
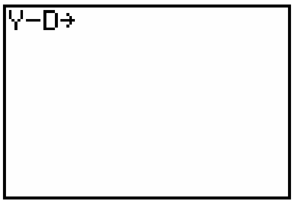
Determining amplitude:

Instruction	Screen Shot
Return to the graph screen: [GRAPH]; draw another horizontal line. In the data collected on the right, amplitude decreases with each swing. Try and place the horizontal line so that it passes through most of the points. **Note:** This value may be changed later to suit the investigation.	
Press [2nd] [MODE] to return to the home screen. You can recall the value for Y by pressing [ALPHA] [1]. The amplitude on the graph is the difference between the highest point(s) and the centre or median position. To determine the amplitude subtract the vertical translation from the current value for Y. ie: Y – A, Y – B, Y – C or Y – D; store this result in the corresponding parameter for amplitude.	









Only one of the screen shots shown above represents the start of the correct answer. The value currently stored in Y represents the maximum value of the trigonometric function. The vertical translation should be subtracted from this value. Decide which procedure is correct and store the result in the appropriate parameter for amplitude.

Question 2

a) Which parameter A, B, C or D represents amplitude in the equation: Y_1=Acos(B(X–C))+D?

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b) What was the amplitude of your graph?

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c) What does this measurement correspond to in terms of the physical characteristics of the pendulum and corresponding data collection?

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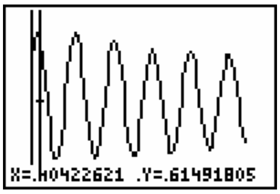
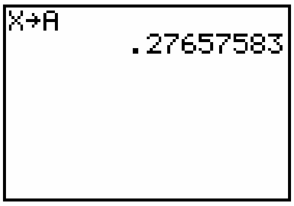
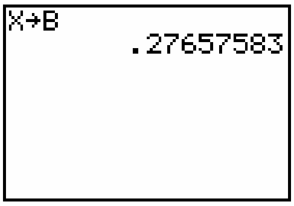
Determining phase shift:

Instruction	Screen Shot
Return to the graph screen: [GRAPH] and draw a vertical line using the Draw menu. Try and place the vertical line so that it passes through the first peak of the graph. If this is not possible choose the next peak.	

Press [2nd] [MODE] to return to the home screen. You can recall the value for X by pressing [X,T,Θ,n]. The X axis represents time. The phase shift in the data above is approximately 0.4 seconds. Determine which parameter in the equation: Y_1=Acos(B(X–C))+D is responsible for the phase shift.





Store your value for X in the appropriate parameter. One of the screen shots shown above represents the correct solution.

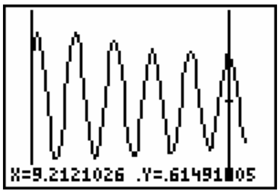
Question 3:

a) Which parameter A, B, C or D represents amplitude in the equation: Y_1=Acos(B(X–C))+D

b) What was your value for the phase shift?

c) What does the phase shift represent in the data collection process?

Determining the Period of the Pendulum:

Instruction	Screen Shot
Return to the graph screen: [GRAPH] and draw another vertical line using the Draw menu. Move the vertical line to the last peak on the graph. Count how many cycles your pendulum completes between your first peak, as previously recorded and the current one. In the screen shots shown here and previously the pendulum has completed 5 full cycles.	

Press [2nd] [MODE] to return to the home screen. Recall the value for X by pressing [X,T,Θ,*n*] and subtract the initial position recorded for the phase shift of your data.

Question 4:

a) Determine the amount of time it took your pendulum to complete one full swing, the period of the pendulum.

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b) Given $P = \frac{2p}{n}$ determine the value of *n.* Store this result in the appropriate parameter in the equation: `Y₁=Acos(B(X–C))+D.` Write down where this value was stored.

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c) Explain, in terms of the physical characteristics of the pendulum what this value represents.

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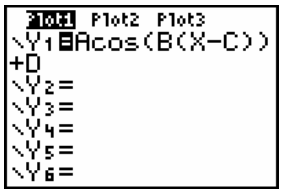
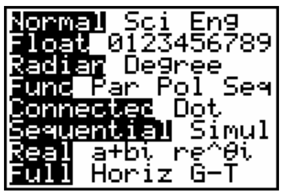
Graphing your equation:

Instruction	Screen Shot
Press [Y=] and switch the equation on by using the [▲] [▼] [◄] [►] arrow keys to move over the top of the '=' and press [ENTER]. Then press [GRAPH]	
Press [MODE] and make sure your calculator is in radian mode.	

Press [GRAPH] to view your completed equation for the motion of the pendulum.

How fast is your Grandfather?

The "Grandfather's Equation" investigation determined the position of a pendulum at any time *t*. In this investigation an equation for the velocity of the pendulum will be determined by studying the position of the pendulum.

Equipment:

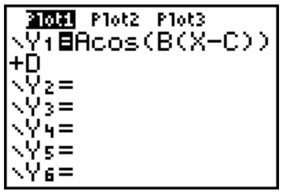
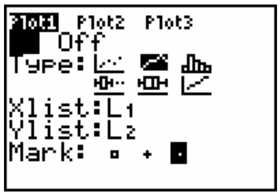
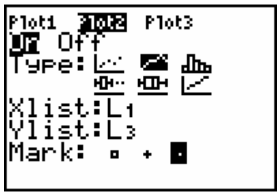
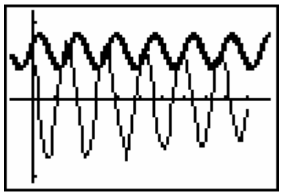
TI-83(Plus) Graphic Calculator
Pendulum data from "Grandfather's Equation" investigation.

Drawing the velocity on the calculator.

It is possible to use the pendulum data to draw a graph of the velocity. When the CBR collects data it records the time, distance, velocity and acceleration of the object. Time is located in L_1, distance in L_2, velocity in L_3 and acceleration in L_4.

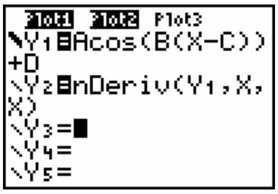
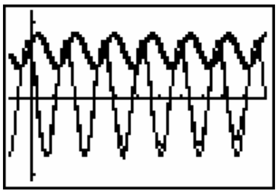
- Return the data to the calculator lists.
- Determine the equation for the position of the pendulum and store the equation in Y_1 as:

$$Y_1=A\cos(B(X-C))+D$$

Instruction	Screen Shot
Press [Y=] and make sure the equation is switched on.	
Switch `Stat-Plot 1` on and match the settings: [2nd] [Y=] [ENTER], use the ◄ ► ▲ ▼ arrow keys and press [ENTER] to make the selections shown.	
Switch `Stat-Plot 2` on and match the settings: [2nd] [Y=] [2], use the ◄ ► ▲ ▼ arrow keys and press [ENTER] to make the selections shown.	
Press [ZOOM] and select option [9] - `Zoomstat`.	

Note: The equation for the velocity of the pendulum can be determined by finding the derivative or gradient at any point of the position equation. For this calculation 'nderiv' can be used. When placed in the y= editor it can determine the gradient for all points specified in the x – window range:

`nderiv(expression, variable, value, [,Dx] )`

Instruction	Screen Shot
Press [Y=] and use the [◄] [►] [▲] [▼] arrow keys to scroll down to Y_2. To determine the numerical derivative for all points in the x – window range type: [MATH] [8] [VARS] [►] [1] [1] [,] [X,T,Θ,n] [,] [X,T,Θ,n] [)] [ENTER]	
Press [GRAPH] to see both the position time data and graph and the velocity time data with the graph.	

Question 2

a) Write down your equation for displacement:

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b) Using similar methods to those from the "Grandfather's Equation" investigation, determine the equation to the velocity in the form: `Y = Asin(B(X-C))+D`

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c) [1]Determine the derivative of your equation *Using calculus*:

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d) Graph your equation on the calculator and compare it to the one produced by the calculator using *nderiv*. (**Hint:** *You may use the lists to compare the equations to the raw data.*)

[1] This question requires calculus